

Package ‘PDEnaiveBayes’

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Type Package

Title Plausible Naive Bayes Classifier Using PDE

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Description A nonparametric, multicore-capable plausible naive Bayes classifier based on the Pareto density estimation (PDE) featuring a plausible approach to a pitfall in the Bayesian theorem covering low evidence cases. Stier, Q., Hoffmann, J., and Thrun, M.C.: ``Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naive Bayes" (2026), Machine Learning and Knowledge Extraction (MAKE), <[DOI:10.3390/make8010013](https://doi.org/10.3390/make8010013)>.

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Imports Rcpp (>= 1.0.8), RcppParallel (>= 5.1.4), pracma, plotly, utils, grDevices, stats, graphics, methods, ggplot2, DatabionicsSwarm, memshare

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VignetteBuilder knitr

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Depends R (>= 2.10)

License GPL-3

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BugReports <https://github.com/Mthrun/PDEbayes/issues>

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PDEnaiveBayes-package *Plausible Naive Bayes Classifier Using PDE*

Description

A nonparametric, multicore-capable plausible naive Bayes classifier based on the Pareto density estimation (PDE) featuring a plausible approach to a pitfall in the Bayesian theorem covering low evidence cases. Stier, Q., Hoffmann, J., and Thrun, M.C.: "Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naive Bayes" (2026), Machine Learning and Knowledge Extraction (MAKE), <DOI:10.3390/make8010013>.

Details

Pareto Density Estimated naive Bayes Classifier Index of help topics:

ApplyBayesTheorem4Likelihoods	ApplyBayesTheorem4Likelihoods
GetLikelihoods	GetLikelihoods
Hepta	Hepta introduced in [Ultsch, 2003]
PDEnaiveBayes-package	Plausible Naive Bayes Classifier Using PDE
PlotBayesianDecision2D	PlotBayesianDecision2D
PlotLikelihoodFuns	PlotLikelihoodFuns

PlotLikelihoods	PlotLikelihoods
PlotNaiveBayes	PlotNaiveBayes
PlotPosteriors	PlotPosteriors
Predict_naiveBayes	Predict_naiveBayes
Train_naiveBayes	Train_naiveBayes
Train_naiveBayes_multicore	Train_naiveBayes_multicore
defineOrEstimateDistribution	defineOrEstimateDistribution
fitParameters	fitParameters
getPriors	getPriors
predict.PDEbayes	predict.PDEbayes

(PDENB) of [Stier et al., 2026].

Author(s)

Michal Thrun

Maintainer: Michael Thrun <mthrun@informatik.uni-marburg.de>

References

- [Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.
- [Thrun et al., 2020] Thrun, M. C., Gehlert, T., & Ultsch, A.: Analyzing the Fine Structure of Distributions, PloS one, Vol. 15(10), pp. e0238835, doi 10.1371/journal.pone.0238835 2020.
- [Thrun/Ultsch, 2020] Thrun, M. C., & Ultsch, A.: Clustering Benchmark Datasets Exploiting the Fundamental Clustering Problems, Data in Brief, Vol. 30(C), pp. 105501, doi 10.1016/j.dib.2020.105501, 2020.
- [Ultsch et al., 2015] Ultsch, A., Thrun, M. C., Hansen-Goos, O., & L?tsch, J.: Identification of Molecular Fingerprints in Human Heat Pain Thresholds by Use of an Interactive Mixture Model R Toolbox (AdaptGauss), International journal of molecular sciences, Vol. 16(10), pp. 25897-25911, doi 10.3390/ijms161025897, 2015.

Examples

```
if(requireNamespace("FCPS")){
V=FCPS::ClusterChallenge("Hepta",1000)
Data=V$Hepta
Cls=V$Cls
ind=1:length(Cls)
indtrain=sample(ind,800)
indtest=setdiff(ind,indtrain)
#parametric
#model=Train_naiveBayes(Data[indtrain,],Cls[indtrain],Gaussian=TRUE)
#ClsTrain=model$ClsTrain
#table(Cls[indtrain],ClsTrain)
```

```
#res=Predict_naiveBayes(Data[indtest,], Model = model)
#table(Cls[indtest],res$ClsTest)

#PDEbayes
model=Train_naiveBayes(Data[indtrain,],Cls[indtrain],Gaussian=FALSE)
ClsTrain=model$ClsTrain
table(Cls[indtrain],ClsTrain)

res=Predict_naiveBayes(Data[indtest,], Model = model)
table(Cls[indtest],res$ClsTest)
}
```

ApplyBayesTheorem4Likelihoods
<i>ApplyBayesTheorem4Likelihoods</i>

Description

Calculates the posteriors, for given likelihoods and priors using the Bayes Theorem

Usage

ApplyBayesTheorem4Likelihoods(Likelihoods,Priors,threshold=.Machine\$double.eps*1000)

Arguments

Likelihoods	List of d numeric matrices, one per feature, each matrix with 1:k columns containing the distribution of class 1:k.
Priors	[1:k] Numeric vector with prior probability for each class.
threshold	(Optional: Default=0.00001).

Value

Posteriors	[1:n, 1:d] Numeric matrix with posterior probability according to the bayes theorem.
------------	--

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```

if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls
  #parametric
  #V=Train_naiveBayes(Data,Cls,Gaussian=TRUE)
  #ClsTrain=V$ClsTrain
  #table(Cls,ClsTrain)

  #non-parametric
  V=Train_naiveBayes(Data,Cls,Gaussian=FALSE)
  ClsTrain=V$ClsTrain
  table(Cls,ClsTrain)
}

```

defineOrEstimateDistribution

defineOrEstimateDistribution

Description

The function estimates the distribution of values within a features that belong to a specific class, i.e., the conditional probability of the likelihood

Usage

```

defineOrEstimateDistribution(Feature,ClassInd,Gaussian=FALSE,ParetoRadius=NULL,
InternalPlotIt=FALSE,SD_Threshold=0.001,...)

```

Arguments

Feature	[1:n] Numeric Vector
ClassInd	Integer Vector with class indices
Gaussian	(Optional: Default=TRUE). Assume gaussian distribution.
ParetoRadius	Optional [1:d] numerical vector for pareto radii computed priorly, see ParetoRadius
InternalPlotIt	Optional: Default=FALSE). Create plot if set to TRUE.
SD_Threshold	Optional: Default=0.001.
...	Robust: Optional: Default=FALSE, TRUE: robust estimation of mean and std in case of Gaussian=TRUE Type: (Optional: Default=2, 1=original PDE, 2= improved PDE na.rm: (Optional: Default=TRUE). Remove na.

Value

Kernels	[1:m] Numeric vector with kernels (x-values) of a 1D pdf.
PDF	[1:m] Numeric vector with the distribution values of a 1D pdf.
Theta	Numeric vector with parameters of gaussian of mean and standard deviation - NULL if no gaussian used.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls
  Priors=getPriors(Cls)
}
```

fitParameters	<i>fitParameters</i>
---------------	----------------------

Description

Fit Gaussian parameters.

Usage

```
fitParameters(Feature,ClassInd,Robust=FALSE,na.rm=TRUE,SD_Threshold=0.0001)
```

Arguments

Feature	[1:n] Numeric Vector
ClassInd	Integer Vector with class indices
Robust	(Optional: Default=FALSE). Robust computation if set to TRUE.
na.rm	(Optional: Default=TRUE). Remove na.
SD_Threshold	(Optional: Default=0.00001).

Value

Parameters [1:2] Numeric vector with Mean and Std.

Author(s)

Michael Thrun

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls
  Priors=getPriors(Cls)
}
```

 GetLikelihoods

GetLikelihoods

Description

Yields the likelihoods per feature and class as values of distribution either defined by Gaussian or estimated from the data using pareto density estimation.

Usage

```
GetLikelihoods(Data,Cls,...)
```

Arguments

Data	[1:n,1:d] matrix of training data. It consists of n cases of d-dimensional data points. Every case has d attributes, variables or features.
Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
...	Further arguments for defineOrEstimateDistribution Robust=TRUE: robustly estimated gaussians na.rm=TRUE: remove NaNs Threshold: threshold for which the standard deviation cannot be smaller (default 0.0001)

Details

Due to pareto density estimation per class and feature, usually the number of rows in each element of `c_Kernels_list` and `ListOfLikelihoods` varies and does not equal the number of rows of data `n`.

Value

c_Kernels_list	List of d numeric matrices, one per feature, each matrix with 1:k columns containing the kernels of class 1:k
ListOfLikelihoods	List of d numeric matrices, one per feature, each matrix with 1:k columns containing distribution values (likelihood) of class 1:k
Thetas	If Gaussian=TRUE: List of d numeric matrices, one per feaature, each matrix with 1:k rows containing the mean in the first column and the standard deviation in teh seconf columd of class 1:k Otherwise: NULL
ParetoRadiusPerFeature	Numeric vector with estimated pareto radius per feature.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls
  Priors=getPriors(Cls)
}
```

getPriors	<i>getPriors</i>
-----------	------------------

Description

Get a prior via class proportions.

Usage

```
getPriors(Cls)
```

Arguments

Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
-----	---

Value

Priors [1:k] Numeric vector with prior probability for each class.

Author(s)

Michael Thrun

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls
  Priors=getPriors(Cls)
}
```

Hepta

Hepta introduced in [Utsch, 2003]

Description

Clearly defined clusters, different variances. Detailed description of dataset and its clustering challenge is provided in [Thrun/Utsch, 2020].

Usage

```
data("Hepta")
```

Details

Size 212, Dimensions 3, stored in Hepta\$Data

Classes 7, stored in Hepta\$Cls

References

[Utsch, 2003] Utsch, A.: Maps for the visualization of high-dimensional data spaces, Proc. Workshop on Self organizing Maps (WSOM), pp. 225-230, Kyushu, Japan, 2003.

[Thrun/Utsch, 2020] Thrun, M. C., & Utsch, A.: Clustering Benchmark Datasets Exploiting the Fundamental Clustering Problems, Data in Brief, Vol. 30(C), pp. 105501, doi:[10.1016/j.dib.2020.105501](https://doi.org/10.1016/j.dib.2020.105501), 2020.

Examples

```
data(Hepta)
str(Hepta)
```

PlotBayesianDecision2D

PlotBayesianDecision2D

Description

Plots estimation of decision boundary in a 2D slice of the data using the posteriors

Usage

```
PlotBayesianDecision2D(X, Y, Posteriors, Class = 1, NoBins,
  CellColorsOrPalette, Showpoints = TRUE, xlim, ylim, xlab, ylab, main,
  PlotIt = TRUE)
```

Arguments

X	Numeric vector with point coordinates of first dimension of data selection.
Y	Numeric vector with point coordinates of second dimension of data selection.
Posteriors	[1:n, 1:Class] matrix of posteriors.
Class	Optional,Integer defining which class to look at.
NoBins	Optional,Number of bins for class posteriori.
CellColorsOrPalette	Optional, Either a function defining the color palette of a character vector or character vector of length NoBins stating colors.
Showpoints	Optional, TRUE, points are displayed.
xlim	Optional,Numeric vector of length 2 stating limits of x axis.
ylim	Optional,Numeric vector of length 2 stating limits of y axis.
xlab	Optional,Character stating name of x axis.
ylab	Optional,Character stating name of y axis.
main	Optional, Character name of title
PlotIt	Optional, TRUE: prints GGPlot2 object, FALSE: not shown plot.

Details

Boundaries are assumed to be zero for plotting.

Value

List of:

Mapping	List containing a map for colors, kernels and bin number.
GGobj	ggplot2 object containing 2D visualization of Posteriori.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
Data = as.matrix(iris[,1:4])
Cls = as.numeric(iris[,5])

TrainIdx = c(17, 73, 46, 29, 68, 35, 131, 62, 132, 127, 71, 72,
144, 99, 93, 13, 38, 21, 102, 53, 36, 111, 114, 96, 57, 74, 145,
86, 3, 16, 52, 59, 140, 40, 122, 109, 6, 91, 79, 15, 108, 139,
37, 76, 20, 115, 66, 28, 100, 117, 44, 78, 80, 150, 146, 142,
9, 90, 45, 58, 134, 11, 87, 125, 141, 118, 136, 48, 124, 47,
8, 27, 33, 92, 130, 54, 65, 104, 23, 98, 129, 123, 34, 128, 135,
51, 64, 5, 94, 83, 42, 116, 101, 43, 7, 12, 82, 1, 84, 138, 2,
56, 4, 106, 120)

TestIdx = c(60, 10, 75, 70, 81, 18, 97, 95, 67, 22, 55, 143,
88, 24, 105, 26, 119, 31, 107, 63, 41, 61, 32, 147, 89, 14, 121,
19, 113, 49, 126, 112, 25, 77, 137, 103, 50, 30, 149, 110, 39,
69, 148, 85, 133)

TrainX = Data[TrainIdx, ]
TestX = Data[TestIdx, ]
TrainY = Cls[TrainIdx]
TestY = Cls[TestIdx]

VPDENB = Train_naiveBayes(Data = TrainX, Cls = TrainY, Plausible = FALSE)

PlotBayesianDecision2D(X = TrainX[, 1], Y = TrainX[, 2],
Posterior = VPDENB$Posteriors, Class = 1)
```

PlotLikelihoodFuns *PlotLikelihoodFuns*

Description

Plots the class-conditional Likelihoods per feature, given the generating likelihood functions.

Usage

```
PlotLikelihoodFuns(LikelihoodFuns,Data,PlausibleLikelihoodFuns=NULL,
Epsilon=NULL,PlausibleCenters=NULL,PlotCutOff=4,xlim)
```

Arguments

LikelihoodFuns	List with Likelihoods generating functions
Data	Numeric matrix with data.
PlausibleLikelihoodFuns	List with plausible Likelihoods.
Epsilon	Numeric scalar defining epsilon fo plausible likelihoods.
PlausibleCenters	Numeric vector [1:k] plausible centers used to compute plausible likelihoods.
PlotCutOff	scalar defining the how many feature starting from 1 should be plotted or numerical vector defining the index of features to be plotted in second case should not be too many otherwise plot yields an error.
xlim	Numeric vector of length 2 stating limits of x axis.

Value

No return value.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
Data = as.matrix(iris[,1:4])
Cls = as.numeric(iris[,5])

TrainIdx = c(17, 73, 46, 29, 68, 35, 131, 62, 132, 127, 71, 72,
144, 99, 93, 13, 38, 21, 102, 53, 36, 111, 114, 96, 57, 74, 145,
86, 3, 16, 52, 59, 140, 40, 122, 109, 6, 91, 79, 15, 108, 139,
37, 76, 20, 115, 66, 28, 100, 117, 44, 78, 80, 150, 146, 142,
9, 90, 45, 58, 134, 11, 87, 125, 141, 118, 136, 48, 124, 47,
8, 27, 33, 92, 130, 54, 65, 104, 23, 98, 129, 123, 34, 128, 135,
51, 64, 5, 94, 83, 42, 116, 101, 43, 7, 12, 82, 1, 84, 138, 2,
56, 4, 106, 120)

TestIdx = c(60, 10, 75, 70, 81, 18, 97, 95, 67, 22, 55, 143,
88, 24, 105, 26, 119, 31, 107, 63, 41, 61, 32, 147, 89, 14, 121,
```

```

19, 113, 49, 126, 112, 25, 77, 137, 103, 50, 30, 149, 110, 39,
69, 148, 85, 133)

TrainX = Data[TrainIdx, ]
TestX  = Data[TestIdx, ]
TrainY = Cls[TrainIdx]
TestY  = Cls[TestIdx]

VPDENB = Train_naiveBayes(Data = TrainX, Cls = TrainY, Plausible = FALSE)

PlotLikelihoodFuns(LikelihoodFuns = VPDENB$Model$PDFs_funs, Data = TrainX)

```

PlotLikelihoods	<i>PlotLikelihoods</i>
-----------------	------------------------

Description

Plots the Likelihoods per feature.

Usage

```
PlotLikelihoods(Likelihoods, Data, PlausibleLikelihoods=NULL, Epsilon=NULL,
PlausibleCenters=NULL, PlotCutOff=4, xlim)
```

Arguments

Likelihoods	List with Likelihoods.
Data	Numeric matrix with data.
PlausibleLikelihoods	List with plausible Likelihoods.
Epsilon	Numeric scalar defining epsilon for plausible likelihoods.
PlausibleCenters	Numeric vector [1:k] plausible centers used to compute plausible likelihoods.
PlotCutOff	scalar defining the how many feature starting from 1 should be plotted or numerical vector defining the index of features to be plotted in second case should not be too many otherwise plot yields an error.
xlim	Numeric vector of length 2 stating limits of x axis.

Details

Boundaries are assumed to be zero for plotting.

Value

No return value.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
Data = as.matrix(iris[,1:4])
Cls = as.numeric(iris[,5])

TrainIdx = c(17, 73, 46, 29, 68, 35, 131, 62, 132, 127, 71, 72,
144, 99, 93, 13, 38, 21, 102, 53, 36, 111, 114, 96, 57, 74, 145,
86, 3, 16, 52, 59, 140, 40, 122, 109, 6, 91, 79, 15, 108, 139,
37, 76, 20, 115, 66, 28, 100, 117, 44, 78, 80, 150, 146, 142,
9, 90, 45, 58, 134, 11, 87, 125, 141, 118, 136, 48, 124, 47,
8, 27, 33, 92, 130, 54, 65, 104, 23, 98, 129, 123, 34, 128, 135,
51, 64, 5, 94, 83, 42, 116, 101, 43, 7, 12, 82, 1, 84, 138, 2,
56, 4, 106, 120)

TestIdx = c(60, 10, 75, 70, 81, 18, 97, 95, 67, 22, 55, 143,
88, 24, 105, 26, 119, 31, 107, 63, 41, 61, 32, 147, 89, 14, 121,
19, 113, 49, 126, 112, 25, 77, 137, 103, 50, 30, 149, 110, 39,
69, 148, 85, 133)

TrainX = Data[TrainIdx, ]
TestX = Data[TestIdx, ]
TrainY = Cls[TrainIdx]
TestY = Cls[TestIdx]

VPDENB = Train_naiveBayes(Data = TrainX, Cls = TrainY, Plausible = FALSE)

PlotLikelihoods(Likelihoods = VPDENB$Model$ListOfLikelihoods, Data = TrainX)
```

PlotNaiveBayes

PlotNaiveBayes

Description

Visualize the class-conditional distributions of the Pareto Density estimated naive Bayes model (PDENB) [Stier et al., 2026].

Usage

```
PlotNaiveBayes(Model, FeatureNames, ClassNames, DatasetName = "Data",
  nrows = 1, FeatureOrder, NumFeaturesPerRow = 4, Colors,
  IndividualFigures = FALSE)
```

Arguments

Model	List with elements Priors,c_2List_Train.
FeatureNames	Character vector of names with a name for each feature contained in the data used to create the naive bayes model.
ClassNames	Character vector of class names to present in the legend of the plots.
DatasetName	Character title for each plot.
nrows	Number of rows inside one plot.
FeatureOrder	Numeric vector representing the order of the features to be displayed.
NumFeaturesPerRow	Maximum number of features to be displayed in one plot.
Colors	Character vector of color names. The length of the vector must be the same as the number of classes within the data modeled by the naive Bayes classifier.
IndividualFigures	Optional boolean: If set to TRUE, it returns a list of the individual figures for customization.

Details

Boundaries are assumed to be zero for plotting.

Value

Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
Posteriors	[1:n, 1:l] Numeric matrices with posterior probabilities.
DataLikelihoodsPerClass	list of length d, each element is a matrix [1:n, 1:k] of interpolated class likelihoods per feature d

Author(s)

Quirin Stier

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
Data = as.matrix(iris[,1:4])
Cls = as.numeric(iris[,5])
DatasetName = "Iris"

TrainIdx = c(17, 73, 46, 29, 68, 35, 131, 62, 132, 127, 71, 72,
144, 99, 93, 13, 38, 21, 102, 53, 36, 111, 114, 96, 57, 74, 145,
86, 3, 16, 52, 59, 140, 40, 122, 109, 6, 91, 79, 15, 108, 139,
37, 76, 20, 115, 66, 28, 100, 117, 44, 78, 80, 150, 146, 142,
9, 90, 45, 58, 134, 11, 87, 125, 141, 118, 136, 48, 124, 47,
8, 27, 33, 92, 130, 54, 65, 104, 23, 98, 129, 123, 34, 128, 135,
51, 64, 5, 94, 83, 42, 116, 101, 43, 7, 12, 82, 1, 84, 138, 2,
56, 4, 106, 120)

TestIdx = c(60, 10, 75, 70, 81, 18, 97, 95, 67, 22, 55, 143,
88, 24, 105, 26, 119, 31, 107, 63, 41, 61, 32, 147, 89, 14, 121,
19, 113, 49, 126, 112, 25, 77, 137, 103, 50, 30, 149, 110, 39,
69, 148, 85, 133)

TrainX = Data[TrainIdx, ]
TestX = Data[TestIdx, ]
TrainY = Cls[TrainIdx]
TestY = Cls[TestIdx]

VPDENB = Train_naiveBayes(Data = TrainX, Cls = TrainY, Plausible = FALSE)

FeatureNames = colnames(Data)

PlotNaiveBayes(Model = VPDENB$Model, FeatureNames = FeatureNames)
```

PlotPosteriors	<i>PlotPosteriors</i>
----------------	-----------------------

Description

Plots posteriors either using a panel of plots based on PlotBayesianDecision2D or in 1D as a line plot [Stier et al., 2026].

Usage

```
PlotPosteriors(Data, Posteriors, Class = 1, CellColorsOrPalette,
Showpoints = TRUE)
```

Arguments

Data	Either numeric matrix [1:n, 1:d] with data or one column of data.
Posteriors	[1:n, 1:Class] matrix of posteriors.
Class	Integer defining which class to look at if numeric matrix is given, for column of data all posteriors are overlayed in line plot.

CellColorsOrPalette

Either a function defining the color palette of a character vector or character vector of length NoBins stating colors.

Showpoints TRUE, points are displayed.

Details

Plotting posteriors in one directions only often does not give any insight. The default option using PlotBayesianDecision2D is often more useful.

Value

GGobj ggplot2 object containing 2D visualization of Posteriori.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

Examples

```
Data = as.matrix(iris[,1:4])
Cls = as.numeric(iris[,5])

TrainIdx = c(17, 73, 46, 29, 68, 35, 131, 62, 132, 127, 71, 72,
144, 99, 93, 13, 38, 21, 102, 53, 36, 111, 114, 96, 57, 74, 145,
86, 3, 16, 52, 59, 140, 40, 122, 109, 6, 91, 79, 15, 108, 139,
37, 76, 20, 115, 66, 28, 100, 117, 44, 78, 80, 150, 146, 142,
9, 90, 45, 58, 134, 11, 87, 125, 141, 118, 136, 48, 124, 47,
8, 27, 33, 92, 130, 54, 65, 104, 23, 98, 129, 123, 34, 128, 135,
51, 64, 5, 94, 83, 42, 116, 101, 43, 7, 12, 82, 1, 84, 138, 2,
56, 4, 106, 120)

TestIdx = c(60, 10, 75, 70, 81, 18, 97, 95, 67, 22, 55, 143,
88, 24, 105, 26, 119, 31, 107, 63, 41, 61, 32, 147, 89, 14, 121,
19, 113, 49, 126, 112, 25, 77, 137, 103, 50, 30, 149, 110, 39,
69, 148, 85, 133)

TrainX = Data[TrainIdx, ]
TestX = Data[TestIdx, ]
TrainY = Cls[TrainIdx]
TestY = Cls[TestIdx]

VPDENB = Train_naiveBayes(Data = TrainX, Cls = TrainY, Plausible = FALSE)
#default option
PlotPosteriors(Data = TrainX, Posteriors = VPDENB$Posteriors, Class = 1)
```

```
# alternative option
PlotPosteriors(Data = TrainX[,3], Posteriors = VPDENB$Posteriors)
```

predict.PDEbayes	<i>predict.PDEbayes</i>
------------------	-------------------------

Description

Predict a classification with the Pareto Density estimated naive Bayes model [Stier et al., 2026] . (PDENB).

Usage

```
predict.PDEbayes(object, newdata, type = c("class", "response", "prob"), ...)
```

Arguments

object	Model obtained from training routine in PDenaiveBayes package.
newdata	[1:n,1:d] matrix of test data. It consists of n cases of d-dimensional data points. Every case has d attributes, variables or features.
type	Optional parameter.
...	Gaussian: Optional: Default=TRUE). Assume gaussian distribution. Plausible: (Optional: TRUE: uses plausible bayesian theorem, FALSE non-plausible bayesian theorem Type: (Optional: default=1, 1 = original PDE, 2 = R native density estimation Threshold: Threshold for which the standard deviation cannot be smaller (default =1e-12) PlotIt: Optional: Default=FALSE, TRUE: Plots Likelihoods PlotCutOff: Optional: Scalar indicating how many features (starting from 1) should be plotted, or a numerical vector specifying the indices of the features to plot. Note: In the second case, avoid selecting too many features, as this may cause the plot to fail ParetoRadiusPerFeauture: Optional [1:d] numerical vector for pareto radii computed priorly, see ParetoRadius or {ParetoRadius_fast} cl: Optional: a cluster object, created by parallel, if given and ParetoRadiusPerFeauture missing, then ParetoRadiusPerFeauture is compputed multicore otherwise single core Robust: Optional: Default=FALSE, TRUE: robust estimation of mean and std in case of Gaussian=TRUE

Details

The function is implemented in a way so that one can combine training and test data although it is intended to be applied on test data only.

Value

Cls	Numeric vector with predicted class associated with newdata.
-----	--

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

See Also

[Train_naiveBayes](#)

Examples

```
if(requireNamespace("FCPS")){
  V=FCPS::ClusterChallenge("Hepta",1000)
  Data=V$Hepta
  Cls=V$Cls
  ind=1:length(Cls)
  indtrain=sample(ind,800)
  indtest=setdiff(ind,indtrain)

  model=Train_naiveBayes(Data[indtrain,],Cls[indtrain],Gaussian=FALSE)
  ClsTrain=model$ClsTrain
  table(Cls[indtrain],ClsTrain)

  ClsTest=predict.PDEbayes(object = model, newdata = Data[indtest,])
  table(Cls[indtest],ClsTest)
}
```

Predict_naiveBayes	<i>Predict_naiveBayes</i>
--------------------	---------------------------

Description

Predict classification with naive Bayes model [Stier et al., 2026].

Usage

```
Predict_naiveBayes(Data, Model, ...)
```

Arguments

Data	[1:n,1:d] matrix of test data. It consists of n cases of d-dimensional data points. Every case has d attributes, variables or features.
Model	Optional, list with elements Priors,c_2List_Train,Thetas, alternative set arguments seperatly
...	Priors: Optional, if Model missing, then [1:k] Numeric vector with prior probability for each class. c_2List_Train: Optional, if Model missing, then c_2List_Train is the output of GetLikelihoods: a list of two three elements of Kernels, Likelihoods per feature and class, optional Thetas or PlausibleCenters depending on parameter setting Thetas: Optional, if Model missing, then If c_2List_Train is missing, alternatively the parameters mean and standard deviation of the gaussian distributions per class and feaures. PlotIt: Optional: Default=FALSE, TRUE: Plots Likelihoods PlotCutoff: Optional: Scalar indicating how many features (starting from 1) should be plotted, or a numerical vector specifying the indices of the features to plot. Note: In the second case, avoid selecting too many features, as this may cause the plot to fail

Details

The function is implemented in a way so that one can combine training and test data although it is intended to be applied on test data only.

Value

Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
Posteriors	[1:n, 1:l] Numeric matrices with posterior probabilities.
DataLikelihoodsPerClass	list of length d, each element is a matrix [1:n, 1:k] of interpolated class likelihoods per feature d

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

See Also

[Train_naiveBayes](#)

Examples

```

if(requireNamespace("FCPS")){
V=FCPS::ClusterChallenge("Hepta",1000)
Data=V$Hepta
Cls=V$Cls
ind=1:length(Cls)
indtrain=sample(ind,800)
indtest=setdiff(ind,indtrain)

#PDEbayes
model=Train_naiveBayes(Data[indtrain,],Cls[indtrain],Gaussian=FALSE)
ClsTrain=model$ClsTrain
table(Cls[indtrain],ClsTrain)

res=Predict_naiveBayes(Data[indtest,], Model = model)
table(Cls[indtest],res$ClsTest)
}

```

Train_naiveBayes	<i>Train_naiveBayes</i>
------------------	-------------------------

Description

Trains a Pareto Density estimated naive Bayes model (PDENB) of [Stier et al., 2026].

Usage

```
Train_naiveBayes(Data,Cls,Predict=TRUE,Priors,...)
```

Arguments

Data	[1:n,1:d] matrix of training data. It consists of n cases of d-dimensional data points. Every case hasd attributes, variables or features.
Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
Predict	Optional, boolean to decide extent of output. In case of TRUE, yields ClsTrain and Posteriors, else it yields only Model and Thetas. Note: Only if Predict is set to TRUE, parameter EvalPlausible can be set true!
Priors	Optional, [1:k] numerical vector defining the prior probabilities of the k classes. If missing, estimated from Cls.
...	Gaussian: Optional: Default=TRUE). Assume gaussian distribution. Plausible: (Optional: TRUE: uses plausible bayesian theorem, FALSE non-plausible bayesian theorem. Type: (Optional: default=1, 1 = original PDE, 2 = R native density estimation. Threshold: Threshold for which the standard deviation cannot be smaller (default =1e-12).

PlotIt: Optional: Default=FALSE, TRUE: Plots Likelihoods.

PlotCutOff: Optional: Scalar indicating how many features (starting from 1) should be plotted, or a numerical vector specifying the indices of the features to plot. Note: In the second case, avoid selecting too many features, as this may cause the plot to fail.

ParetoRadiusPerFeature: Optional [1:d] numerical vector for pareto radii computed priorly, see [ParetoRadius](#) or {ParetoRadius_fast}

cl: Optional: a cluster object, created by parallel, if given and ParetoRadiusPerFeature missing, then ParetoRadiusPerFeature is computed multicore otherwise single core

Robust: Optional: Default=FALSE, TRUE: robust estimation of mean and std in case of Gaussian=TRUE.

GlobalPR: Optional: Default=TRUE, FALSE: estimation of pareto radius for each class individually.

Details

Precomputation of ParetoRadiusPerFeature can be useful to make cross-validation faster although it should be only done on the training data.

If Plausible is not given, both options are evaluated using shannon information.

c_Kernels_list and ListOfLikelihoods have d elements each storing a matrix [1:m, 1:k], usually m!=n. In contrast to DataLikelihoodsPerClass in which by interpolation the matrix are of size [1:n, 1:k]

Value

Model	List of model parameters and results.
c_Kernels_list	List of matrices, where each matrix represent the kernels of one feature for all classes.
ListOfLikelihoods	List of matrices, where each matrix represent the likelihood of one feature for all classes.
PDFs_funs	Nested list of depth 1, where the first index assigns the feature index and the second index assigns the class. The elements are functions for the density estimation for each feature and each class.
ParetoRadiusPerFeature	Numeric vector which stores the pareto radius for each feature.
Theta	Parameters mean and standard deviation of the Gaussian distributions per class and features.
Priors	Numeric vector which stores the prior probability of each class to appear.
PlausibleCenters	[1:k, 1:f] Numeric matrix which stores the centers for each feature and each class, where the row index assigns features and the column index assigns classes.
ClsTrain	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
Posteriors	[1:n, 1:k] Numeric matrices with posterior probabilities.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q., Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

See Also

[Predict_naiveBayes](#)

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls

  #non-parametric
  V=Train_naiveBayes(Data,Cls,Gaussian=FALSE)
  ClsTrain=V$ClsTrain
  table(Cls,ClsTrain)
}
```

Train_naiveBayes_multicore

Train_naiveBayes_multicore

Description

Trains a Pareto Density estimated naive Bayes model (PDENB) with multicore parallelity

Usage

```
Train_naiveBayes_multicore(cl=NULL,Data,Cls,Predict=FALSE,Priors,UseMemshare=FALSE,...)
```

Arguments

cl	Object instance of package parallel.
Data	[1:n,1:d] matrix of training data. It consists of n cases of d-dimensional data points. Every case hasd attributes, variables or features.
Cls	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.

Predict	Optional, boolean to decide extent of output. In case of TRUE, yields ClsTrain and Posteriors, else it yields only Model and Thetas. Note: Only if Predict is set to TRUE, parameter EvalPlausible can be set true!
Priors	Optional, [1:k] numerical vector defining the prior probabilities of the k classes. If missing, estimated from Cls.
UseMemshare	Optional boolean. If set to TRUE, then package functionality from Memshare is used, else classic library parallel is used.
...	Gaussian: Optional: Default=TRUE). Assume gaussian distribution. Plausible: (Optional: TRUE: uses plausible bayesian theorem, FALSE non-plausible bayesian theorem Type: (Optional: default=1, 1 = original PDE, 2 = R native density estimation Threshold: Threshold for which the standard deviation cannot be smaller (default =1e-12) PlotIt: Optional: Default=FALSE, TRUE: Plots Likelihoods PlotCutOff: Optional: Scalar indicating how many features (starting from 1) should be plotted, or a numerical vector specifying the indices of the features to plot. Note: In the second case, avoid selecting too many features, as this may cause the plot to fail ParetoRadiusPerFeature: Optional [1:d] numerical vector for pareto radii computed priorly, see ParetoRadius or {ParetoRadius_fast} cl: Optional: a cluster object, created by parallel, if given and ParetoRadiusPerFeature missing, then ParetoRadiusPerFeature is computed multicore otherwise single core Robust: Optional: Default=FALSE, TRUE: robust estimation of mean and std in case of Gaussian=TRUE GlobalPR: Optional: Default=TRUE, FALSE: estimation of pareto radius for each class individually.

Details

Precomputation of ParetoRadiusPerFeature can be usefull to make cross-validation faster although it should be only done on the training data.

If Plausible is not given, both options are evalauted using shannon information.

c_Kernels_list and ListOfLikelihoods have d elements each storing a matrix [1:m, 1:k], usually m!=n. In contrast to DataLikelihoodsPerClass in which by interpolation the matrix are of size [1:n, 1:k]

Value

Model	List of model parameters and results.
c_Kernels_list	List of matrices, where each matrix represent the kernels of one feature for all classes.
ListOfLikelihoods	List of matrices, where each matrix represent the likelihood of one feature for all classes.

PDFs_funs	Nested list of depth 1, where the first index assigns the feature index and the second index assigns the class. The elements are functions for the density estimation for each feature and each class.
ParetoRadiusPerFeature	Numeric vector which stores the pareto radius for each feature.
Theta	Parameters mean and standard deviation of the Gaussian distributions per class and feaures.
Priors	Numeric vector which stores the prior probability of each class to appear.
PlausibleCenters	[1:k, 1:f] Numeric matrix which stores the centers for each feature and each class, where the row index assigns features and the column index assigns classes.
ClsTrain	[1:n] numerical vector with n numbers defining the classification. It has k unique numbers representing the arbitrary labels of the classification.
Posteriors	[1:n, 1:k] Numeric matrices with posterior probabilities.

Author(s)

Michael Thrun

References

[Stier et al., 2026] Stier, Q.,Hoffmann, J. & Thrun, M. C.: Classifying with the Fine Structure of Distributions: Leveraging Distributional Information for Robust and Plausible Naïve Bayes, Machine Learning and Knowledge Extraction (MAKE), Vol. 8(1), 13, doi 10.3390/make8010013, MDPI, 2026.

See Also

[Predict_naiveBayes](#)

Examples

```
if(requireNamespace("FCPS")){
  data(Hepta)
  Data=Hepta$Data
  Cls=Hepta$Cls

  #non-parametric
  V=Train_naiveBayes_multicore(cl=NULL,Data=Data,Cls=Cls,Gaussian=FALSE,Predict=TRUE)
  ClsTrain=V$ClsTrain
  table(Cls,ClsTrain)
}
```

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