

# Package ‘L2E’

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**Type** Package

**Title** Robust Structured Regression via the L2 Criterion

**Version** 2.0

**Description** An implementation of a computational framework for performing robust structured regression with the L2 criterion from Chi and Chi (2021+). Improvements using the majorization-minimization (MM) principle from Liu, Chi, and Lange (2022+) added in Version 2.0.

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 bank

*Bank data*


---

### Description

Data from an Italian bank on 1,949 customers. The response  $y$  is the amount of money made over a year. The 13 covariates are possible macroservices the customers can sign up for.

**Format**

A data frame with 1949 rows and 14 variables

**References**

Marco Riani, Andrea Cerioli, and Anthony C. Atkinson (2014). Monitoring robust regression. *Electronic Journal of Statistics*, Volume 8, 646-677.

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CV\_L2E\_sparse\_dist      *Cross validation for L2E sparse regression with distance penalization*

---

**Description**

CV\_L2E\_sparse\_dist performs k-fold cross-validation for robust sparse regression under the L2 criterion with distance penalty

**Usage**

```
CV_L2E_sparse_dist(
  y,
  X,
  beta0,
  tau0,
  kSeq,
  rhoSeq,
  nfolds = 5,
  seed = 1234,
  method = "median",
  max_iter = 100,
  tol = 1e-04,
  trace = TRUE
)
```

**Arguments**

y	Response vector
X	Design matrix
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
kSeq	A sequence of tuning parameter k, the number of nonzero entries in the estimated coefficients
rhoSeq	A sequence of tuning parameter rho, can be omitted
nfolds	The number of cross-validation folds. Default is 5.
seed	Users can set the seed of the random number generator to obtain reproducible results.

method	Median or mean to compute the objective
max_iter	Maximum number of iterations
tol	Relative tolerance
trace	Whether to trace the progress of the cross-validation

### Value

Returns a list object containing the mean and standard error of the cross-validation error (vectors) – CVE and CVSE – for each value of k, the index of the k value with the minimum CVE and the k value itself (scalars), the index of the k value with the 1SE CVE and the k value itself (scalars), the sequence of rho and k used in the regression (vectors), and a vector listing which fold each element of y was assigned to

### Examples

```
## Completes in 15 seconds

set.seed(12345)
n <- 100
tau <- 1
f <- matrix(c(rep(2,5), rep(0,45)), ncol = 1)
X <- X0 <- matrix(rnorm(n*50), nrow = n)
y <- y0 <- X0 %*% f + (1/tau)*rnorm(n)

## Clean Data
k <- c(6,5,4)
# (not run)
# cv <- CV_L2E_sparse_dist(y=y, X=X, kSeq=k, nfolds=2, seed=1234)
# (k_min <- cv$k.min) ## selected number of nonzero entries

# sol <- L2E_sparse_dist(y=y, X=X, kSeq=k_min)
# r <- y - X %*% sol$Beta
# ix <- which(abs(r) > 3/sol$Tau)
# l2e_fit <- X %*% sol$Beta

# plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
# points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

## Contaminated Data
i <- 1:5
y[i] <- 2 + y0[i]
X[i,] <- 2 + X0[i,]

# (not run)
# cv <- CV_L2E_sparse_dist(y=y, X=X, kSeq=k, nfolds=2, seed=1234)
# (k_min <- cv$k.min) ## selected number of nonzero entries

# sol <- L2E_sparse_dist(y=y, X=X, kSeq=k_min)
# r <- y - X %*% sol$Beta
# ix <- which(abs(r) > 3/sol$Tau)
# l2e_fit <- X %*% sol$Beta
```

```
# plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
# points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)
```

---

CV\_L2E\_sparse\_ncv      *Cross validation for L2E sparse regression with existing penalization methods*

---

### Description

CV\_L2E\_sparse\_ncv performs k-fold cross-validation for robust sparse regression under the L2 criterion. Available penalties include lasso, MCP and SCAD.

### Usage

```
CV_L2E_sparse_ncv(
  y,
  X,
  beta0,
  tau0,
  lambdaSeq,
  penalty = "MCP",
  nfold = 5,
  seed = 1234,
  method = "median",
  max_iter = 100,
  tol = 1e-04,
  trace = TRUE
)
```

### Arguments

y	Response vector
X	Design matrix
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
lambdaSeq	A decreasing sequence of tuning parameter lambda, can be omitted
penalty	Available penalties include lasso, MCP and SCAD.
nfold	The number of cross-validation folds. Default is 5.
seed	Users can set the seed of the random number generator to obtain reproducible results.
method	Median or mean to compute the objective
max_iter	Maximum number of iterations
tol	Relative tolerance
trace	Whether to trace the progress of the cross-validation

**Value**

Returns a list object containing the mean and standard error of the cross-validation error – CVE and CVSE – for each value of  $k$  (vectors), the index of the lambda with the minimum CVE and the lambda value itself (scalars), the index of the lambda value with the 1SE CVE and the lambda value itself (scalars), the sequence of lambda used in the regression (vector), and a vector listing which fold each element of  $y$  was assigned to

**Examples**

```
## Completes in 20 seconds

set.seed(12345)
n <- 100
tau <- 1
f <- matrix(c(rep(2,5), rep(0,45)), ncol = 1)
X <- X0 <- matrix(rnorm(n*50), nrow = n)
y <- y0 <- X0 %*% f + (1/tau)*rnorm(n)

## Clean Data
lambda <- 10^seq(-1, -2, length.out=20)
# (not run)
# cv <- CV_L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda, penalty="SCAD", seed=1234, nolds=2)
# (lambda_min <- cv$lambda.min)

# sol <- L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda_min, penalty="SCAD")
# r <- y - X %*% sol$Beta
# ix <- which(abs(r) > 3/sol$Tau)
# l2e_fit <- X %*% sol$Beta

# plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
# points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

## Contaminated Data
i <- 1:5
y[i] <- 2 + y0[i]
X[i,] <- 2 + X0[i,]

# (not run)
# cv <- CV_L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda, penalty="SCAD", seed=1234, nolds=2)
# (lambda_min <- cv$lambda.min)

# sol <- L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda_min, penalty="SCAD")
# r <- y - X %*% sol$Beta
# ix <- which(abs(r) > 3/sol$Tau)
# l2e_fit <- X %*% sol$Beta

# plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
# points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)
```

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CV_L2E_TF_dist	<i>Cross validation for L2E trend filtering regression with distance penalization</i>
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### Description

CV\_L2E\_TF\_dist performs k-fold cross-validation for robust trend filtering regression under the L2 criterion with distance penalty

### Usage

```
CV_L2E_TF_dist(
  y,
  X,
  beta0,
  tau0,
  D,
  kSeq,
  rhoSeq,
  nfold = 5,
  seed = 1234,
  method = "median",
  max_iter = 100,
  tol = 1e-04,
  trace = TRUE
)
```

### Arguments

y	Response vector
X	Design matrix. Default is the identity matrix.
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
D	The fusion matrix
kSeq	A sequence of tuning parameter k, the number of nonzero entries in Dbeta
rhoSeq	A sequence of tuning parameter rho, can be omitted
nfold	The number of cross-validation folds. Default is 5.
seed	Users can set the seed of the random number generator to obtain reproducible results.
method	Median or mean to calculate the objective value
max_iter	Maximum number of iterations
tol	Relative tolerance
trace	Whether to trace the progress of the cross-validation

**Value**

Returns a list object containing the mean and standard error of the cross-validation error – CVE and CVSE – for each value of  $k$  (vectors), the index of the  $k$  value with the minimum CVE and the  $k$  value itself (scalars), the index of the  $k$  value with the 1SE CVE and the  $k$  value itself (scalars), the sequence of  $\rho$  and  $k$  used in the regression (vectors), and a vector listing which fold each element of  $y$  was assigned to

**Examples**

```
## Completes in 20 seconds

set.seed(12345)
n <- 100
x <- 1:n
f <- matrix(rep(c(-2,5,0,-10), each=n/4), ncol=1)
y <- y0 <- f + rnorm(length(f))

## Clean Data
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

D <- myGetDkn(1, n)
k <- c(4,3,2)
rho <- 10^8
# (not run)
# cv <- CV_L2E_TF_dist(y=y0, D=D, kSeq=k, rhoSeq=rho, nfolds=2, seed=1234)
# (k_min <- cv$k.min)

# sol <- L2E_TF_dist(y=y0, D=D, kSeq=k_min, rhoSeq=rho)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta, col='blue', lwd=3)

## Contaminated Data
ix <- sample(1:n, 10)
y[ix] <- y0[ix] + 2

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

# (not run)
# cv <- CV_L2E_TF_dist(y=y, D=D, kSeq=k, rhoSeq=rho, nfolds=2, seed=1234)
# (k_min <- cv$k.min)

# sol <- L2E_TF_dist(y=y, D=D, kSeq=k_min, rhoSeq=rho)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta, col='blue', lwd=3)
```



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CV_L2E_TF_lasso	<i>Cross validation for L2E trend filtering regression with Lasso penalization</i>
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---

### Description

CV\_L2E\_TF\_lasso performs k-fold cross-validation for robust trend filtering regression under the L2 criterion with the Lasso penalty

### Usage

```
CV_L2E_TF_lasso(
  y,
  X,
  beta0,
  tau0,
  D,
  lambdaSeq,
  nfolds = 5,
  seed = 1234,
  method = "median",
  max_iter = 100,
  tol = 1e-04,
  trace = TRUE
)
```

### Arguments

y	Response vector
X	Design matrix. Default is the identity matrix.
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
D	The fusion matrix
lambdaSeq	A decreasing sequence of tuning parameter lambda, can be omitted
nfolds	The number of cross-validation folds. Default is 5.
seed	Users can set the seed of the random number generator to obtain reproducible results.
method	Median or mean to calculate the objective value
max_iter	Maximum number of iterations
tol	Relative tolerance
trace	Whether to trace the progress of the cross-validation

**Value**

Returns a list object containing the mean and standard error of the cross-validation error – CVE and CVSE – for each value of  $k$  (vectors), the index of the lambda with the minimum CVE and the lambda value itself (scalars), the index of the lambda value with the 1SE CVE and the lambda value itself (scalars), the sequence of lambda used in the regression (vector), and a vector listing which fold each element of  $y$  was assigned to

**Examples**

```
## Completes in 30 seconds

set.seed(12345)
n <- 100
x <- 1:n
f <- matrix(rep(c(-2,5,0,-10), each=n/4), ncol=1)
y <- y0 <- f + rnorm(length(f))

## Clean Data
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

D <- myGetDkn(1, n)
lambda <- 10^seq(-1, -2, length.out=20)
# (not run)
# cv <- CV_L2E_TF_lasso(y=y0, D=D, lambdaSeq=lambda, nfolds=2, seed=1234)
# (lambda_min <- cv$lambda.min)

# sol <- L2E_TF_lasso(y=y0, D=D, lambdaSeq=lambda_min)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta, col='blue', lwd=3)

## Contaminated Data
ix <- sample(1:n, 10)
y[ix] <- y0[ix] + 2

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

# (not run)
# cv <- CV_L2E_TF_lasso(y=y, D=D, lambdaSeq=lambda, nfolds=2, seed=1234)
# (lambda_min <- cv$lambda.min)

# sol <- L2E_TF_lasso(y=y, D=D, lambdaSeq=lambda_min)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta, col='blue', lwd=3)
```

---

L2E

*L2E*


---

### Description

The L2E package is an R implementation of a user-friendly computational framework for performing a wide variety of robust structured regression methods via the L2 criterion.

### Details

Please refer to the vignette for examples of how to use this package.

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L2E\_convex

*L2E convex regression*


---

### Description

L2E\_convex performs convex regression under the L2 criterion. Available methods include proximal gradient descent (PG) and majorization-minimization (MM).

### Usage

```
L2E_convex(
  y,
  beta,
  tau,
  method = "MM",
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

### Arguments

y	Response vector
beta	Initial vector of regression coefficients
tau	Initial precision estimate
method	Available methods include PG and MM. MM by default.
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

**Value**

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta (vector) and tau or eta (vector)

**Examples**

```

set.seed(12345)
n <- 200
tau <- 1
x <- seq(-2, 2, length.out=n)
f <- x^4 + x
y <- f + (1/tau) * rnorm(n)

## Clean Data
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
## Least Squares method
cvx <- fitted(cobs::conreg(y, convex=TRUE))
## MM method
sol_mm <- L2E_convex(y, b, tau)
## PG method
sol_pg <- L2E_convex(y, b, tau, method='PG')

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, cvx, col='blue', lwd=3) ## LS
lines(x, sol_mm$beta, col='red', lwd=3) ## MM
lines(x, sol_pg$beta, col='dark green', lwd=3) ## PG

## Contaminated Data
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
cvx <- fitted(cobs::conreg(y, convex=TRUE))
sol_mm <- L2E_convex(y, b, tau)
sol_pg <- L2E_convex(y, b, tau, method='PG')

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, cvx, col='blue', lwd=3) ## LS
lines(x, sol_mm$beta, col='red', lwd=3) ## MM
lines(x, sol_pg$beta, col='dark green', lwd=3) ## PG

```

---

L2E_isotonic	<i>L2E isotonic regression</i>
--------------	--------------------------------

---

### Description

L2E\_isotonic performs isotonic regression under the L2 criterion. Available methods include proximal gradient descent (PG) and majorization-minimization (MM).

### Usage

```
L2E_isotonic(
  y,
  beta,
  tau,
  method = "MM",
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

### Arguments

y	Response vector
beta	Initial vector of regression coefficients
tau	Initial precision estimate
method	Available methods include PG and MM. MM by default.
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta (vector) and tau or eta (vector)

### Examples

```
set.seed(12345)
n <- 200
tau <- 1
x <- seq(-2.5, 2.5, length.out=n)
f <- x^3
y <- f + (1/tau) * rnorm(n)

## Clean Data
```

```

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
## Least Squares method
iso <- isotone::gpava(1:n, y)$x
## MM method
sol_mm <- L2E_isotonic(y, b, tau)
## PG method
sol_pg <- L2E_isotonic(y, b, tau, method='PG')

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, iso, col='blue', lwd=3) ## LS
lines(x, sol_mm$beta, col='red', lwd=3) ## MM
lines(x, sol_pg$beta, col='dark green', lwd=3) ## PG

## Contaminated Data
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
iso <- isotone::gpava(1:n, y)$x
sol_mm <- L2E_isotonic(y, b, tau)
sol_pg <- L2E_isotonic(y, b, tau, method='PG')

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, iso, col='blue', lwd=3) ## LS
lines(x, sol_mm$beta, col='red', lwd=3) ## MM
lines(x, sol_pg$beta, col='dark green', lwd=3) ## PG

```

---

L2E\_multivariate

*L2E multivariate regression*


---

### Description

L2E\_multivariate performs multivariate regression under the L2 criterion. Available methods include proximal gradient descent (PG) and majorization-minimization (MM).

### Usage

```

L2E_multivariate(
  y,

```

```

X,
beta,
tau,
method = "MM",
max_iter = 100,
tol = 1e-04,
Show.Time = TRUE
)

```

### Arguments

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
method	Available methods include PG and MM. MM by default.
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta (vector) and tau or eta (vector)

### Examples

```

# Bank data example
y <- bank$y
X <- as.matrix(bank[,1:13])
X0 <- as.matrix(cbind(rep(1,length(y)), X))

tau <- 1/mad(y)
b <- matrix(0, 14, 1)

# MM method
sol_mm <- L2E_multivariate(y, X0, b, tau)
r_mm <- y - X0 %*% sol_mm$beta
ix_mm <- which(abs(r_mm) > 3/sol_mm$tau)
l2e_fit_mm <- X0 %*% sol_mm$beta

# PG method
sol_pg <- L2E_multivariate(y, X0, b, tau, method="PG")
r_pg <- y - X0 %*% sol_pg$beta
ix_pg <- which(abs(r_pg) > 3/sol_pg$tau)
l2e_fit_pg <- X0 %*% sol_pg$beta

plot(y, l2e_fit_mm, ylab='Predicted values', main='MM', pch=16, cex=0.8) # MM

```

```

points(y[ix_mm], l2e_fit_mm[ix_mm], pch=16, col='blue', cex=0.8) # MM
plot(y, l2e_fit_pg, ylab='Predicted values', main='PG', pch=16, cex=0.8) # PG
points(y[ix_pg], l2e_fit_pg[ix_pg], pch=16, col='blue', cex=0.8) # PG

```

---

l2e_regression	<i>L2E multivariate regression - PG</i>
----------------	---

---

### Description

l2e\_regression performs L2E multivariate regression via block coordinate descent with proximal gradient for updating both beta and tau.

### Usage

```
l2e_regression(y, X, b, tau, max_iter = 100, tol = 1e-04, Show.Time = TRUE)
```

### Arguments

y	Response vector
X	Design matrix
b	Initial vector of regression coefficients
tau	Initial precision estimate
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and tau (vectors)

### Examples

```

# Bank data example
y <- bank$y
X <- as.matrix(bank[,1:13])
X0 <- as.matrix(cbind(rep(1,length(y)), X))
tauinit <- 1/mad(y)
binit <- matrix(0, 14, 1)

sol <- l2e_regression(y, X0, binit, tauinit)
r <- y - X0 %*% sol$beta
ix <- which(abs(r) > 3/sol$tau)
l2e_fit <- X0 %*% sol$beta

```



```
plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)
```

---

l2e\_regression\_convex *L2E convex regression - PG*

---

### Description

l2e\_regression\_convex performs L2E convex regression via block coordinate descent with proximal gradient for updating both beta and tau.

### Usage

```
l2e_regression_convex(y, b, tau, max_iter = 100, tol = 1e-04, Show.Time = TRUE)
```

### Arguments

y	Response vector
b	Initial vector of regression coefficients
tau	Initial precision estimate
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and tau (vectors)

### Examples

```
set.seed(12345)
n <- 200
tau <- 1
x <- seq(-2, 2, length.out=n)
f <- x^4 + x
y <- f + (1/tau) * rnorm(n)

## Clean data example
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_convex(y, b, tau)
```

```

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
cvx <- fitted(cobs::conreg(y, convex=TRUE))
lines(x, cvx, col='blue', lwd=3)
lines(x, sol$beta, col='dark green', lwd=3)

## Contaminated data example
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_convex(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
cvx <- fitted(cobs::conreg(y, convex=TRUE))
lines(x, cvx, col='blue', lwd=3)
lines(x, sol$beta, col='dark green', lwd=3)

```

---

l2e\_regression\_convex\_MM

*L2E convex regression - MM*

---

## Description

l2e\_regression\_convex\_MM performs L2E convex regression via block coordinate descent with MM for updating beta and modified Newton for updating tau.

## Usage

```

l2e_regression_convex_MM(
  y,
  beta,
  tau,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)

```

## Arguments

y	response
beta	initial vector of regression coefficients

tau	initial precision estimate
max_iter	maximum number of iterations
tol	relative tolerance
Show.Time	Report the computing time

**Value**

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and eta (vectors)

**Examples**

```

set.seed(12345)
n <- 200
tau <- 1
x <- seq(-2, 2, length.out=n)
f <- x^4 + x
y <- f + (1/tau)*rnorm(n)

## Clean
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_convex_MM(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
cvx <- fitted(cobs::conreg(y, convex=TRUE))
lines(x, cvx, col='blue', lwd=3)
lines(x, sol$beta, col='red', lwd=3)

## Contaminated
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_convex_MM(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
cvx <- fitted(cobs::conreg(y, convex=TRUE))
lines(x, cvx, col='blue', lwd=3)
lines(x, sol$beta, col='red', lwd=3)

```

---

`l2e_regression_isotonic`*L2E isotonic regression - PG*

---

**Description**

`l2e_regression_isotonic` performs L2E isotonic regression via block coordinate descent with proximal gradient for updating both beta and tau.

**Usage**

```
l2e_regression_isotonic(  
  y,  
  b,  
  tau,  
  max_iter = 100,  
  tol = 1e-04,  
  Show.Time = TRUE  
)
```

**Arguments**

<code>y</code>	Response vector
<code>b</code>	Initial vector of regression coefficients
<code>tau</code>	Initial precision estimate
<code>max_iter</code>	Maximum number of iterations
<code>tol</code>	Relative tolerance
<code>Show.Time</code>	Report the computing time

**Value**

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and tau (vectors)

**Examples**

```
set.seed(12345)  
n <- 200  
tau <- 1  
x <- seq(-2.5, 2.5, length.out=n)  
f <- x^3  
y <- f + (1/tau)*rnorm(n)  
  
# Clean Data  
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
```

```

lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_isotonic(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
iso <- isotone::gpava(1:n, y)$x
lines(x, iso, col='blue', lwd=3)
lines(x, sol$beta, col='dark green', lwd=3)

# Contaminated Data
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_isotonic(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
iso <- isotone::gpava(1:n, y)$x
lines(x, iso, col='blue', lwd=3)
lines(x, sol$beta, col='dark green', lwd=3)

```

---

l2e\_regression\_isotonic\_MM

*L2E isotonic regression - MM*


---

## Description

l2e\_regression\_isotonic\_MM performs L2E isotonic regression via block coordinate descent with MM for updating beta and modified Newton for updating tau.

## Usage

```

l2e_regression_isotonic_MM(
  y,
  beta,
  tau,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)

```

**Arguments**

y	Response vector
beta	Initial vector of regression coefficients
tau	Initial precision estimate
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

**Value**

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and eta (vectors)

**Examples**

```

set.seed(12345)
n <- 200
tau <- 1
x <- seq(-2.5, 2.5, length.out=n)
f <- x^3
y <- f + (1/tau)*rnorm(n)

## Clean
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_isotonic_MM(y, b, tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
iso <- isotone::gpava(1:n, y)$x
lines(x, iso, col='blue', lwd=3)
lines(x, sol$beta, col='red', lwd=3)

## Contaminated
ix <- 0:9
y[45 + ix] <- 14 + rnorm(10)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

tau <- 1
b <- y
sol <- l2e_regression_isotonic_MM(y,b,tau)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

```

```
iso <- isotone::gpava(1:n, y)$x
lines(x, iso, col='blue', lwd=3)
lines(x, sol$beta, col='red', lwd=3)
```

---

l2e\_regression\_MM      *L2E multivariate regression - MM*

---

### Description

l2e\_regression\_MM performs L2E multivariate regression via block coordinate descent with MM for updating beta and modified Newton for updating tau.

### Usage

```
l2e_regression_MM(  
  y,  
  X,  
  beta,  
  tau,  
  max_iter = 100,  
  tol = 1e-04,  
  Show.Time = TRUE  
)
```

### Arguments

y	response
X	Design matrix
beta	initial vector of regression coefficients
tau	initial precision estimate
max_iter	maximum number of iterations
tol	relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (vector) and tau (scalar), the number of outer block descent iterations until convergence (scalar), and the number of inner iterations per outer iteration for updating beta and eta (vectors)

**Examples**

```
# Bank data example
y <- bank$y
X <- as.matrix(bank[,1:13])
X0 <- as.matrix(cbind(rep(1,length(y)), X))
tau <- 1/mad(y)
b <- matrix(0, 14, 1)

sol <- l2e_regression_MM(y, X0, b, tau)
r <- y - X0 %*% sol$beta
ix <- which(abs(r) > 3/sol$tau)
l2e_fit <- X0 %*% sol$beta

plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)
```

---

l2e\_regression\_sparse\_dist

*L2E sparse regression with distance penalization*


---

**Description**

l2e\_regression\_sparse\_dist performs robust sparse regression under the L2 criterion with the distance penalty

**Usage**

```
l2e_regression_sparse_dist(
  y,
  X,
  beta,
  tau,
  k,
  rho = 1,
  stepsize = 0.9,
  sigma = 0.5,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

**Arguments**

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients



tau	Initial precision estimate
k	The number of nonzero entries in the estimated coefficients
rho	The parameter in the proximal distance algorithm
stepsize	The stepsize parameter for the MM algorithm (0, 1)
sigma	The halving parameter sigma (0, 1)
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

---

l2e\_regression\_sparse\_ncv

*L2E sparse regression with existing penalization methods*


---

### Description

l2e\_regression\_sparse\_ncv performs robust sparse regression under the L2 criterion. Available penalties include lasso, MCP and SCAD.

### Usage

```
l2e_regression_sparse_ncv(
  y,
  X,
  beta,
  tau,
  lambda,
  penalty,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

### Arguments

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
lambda	Tuning parameter
penalty	Available penalties include lasso, MCP and SCAD.
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

---

`l2e_regression_TF_dist`*L2E trend filtering regression with distance penalization*

---

**Description**

`l2e_regression_TF_dist` performs robust trend filtering regression under the L2 criterion with distance penalty

**Usage**

```
l2e_regression_TF_dist(  
  y,  
  X,  
  beta,  
  tau,  
  D,  
  k,  
  rho = 1,  
  max_iter = 100,  
  tol = 1e-04,  
  Show.Time = TRUE  
)
```

**Arguments**

<code>y</code>	Response vector
<code>X</code>	Design matrix
<code>beta</code>	Initial vector of regression coefficients
<code>tau</code>	Initial precision estimate
<code>D</code>	The fusion matrix
<code>k</code>	The number of nonzero entries in $D \cdot \text{beta}$
<code>rho</code>	The parameter in the proximal distance algorithm
<code>max_iter</code>	Maximum number of iterations
<code>tol</code>	Relative tolerance
<code>Show.Time</code>	Report the computing time

---

`l2e_regression_TF_lasso`*L2E trend filtering regression with Lasso penalization*

---

**Description**

`l2e_regression_TF_lasso` performs robust trend filtering regression under the L2 criterion with Lasso penalty

**Usage**

```
l2e_regression_TF_lasso(  
  y,  
  X,  
  beta,  
  tau,  
  D,  
  lambda = 1,  
  max_iter = 100,  
  tol = 1e-04,  
  Show.Time = TRUE  
)
```

**Arguments**

<code>y</code>	Response vector
<code>X</code>	Design matrix
<code>beta</code>	Initial vector of regression coefficients
<code>tau</code>	Initial precision estimate
<code>D</code>	The fusion matrix
<code>lambda</code>	The tuning parameter
<code>max_iter</code>	Maximum number of iterations
<code>tol</code>	Relative tolerance
<code>Show.Time</code>	Report the computing time

---

L2E\_sparse\_dist      *Solution path of L2E sparse regression with distance penalization*

---

### Description

L2E\_sparse\_dist computes the solution path of the robust sparse regression under the L2 criterion with distance penalty

### Usage

```
L2E_sparse_dist(
  y,
  X,
  beta0,
  tau0,
  kSeq,
  rhoSeq,
  stepsize = 0.9,
  sigma = 0.5,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

### Arguments

y	Response vector
X	Design matrix
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
kSeq	A sequence of tuning parameter k, the number of nonzero entries in the estimated coefficients
rhoSeq	An increasing sequence of tuning parameter rho, can be omitted
stepsize	The stepsize parameter for the MM algorithm (0, 1)
sigma	The halving parameter sigma (0, 1)
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (matrix) and tau (vector) for each value of the tuning parameter k, the path of estimates for beta (list of matrices) and tau (matrix) for each value of rho, the run time (vector) for each k, and the sequence of rho and k used in the regression (vectors)

**Examples**

```

set.seed(12345)
n <- 100
tau <- 1
f <- matrix(c(rep(2,5), rep(0,45)), ncol = 1)
X <- X0 <- matrix(rnorm(n*50), nrow = n)
y <- y0 <- X0 %>% f + (1/tau)*rnorm(n)

## Clean Data
k <- 5
sol <- L2E_sparse_dist(y=y, X=X, kSeq=k)
r <- y - X %>% sol$Beta
ix <- which(abs(r) > 3/sol$Tau)
l2e_fit <- X %>% sol$Beta

plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

## Contaminated Data
i <- 1:5
y[i] <- 2 + y0[i]
X[i,] <- 2 + X0[i,]

sol <- L2E_sparse_dist(y=y, X=X, kSeq=k)
r <- y - X %>% sol$Beta
ix <- which(abs(r) > 3/sol$Tau)
l2e_fit <- X %>% sol$Beta

plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

```

---

L2E\_sparse\_ncv

*Solution path of L2E sparse regression with existing penalization methods*


---

**Description**

L2E\_sparse\_ncv computes the solution path of robust sparse regression under the L2 criterion. Available penalties include lasso, MCP and SCAD.

**Usage**

```

L2E_sparse_ncv(
  y,
  X,
  b,
  tau,
  lambdaSeq,

```

```

penalty = "MCP",
max_iter = 100,
tol = 1e-04,
Show.Time = TRUE
)

```

### Arguments

y	Response vector
X	Design matrix
b	Initial vector of regression coefficients, can be omitted
tau	Initial precision estimate, can be omitted
lambdaSeq	A decreasing sequence of values for the tuning parameter lambda, can be omitted
penalty	Available penalties include lasso, MCP and SCAD.
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

### Value

Returns a list object containing the estimates for beta (matrix) and tau (vector) for each value of the tuning parameter lambda, the run time (vector) for each lambda, and the sequence of lambda used in the regression (vector)

### Examples

```

set.seed(12345)
n <- 100
tau <- 1
f <- matrix(c(rep(2,5), rep(0,45)), ncol = 1)
X <- X0 <- matrix(rnorm(n*50), nrow = n)
y <- y0 <- X0 %*% f + (1/tau)*rnorm(n)

## Clean Data
lambda <- 10^(-1)
sol <- L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda, penalty="SCAD")
r <- y - X %*% sol$Beta
ix <- which(abs(r) > 3/sol$Tau)
l2e_fit <- X %*% sol$Beta

plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

## Contaminated Data
i <- 1:5
y[i] <- 2 + y0[i]
X[i,] <- 2 + X0[i,]

```

```

sol <- L2E_sparse_ncv(y=y, X=X, lambdaSeq=lambda, penalty="SCAD")
r <- y - X %*% sol$Beta
ix <- which(abs(r) > 3/sol$Tau)
l2e_fit <- X %*% sol$Beta

plot(y, l2e_fit, ylab='Predicted values', pch=16, cex=0.8)
points(y[ix], l2e_fit[ix], pch=16, col='blue', cex=0.8)

```

---

L2E\_TF\_dist

*Solution path of the L2E trend filtering regression with distance penalization*


---

### Description

L2E\_TF\_dist computes the solution path of the robust trend filtering regression under the L2 criterion with distance penalty

### Usage

```

L2E_TF_dist(
  y,
  X,
  beta0,
  tau0,
  D,
  kSeq,
  rhoSeq,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)

```

### Arguments

y	Response vector
X	Design matrix. Default is the identity matrix.
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
D	The fusion matrix
kSeq	A sequence of tuning parameter k, the number of nonzero entries in $D \cdot \beta$
rhoSeq	An increasing sequence of tuning parameter rho, can be omitted
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

**Value**

Returns a list object containing the estimates for beta (matrix) and tau (vector) for each value of the tuning parameter k, the path of estimates for beta (list of matrices) and tau (matrix) for each value of rho, the run time (vector) for each k, and the sequence of rho and k used in the regression (vectors)

**Examples**

```
## Completes in 15 seconds

set.seed(12345)
n <- 100
x <- 1:n
f <- matrix(rep(c(-2,5,0,-10), each=n/4), ncol=1)
y <- y0 <- f + rnorm(length(f))

## Clean Data
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

D <- myGetDkn(1, n)
k <- c(4,3,2)
rho <- 10^8
# (not run)
# sol <- L2E_TF_dist(y=y, D=D, kSeq=k, rhoSeq=rho)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta[,3], col='blue', lwd=3) ## k=2
# lines(x, sol$Beta[,2], col='red', lwd=3) ## k=3
# lines(x, sol$Beta[,1], col='dark green', lwd=3) ## k=4

## Contaminated Data
ix <- sample(1:n, 10)
y[ix] <- y0[ix] + 2

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

# (not run)
# sol <- L2E_TF_dist(y=y, D=D, kSeq=k, rhoSeq=rho)

# plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
# lines(x, f, lwd=3)
# lines(x, sol$Beta[,3], col='blue', lwd=3) ## k=2
# lines(x, sol$Beta[,2], col='red', lwd=3) ## k=3
# lines(x, sol$Beta[,1], col='dark green', lwd=3) ## k=4
```



**Description**

L2E\_TF\_lasso computes the solution path of the robust trend filtering regression under the L2 criterion with Lasso penalty

**Usage**

```
L2E_TF_lasso(
  y,
  X,
  beta0,
  tau0,
  D,
  lambdaSeq,
  max_iter = 100,
  tol = 1e-04,
  Show.Time = TRUE
)
```

**Arguments**

y	Response vector
X	Design matrix. Default is the identity matrix.
beta0	Initial vector of regression coefficients, can be omitted
tau0	Initial precision estimate, can be omitted
D	The fusion matrix
lambdaSeq	A decreasing sequence of values for the tuning parameter lambda, can be omitted
max_iter	Maximum number of iterations
tol	Relative tolerance
Show.Time	Report the computing time

**Value**

Returns a list object containing the estimates for beta (matrix) and tau (vector) for each value of the tuning parameter lambda, the run time (vector) for each lambda, and the sequence of lambda used in the regression (vector)

**Examples**

```
## Completes in 10 seconds

set.seed(12345)
n <- 100
x <- 1:n
f <- matrix(rep(c(-2,5,0,-10), each=n/4), ncol=1)
y <- y0 <- f + rnorm(length(f))
```

```

## Clean Data
plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

D <- myGetDkn(1, n)
lambda <- 10^seq(-1, -2, length.out=20)
sol <- L2E_TF_lasso(y=y, D=D, lambdaSeq=lambda)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, sol$Beta[,1], col='blue', lwd=3) ## 1st lambda

## Contaminated Data
ix <- sample(1:n, 10)
y[ix] <- y0[ix] + 2

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)

sol <- L2E_TF_lasso(y=y, D=D, lambdaSeq=lambda)

plot(x, y, pch=16, cex.lab=1.5, cex.axis=1.5, cex.sub=1.5, col='gray')
lines(x, f, lwd=3)
lines(x, sol$Beta[,1], col='blue', lwd=3) ## 1st lambda

```

---

myGetDkn

*Compute kth order differencing matrix*


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### Description

myGetDkn computes the kth order differencing matrix for use in trend filtering regression

### Usage

```
myGetDkn(k, n)
```

### Arguments

k	Order of the differencing matrix
n	Number of time points

### Value

Returns a Matrix object as the kth order differencing matrix

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objective	<i>Objective function of the L2E regression - eta</i>
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**Description**

objective computes the objective of the L2E regression in terms of eta

**Usage**

```
objective(eta, r, method = "mean")
```

**Arguments**

eta	The current estimate of eta
r	Vector of residuals
method	Mean or median

**Value**

Returns the output of the objective function (scalar)

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objective_tau	<i>Objective function of the L2E regression - tau</i>
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**Description**

objective\_tau computes the objective of the L2E regression in terms of tau

**Usage**

```
objective_tau(tau, r, method = "mean")
```

**Arguments**

tau	The current estimate of tau
r	Vector of residuals
method	Mean or median

**Value**

Returns the output of the objective function (scalar)

---

update\_beta\_convex     *Beta update in L2E convex regression - PG*

---

### Description

update\_beta\_convex updates beta for L2E convex regression using PG

### Usage

```
update_beta_convex(y, b, tau, max_iter = 100, tol = 1e-04)
```

### Arguments

y	Response vector
b	Current estimate for beta
tau	Current estimate for tau
max_iter	Maximum number of iterations
tol	Relative tolerance

### Value

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_isotonic     *Beta update in L2E isotonic regression - PG*

---

### Description

update\_beta\_isotonic updates beta for L2E isotonic regression using PG

### Usage

```
update_beta_isotonic(y, b, tau, max_iter = 100, tol = 1e-04)
```

### Arguments

y	Response vector
b	Current estimate for beta
tau	Current estimate for tau
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_MM\_convex *Beta update in L2E convex regression - MM*

---

**Description**

update\_beta\_MM\_convex updates beta for L2E convex regression using MM

**Usage**

```
update_beta_MM_convex(y, beta, tau, max_iter = 100, tol = 1e-04)
```

**Arguments**

y	Response
beta	Initial vector of regression coefficients
tau	Precision estimate
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_MM\_isotonic  
*Beta update in L2E isotonic regression - MM*

---

**Description**

update\_beta\_MM\_isotonic updates beta for L2E isotonic regression using MM

**Usage**

```
update_beta_MM_isotonic(y, beta, tau, max_iter = 100, tol = 1e-04)
```

**Arguments**

y	response
beta	initial vector of regression coefficients
tau	precision estimate
max_iter	maximum number of iterations
tol	relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_MM\_ls      *Beta update in L2E multivariate regression - MM*

---

**Description**

update\_beta\_MM\_ls updates beta for L2E multivariate regression using MM

**Usage**

```
update_beta_MM_ls(y, X, beta, tau, max_iter = 100, tol = 1e-04)
```

**Arguments**

y	Response
X	Design matrix
beta	Initial vector of regression coefficients
tau	Precision estimate
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_MM\_sparse *Beta update in L2E sparse regression - MM*

---

### Description

update\_beta\_MM\_sparse updates beta for L2E sparse regression using the distance penalty

### Usage

```
update_beta_MM_sparse(  
  y,  
  X,  
  beta,  
  tau,  
  k,  
  rho,  
  stepsize = 0.9,  
  sigma = 0.5,  
  max_iter = 100,  
  tol = 1e-04  
)
```

### Arguments

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
k	The number of nonzero entries in the estimated coefficients
rho	The parameter in the proximal distance algorithm
stepsize	The stepsize parameter for the MM algorithm (0, 1)
sigma	The halving parameter sigma (0, 1)
max_iter	Maximum number of iterations
tol	Relative tolerance

### Value

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_MM\_TF      *Beta update in L2E trend filtering regression - MM*

---

### Description

update\_beta\_MM\_TF updates beta in L2E trend filtering regression using the distance penalty

### Usage

update\_beta\_MM\_TF(y, X, beta, tau, D, k, rho, max\_iter = 100, tol = 1e-04)

### Arguments

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
D	The fusion matrix
k	The number of nonzero entries in D*beta
rho	The parameter in the proximal distance algorithm
max_iter	Maximum number of iterations
tol	Relative tolerance

### Value

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_qr      *Beta update in L2E multivariate regression - PG*

---

### Description

update\_beta\_qr updates beta for L2E multivariate regression via a QR solve

### Usage

update\_beta\_qr(y, X, QRF, tau, b, max\_iter = 100, tol = 1e-04)



**Arguments**

y	Response vector
X	Design matrix
QRF	QR factorization object for X (obtained via 'QRF=qr(X)')
tau	Current estimate for tau
b	Current estimate for beta
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_sparse\_ncv

*Beta update in L2E sparse regression - NCV*

---

**Description**

update\_beta\_sparse\_ncv updates beta for L2E sparse regression using existing penalization methods

**Usage**

```
update_beta_sparse_ncv(
  y,
  X,
  beta,
  tau,
  lambda,
  penalty,
  max_iter = 100,
  tol = 1e-04
)
```

**Arguments**

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
lambda	Tuning parameter
penalty	Available penalties include lasso, MCP and SCAD.
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_beta\_TF\_lasso    *Beta update in L2E trend filtering regression using Lasso*

---

**Description**

update\_beta\_TF\_lasso updates beta in L2E trend filtering regression using the Lasso penalty

**Usage**

```
update_beta_TF_lasso(y, X, beta, tau, D, lambda, max_iter = 100, tol = 1e-04)
```

**Arguments**

y	Response vector
X	Design matrix
beta	Initial vector of regression coefficients
tau	Initial precision estimate
D	The fusion matrix
lambda	The tuning parameter
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for beta (vector) and the number of iterations (scalar) the update step utilized

---

update\_eta\_bktk    *Eta update using Newton's method with backtracking*

---

**Description**

update\_eta\_bktk updates the precision parameter  $\tau = e^{\eta}$  for L2E regression using Newton's method

**Usage**

```
update_eta_bktk(r, eta, max_iter = 100, tol = 1e-10)
```

**Arguments**

r	Vector of residual
eta	Initial estimate of eta
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for eta (scalar), the number of iterations (scalar) the update step utilized, the eta and objective function solution paths (vectors), and the first and second derivatives calculated via Newton's method (vectors)

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update_tau_R	<i>Tau update function</i>
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**Description**

update\_tau\_R updates the precision parameter tau

**Usage**

```
update_tau_R(r, tau, sd_y, max_iter = 100, tol = 1e-10)
```

**Arguments**

r	Residual vector
tau	Current estimate for tau
sd_y	Standard deviation of y
max_iter	Maximum number of iterations
tol	Relative tolerance

**Value**

Returns a list object containing the new estimate for tau (scalar) and the number of iterations (scalar) the update step utilized

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