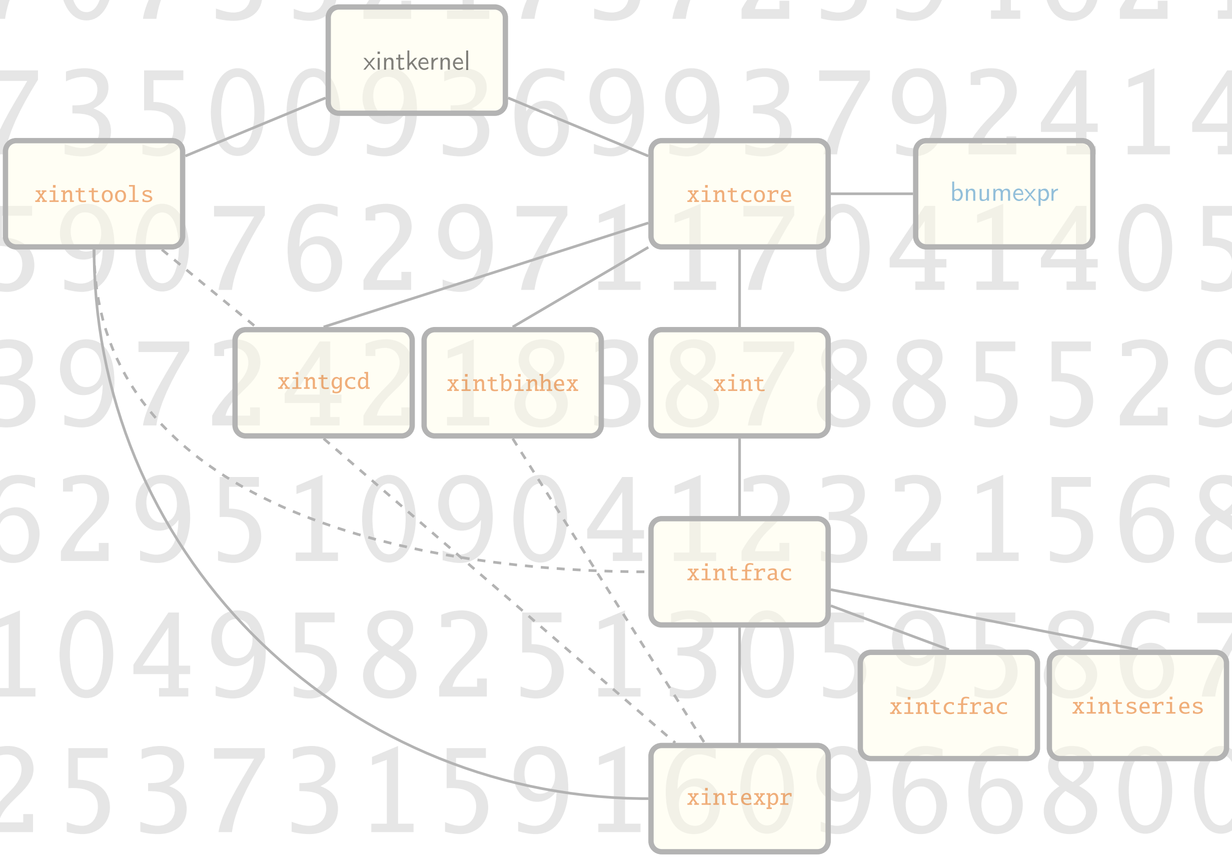
The xint bundle

JEAN-FRANÇOIS BURNOL
jfbu (at) free (dot) fr

Package version: 1.2g (2016/03/19); documentation date: 2016/03/19.
From source file xint.dtx. Time-stamp: <19-03-2016 at 14:52:58 CET>.

Contents



Dependency graph for the `xint` bundle components: modules at the bottom import modules at the top when connected by a continuous line. No module will be loaded twice, this is managed internally under Plain as well as LaTeX. Dashed lines indicate a partial dependency, and to enable the corresponding functionalities of the lower module it is necessary for the user to issue the suitable `\usepackage{top_module}` in the preamble (or `\input top_module.sty\relax` in Plain TeX). The bnumexpr package is a separate package (LaTeX only) by the author.

1 Read this first

This section provides recommended reading on first discovering the package.

xinttools provides utilities of independent interest such as expandable and non-expandable loops. It is not loaded automatically (nor needed) by the other bundle packages, apart from xintexpr.

xintcore provides the expandable TeX macros doing additions, subtractions, multiplications, divisions, and powers on arbitrarily long numbers (loaded automatically by xint, and also by package bnumexpr in its default configuration).

xint extends xintcore with additional operations on big integers. Loads automatically xintcore.

xintfrac extends the scope of xint to decimal numbers, to numbers in scientific notation and also to fractions with arbitrarily long such numerators and denominators separated by a forward slash. Loads automatically xint.

xintexpr extends xintfrac with expandable parsers doing algebra (exact, float, or limited to integers) on comma separated expressions using standard infix notations with parentheses, numbers in decimal notation, scientific notation, comparison operators, Boolean logic, twofold and threefold way conditionals, sub-expressions, some functions with one or many arguments, user-definable variables, user-definable functions, nestable use of dummy variables for evaluation of sub-expressions, with iterations admitting omit, abort, and break instructions. Automatically loads xinttools and xintfrac (hence xint and xintcore too).

Further modules:

xintbinhex is for conversions to and from binary and hexadecimal bases.

xintseries provides some basic functionality for computing in an expandable manner partial sums of series and power series with fractional coefficients.

xintgcd implements the Euclidean algorithm and its typesetting.

xintcfrac deals with the computation of continued fractions.

All macros from the xint packages doing computations are *expandable*, and naturally also the parsers provided by xintexpr.

The reasonable range of use of the package arithmetics is with numbers of *up to a few hundred digits*. Although numbers up to about 19950 digits are acceptable inputs, the package is not at his peak efficiency when confronted with such really big numbers having thousands of digits.[1]

1.1 First examples

With `\usepackage{xintexpr}` if using LaTeX, or `\input xintexpr.sty\relax` for other formats, you can do computations such as the following.

with floats:

[1] The maximal handled size for inputs to multiplication is 19959 digits. This limit is observed with the current default values of some parameters of the tex executable (input save stack size at 5000, maximal expansion depth at 10000). Nesting of macros will reduce it and it is best to restrain numbers to at most 19900 digits. The output, as naturally is the case with multiplication, may exceed the bound.

```
\xintthefloatexpr 3.25^100/3.2^100, 2^1000000, sqrt(1000!), 10^-3.5\relax
```

4.713443069476886, 9.900656229295898e301029, 6.343400192933548e1283, 0.0003162277660168379

with fractions:

```
\xinttheexpr reduce(add((-1)^(i-1)/i**2, i=1..25))\relax
```

19666926252042445851 7/2388980574952171 20000

with integers:

```
\xinttheiiexpr 3^159+2^234\relax
```

72825109579317913703320352400201941556241598394068059751504952996512059822 51

Float computations are done by default with 16 digits of precision. This can be changed by a prior assignment to \xintDigits:

```
% use braces (or a LaTeX environment) to limit the scope of the \xintDigits assignment
{\xintDigits := 88;\xintthefloatexpr  3.25^100-3.2^100\relax}\par
```

1.215554966658265430322806638672591886136929518808939313995673252222064394651297590367526e51

We can even try daring things:[2]

```
{\xintDigits:=500;\printnumber{\xintthefloatexpr sqrt(2)\relax}}
```

1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875343276415727350138462309122970249248360558507372126441214970999358314132226659275055927557999505011527820605714701095599716059702745345968620147285174186408891986095523292304843087143214508397626036279952514079896872533965463318088296406206152583523950547457502877599617298355752203375318570113543746034084988471603868999706990048150305440277903164542478230684929369186215805784631115966687130130156185689872372

This is release 1.2f.

1. exp, cos, sin, etc... are *yet to be implemented*,
2. NaN, +Infty, -Infty, etc... are *yet to be implemented*,
3. New with 1.2f powers work currently only with integral *or also half-integral* exponents (but the latter only for float expressions),
4. xint can handle numbers with thousands of digits, but execution times limit the practical range to a few hundreds (if many such computations are needed),
5. computations in \xinttheexpr and \xinttheiiexpr are exact (except if using sqrt, naturally),
6. fractions are not systematically reduced to smallest terms, use reduce function,
7. for producing fixed point numbers with d digits after decimal mark, use (note the extra `i' in the parser name!) \xinttheiexpr [d] ...\relax. This is actually essentially synonymous with \xinttheexpr round(..,d)\relax (for d=0, \xinttheiexpr [0] is the same as \xinttheiexpr without optional argument, and is like \xinttheexpr round(..)\relax). If truncation rather than rounding is needed use thus \xinttheexpr trunc(..,d)\relax (and \xinttheexpr trunc(..)\relax for truncation to integers),
8. all three parsers allow some constructs with dummy variables as seen above; it is possible to define new functions or to declare variables for use in upcoming computations,
9. \xinttheiiexpr is slightly faster than \xinttheexpr, but usually one can use the latter with no significant time penalty also for integer-only computations.

[2] The \printnumber is not part of the package, see subsection 1.3.

All operations executed by the parsers are based on underlying macros from packages xintfrac and xint which are loaded automatically by xintexpr. With extra packages xintbinhex and xintgcd the parsers can handle hexadecimal notation on input (even fractional) and compute gcd's or lcm's of integers.

All macros doing computations ultimately rely on (and reduce to) the \numexpr primitive from ε-TeX. These ε-TeX extensions date back to 1999 and are by default incorporated into the pdftex etc... executables from major modern TeX installations since more than ten years now. Only the tex binary does not benefit from them, as it has to remain the original D. KNUTH's software, but one can then use etex on the command line. PDFTeX (in pdf or dvi output mode), LuaTeX, XeTeX all include the ε-TeX extensions.

1.2 Quick overview (expressions with xintexpr)

This section gives a first few examples of using the expression parsers which are provided by package xintexpr. Loading xintexpr automatically also loads packages xinttools and xintfrac. The latter loads xint which loads xintcore. All three provide the macros which ultimately do the computations associated in expressions with the various symbols like +, *, ^, ! and functions such as max, sqrt, gcd (the latter requires explicit loading of xintgcd). The package xinttools does not handle computations but provides some useful utilities.

There are three expression parsers and two subsidiary ones. They all admit comma separated expressions, and will then output a comma separated list of results.

- \xinttheiiexpr ... \relax does exact computations *only on integers*. The forward slash / does the rounded integer division (// does truncated division, and /: is the associated modulo). There are two square root extractors sqrt and sqrtr for truncated and rounded square roots. Scientific notation 6.02e23 is *not* accepted on input, one needs to wrap it as num(6.02e23) which will convert to an integer notation 602000000000000000000000.
- \xintthefloatexpr ... \relax does computations with a given precision P, as specified via a prior assignment \xintDigits:=P;. The default is P=16 digits. An optional argument controls the precision for *formatting the output* (this is not the precision of the computations themselves). The four basic operations and the square root realize *correct rounding*.[3]
- \xinttheexpr ... \relax handles integers, decimal numbers, numbers in scientific notation and fractions. The algebraic computations are done *exactly*. The sqrt function is available and works according to the \xintDigits precision or according to its second optional argument.

Currently, the sole available non-algebraic function is the square root extraction sqrt. It is allowed in \xintexpr..\relax but naturally can't return an *exact* value, hence computes as if it was in \xintfloatexpr..\relax. The power operator ^ (equivalently **) works with integral exponents only in \xintiiexpr (non-negative) and \xintexpr (negative exponents allowed, of course) and also with half-integral exponents in \xintfloatexpr (it proceeds via an integral power followed by a square-root extraction).

Two derived parsers:

- \xinttheiexpr ... \relax does all computations like \xinttheexpr ... \relax but rounds the result to the nearest integer. With an optional positive argument [D], the rounding is to the nearest fixed point number with D digits after the decimal mark.
- \xinttheboolexpr ... \relax does all computations like \xinttheexpr ... \relax but converts the result to 1 if it is not zero (works also on comma separated expressions). See also the booleans \xintifboolexpr, \xintifbooliiexpr, \xintifboolfloatexpr (which do not handle comma separated expressions).

Here is a (partial) list of the recognized symbols:

- the comma (to separate distinct computations or arguments to a function),

[3] when the inputs are already floating point numbers with at most P-digits mantissas.

- parentheses,
- infix operators +, -, *, /, ^ (or **),
- branching operators (x)?{x non zero}{x zero}, (x)??{x<0}{x=0}{x>0},
- boolean operators !, && or 'and', || or 'or',
- comparison operators = (or ==), <, >, <=, >=, !=,
- factorial post-fix operator !,
- " for hexadecimal input (uppercase only; package xintbinhex must be loaded additionally to xintexpr),
- functions num, reduce, abs, sgn, frac, floor, ceil, sqr, sqrt, sqrtr, float, round, trunc, mod, quo, rem, gcd, lcm, max, min, \`+\`, \`*\`, not, all, any, xor, if, ifsgn, even, odd, first, last, reversed, bool, togl, factorial, binomial, pfactorial,
- functions with dummy variables add, mul, seq, subs, rseq, iter, rrseq, iterr.

See section 4 (and subsection 10.2) for the complete syntax, as well as subsection 10.3 which contains examples illustrating further some features which were added at the time of release 1.1 2014/10/28.

The normal mode of operation of the parsers is to unveil the parsed material token by token. This means that all elements may arise from expansion of encountered macros (or active characters). For example a closing parenthesis does not have to be immediately visible, it may arise later from expansion. This general behavior has exceptions, in particular constructs with dummy variables need immediately visible balanced parentheses and commas. The expansion stops only when the ending \relax has been found; it is then removed from the token stream, and the final computation result is inserted.

Release 1.2 added the (pseudo) functions qint, qfrac, qfloat to allow swallowing in one-go all digits of a big number, fraction, or float, skipping the token by token expansion.

Here is an example of a computation:

```
\xinttheexpr (31.567^2 - 21.56*52)^3/13.52^5\relax
```

-1936508797861911919204831/4517347060908032[-8]

This illustrates that \xinttheexpr..\relax does its computations *exactly*. The same example as a floating point evaluation:

```
\xintthefloatexpr (31.567^2 - 21.56*52)^3/13.52^5\relax
```

-4.286827582100044

Again, all computations done by \xinttheexpr..\relax are completely exact. Thus, very quickly very big numbers are created (and computation times increase, not to say explode if one goes into handling numbers with thousands of digits). To compute something like 1.23456789^10000 it is thus better to opt for the floating point version:

```
\xintthefloatexpr 1.23456789^10000\relax
```

1.411795173056392e915

(we can deduce that the exact value has 80000+916=80916 digits). A bigger example (the scope of the assignment to \xintDigits is limited by the braces):

```
{\xintDigits:=24; \xintthefloatexpr 1.23456789123456789^123456789\relax }
```

1.90696640042856610942910e11298145 (<- notice the size of the power of ten: this surely largely exceeds your pocket calculator abilities).

It is also possible to do some computer algebra like evaluations (only numerically though):

```
\xinttheiiexpr add(i^5, i=100..200)\relax\par
\noindent\xinttheexpr reduce(add(x/(x+1), x = 1000..1014))\relax
```

10665624997500
4648482709767835886400149017599415343/310206597612274815392155150733157360

Were it not for the reduce function, the latter fraction would not have been obtained in reduced terms:

> By default, the basic operations on fractions are not followed in an automatic manner by reduction to smallest terms: A/B multiplied by C/D returns AC/BD, A/B added to C/D returns

(AD+BC)/BD except if either B divides D or D divides B.

Make sure to read subsection 2.5, section 4 and also section 10.

1.3 Printing big numbers on the page

When producing very long numbers there is the question of printing them on the page, without going beyond the page limits. In this document, I have most of the time made use of these macros (not provided by the package:)

```
\def\allowsplits #1{\ifx #1\relax \else #1\hskip 0pt plus 1pt\relax
                    \expandafter\allowsplits\fi}%
\def\printnumber #1{\expandafter\allowsplits \romannumeral-`0#1\relax }%
% \printnumber thus first ``fully'' expands its argument.
```

It may be used like this:

```
\printnumber {\xintiiQuo{\xintiiPow {2}{1000}}{\xintiFac{100}}}
```

or as \printnumber\mybiginteger or \printnumber{\mybiginteger} if \mybiginteger was previously defined via a \newcommand, a \def or an \edef.

An alternative is to suitably configure the thousand separator with the numprint package (see footnote 7. This will not allow linebreaks when used in math mode; I also tried siunitx but even in text mode could not get it to break numbers accross lines). Recently I became aware of the seqsplit package[4] which can be used to achieve this splitting accross lines, and does work in inline math mode (however it doesn't allow to separate digits by groups of three, for example).

1.4 Randomly chosen examples

Here are some examples of use of the package macros. The first one uses only the base module xint, the next one requires the xintfrac package, which deals with decimal numbers, scientific numbers (lowercase e), and also fractions (it loads automatically xint). Then some examples with expressions, which require the xintexpr package (it loads automatically xintfrac). And finally some examples using xintseries, xintgcd which are among the extra packages included in the xint distribution.

The printing of the outputs will either use a custom \printnumber macro as described in the previous section, or sometimes the \np command from the numprint package (see footnote 7).

- 123456^{99}:
  \xintiiPow {123456}{99}: 1147381811662665566332733300084545867470254804234261029758895452
  4373590894697032027622647054266320583469027086822116813341525003240387627761689532221176
  3429587203376221608860691585075716801971671071208769703353650737748777873778498781606749
  9997983665812517232752154970541659566738491153332674854107560766971890623518995832377826
  3699981109532393993235189992220564587812701495877679143167735437253858445948715594121519
  7416398666125896983737258716757394949435520170950261865801665199030718414432231169678376
  96
- 1234/56789 with 1500 digits after the decimal point:
  \xintTrunc {1500}{1234/56789}\dots: 0.021729560302171195125816619415731919914067865255592
  5273732589057740055292398175703041081899663667259504481501699272746482593460001056542640
  3000581098452165031960414869076757822817799221680254978957192414023842645582771311345507
  0524221240028878832168201588335769251087358467308809804715701984539259363609149659264998
  5032312595749176777192766204722745602141259751008117769286305446477310746799556252091073
  9755938650090686576625754987761714416524326894292908837979186109986088855236049234887037
  9827079187870890489355332898976914543309443730299882019405166493511067284157143108700628

[4] http://ctan.org/pkg/seqsplit

6428709785345753579038194016446847100670904576590536899751712479529486344186374121748930
2505766961911637817182905140079945059782704396978288048741833805842680800859321347444047
2626741094225994470760182429695891810033633274049551849830072725351740653999894345735969
9941890154783496803958513092324217718220077831974502104280758597615735441722868865449294
7577875997112116783179841166423074891264153269119019528429801546074063639085034073500149
6768740425082322280723379527725439785874024899188223071369455352268925320044374790892602
4406134990931342337424501223828558347567310570709116202081389001391114476395076511296201
7292081212910951064466710102308545669055626970011798059483350648893271584285689129937135
7129021465424642096180598355315289932909542340946310024828752047051365581362587825106974
9423303808836218281709485992005494021729560302171195125816619415731919914067865255595273
73258905774005529239817570304108189966366 7...

- 0.99^{-100} with 200 (+1) digits after the decimal point.
  `\xinttheiexpr [201] .99^-100\relax`: 2.731999026429026003846671721257837435505351642938572070833430572508246455518705343044814301378480614036805562476501925307034269685489153194616612271015920671913840348851485747943086470963920731779793038
  Notice that this is rounded, hence we asked `\xinttheiexpr` for one additional digit. To get a truncated result with 200 digits after the decimal mark, we should have issued `\xinttheexpr trunc(.99^-100,200)\relax`, rather.

> The fraction `0.99^-100`'s denominator is first evaluated *exactly* (*i.e.* the integer `99^100` is evaluated exactly and then used to divide the suitable power of ten to get the requested digits); for some longer inputs, such as for example `0.7123045678952^-243`, the exact evaluation before truncation would be costly, and it is more efficient to use floating point numbers:
>
> ```
> \xintDigits:=20; \np{\xintthefloatexpr .7123045678952^-243\relax}
> ```
>
> $6.342,022,117,488,416,127,3 \times 10^{35}$
>
> Side note: the exponent `-243` didn't have to be put inside parentheses, contrarily to what happens with some professional computational software. ;-)

- 200!:
  `\xinttheiiexpr 200!\relax`: 788657867364790503552363213932185062295135977687173263294742533244359449963403342920304284011984623904177212138919638830257642790242637105061926624952829931113462857270763317237396988943922445621451664240254033291864131227428294853277524242407573903240321257405579568660226031904170324062351700858796178922222789623703897374720000000000000000000000000000000000000000000000000

- 2000! as a float. As xintexpr does not handle exp/log so far, the computation is done internally without the Stirling formula, by repeated multiplications truncated suitably:
  `\xintDigits:=50;`
  `\xintthefloatexpr 2000!\relax`: 3.3162750924506332411753933805763240382811172081058e5735

- Just to show off (again), let's print 300 digits (after the decimal point) of the decimal expansion of 0.7^{-25}:[5]

```
% % in the preamble:
% \usepackage[english]{babel}
% \usepackage[autolanguage,np]{numprint}
% \npthousandsep{,\hskip 1pt plus .5pt minus .5pt}
% \usepackage{xintexpr}
% in the body:
\np {\xinttheexpr trunc(.7^-25,300)\relax}\dots
```

[5] the `\np` typesetting macro is from the numprint package.

7,456.739,985,837,358,837,609,119,727,341,853,488,853,339,101,579,533,584,812,792,108,
394,305,337,246,328,231,852,818,407,506,767,353,741,490,769,900,570,763,145,015,081,436,
139,227,188,742,972,826,645,967,904,896,381,378,616,815,228,254,509,149,848,168,782,309,
405,985,245,368,923,678,816,256,779,083,136,938,645,362,240,130,036,489,416,562,067,450,
212,897,407,646,036,464,074,648,484,309,937,461,948,589...

This computation is with \xinttheexpr from package xintexpr, which allows to use standard infix notations and function names to access the package macros, such as here trunc which corresponds to the xintfrac macro \xintTrunc. Regarding this computation, please keep in mind that \xinttheexpr computes *exactly* the result before truncating. As powers with fractions lead quickly to very big ones, it is good to know that xintexpr also provides \xintthefloatexpr which does computations with floating point numbers.

- Computation of a Bezout identity with 7^200-3^200 and 2^200-1: (with xintgcd)

```
\xintAssign \xintBezout {\xinttheiiexpr 7^200-3^200\relax}
                        {\xinttheiiexpr 2^200-1\relax}\to\A\B\U\V\D
$\U\times(7^{200}-3^{200})+\xintiOpp\V\times(2^{200}-1)=\D$
```

$-220045702773594816771390169652074193009609478853\times(7^{200}-3^{200})+143258949362763693185913068326832046547441686338771408915838167247899192113282011912746243715803917775497685719128769314424060506699145633614320567769677489\times(2^{200}-1)=1803403947125$

- The Euclide algorithm applied to 22,206,980,239,027,589,097 and 8,169,486,210,102,119,257: (with xintgcd)[6]

\xintTypesetEuclideAlgorithm {22206980239027589097}{8169486210102119257}

$$
\begin{aligned}
22206980239027589097 &= 2\times 8169486210102119257 + 5868007818823350583\\
8169486210102119257 &= 1\times 5868007818823350583 + 2301478391278768674\\
5868007818823350583 &= 2\times 2301478391278768674 + 1265051036265813235\\
2301478391278768674 &= 1\times 1265051036265813235 + 1036427355012955439\\
1265051036265813235 &= 1\times 1036427355012955439 + 228623681252857796\\
1036427355012955439 &= 4\times 228623681252857796 + 121932630001524255\\
228623681252857796 &= 1\times 121932630001524255 + 106691051251333541\\
121932630001524255 &= 1\times 106691051251333541 + 15241578750190714\\
106691051251333541 &= 6\times 15241578750190714 + 15241578750189257\\
15241578750190714 &= 1\times 15241578750189257 + 1457\\
15241578750189257 &= 10460932567048\times 1457 + 321\\
1457 &= 4\times 321 + 173\\
321 &= 1\times 173 + 148\\
173 &= 1\times 148 + 25\\
148 &= 5\times 25 + 23\\
25 &= 1\times 23 + 2\\
23 &= 11\times 2 + 1\\
2 &= 2\times 1 + 0
\end{aligned}
$$

- $\sum_{n=1}^{500}(4n^2-9)^{-2}$ with each term rounded to twelve digits, and the sum to nine digits:

```
\def\coeff #1{\xintiRound {12}{1/\xintiiSqr{\the\numexpr 4*#1*#1-9\relax }[0]}}
\xintRound {9}{\xintiSeries {1}{500}{\coeff}[-12]}
```

0.062366080

The complete series, extended to infinity, has value $\frac{\pi^2}{144}-\frac{1}{162}=$ 0.062,366,079,945,836,595, 346,844,45...[7] I also used (this is a lengthier computation than the one above) xintseries

[6] this example is computed tremendously faster than the other ones, but we had to limit the space taken by the output hence picked up rather small big integers as input. [7] This number is typeset using the numprint package, with \npthousandsep {,\hskip 1pt plus .5pt minus .5pt}. But the breaking across lines works only in text mode. The number itself was (of course...) computed initially with xint, with 30 digits of π as input. See how xint may compute π from scratch.

to evaluate the sum with 100,000 terms, obtaining 16 correct decimal digits for the complete sum. The coefficient macro must be redefined to avoid a \numexpr overflow, as \numexpr inputs must not exceed $2^{31}-1$; my choice was:

```
\def\coeff #1%
{\xintiRound {22}{1/\xintiiSqr{\xintiiMul{\the\numexpr 2*#1-3\relax}
                                          {\the\numexpr 2*#1+3\relax}}[0]}}
```

- Computation of $2^{999,999,999}$ with 24 significant figures:

  ```
  \numprint{\xintFloatPow [24]{2}{999999999}}
  ```

  $2.306,488,000,584,534,696,558,06 \times 10^{301,029,995}$

  where the numprint package was used (footnote 7), directly in text mode (it can also naturally be used from inside math mode). xint provides a simple-minded \xintFrac typesetting macro,[8] which is math-mode only:

  ```
  $\xintFrac{\xintFloatPow [24]{2}{999999999}}$
  ```

  $230648800058453469655806 \cdot 10^{301029972}$

  The exponent differs, but this is because \xintFrac does not use a decimal mark in the significand of the output. Admittedly most users will have the need of more powerful (and customizable) number formatting macros than \xintFrac. [9] We have already mentioned \numprint which is used above, there is also \num from package siunitx. The raw output from

  ```
  \xintFloatPow [24]{2}{999999999}
  ```

  is 2.30648800058453469655806e301029995.

- As an example of nesting package macros, let us consider the following code snippet within a file with filename myfile.tex:

  ```
  \newwrite\outstream
  \immediate\openout\outstream \jobname-out\relax
  \immediate\write\outstream {\xintiiQuo{\xintiiPow{2}{1000}}{\xintiFac{100}}}
  % \immediate\closeout\outstream
  ```

  The tex run creates a file myfile-out.tex, and then writes to it the quotient from the euclidean division of 2^{1000} by 100!. The number of digits is \xintLen{\xintiiQuo{\xintiiPow{2}{1000}}{\xintiFac{100}}} which expands (in two steps) and tells us that $[2^{1000}/100!]$ has 144 digits. This is not so many, let us print them here: 114813249641507505482278393872551066259805517784186172883663478065826541894704737970419535798876630484358265060061503749531707793118627774829601.

1.5 More examples, some quite elaborate, within this document

- The utilities provided by xinttools (section 6), some completely expandable, others not, are of independent interest. Their use is illustrated through various examples: among those, it is shown in subsection 6.30 how to implement in a completely expandable way the Quick Sort algorithm and also how to illustrate it graphically. Other examples include some dynamically constructed alignments with automatically computed prime number cells: one using a completely expandable prime test and \xintApplyUnbraced (subsection 6.13), another one with \xintFor* (subsection 6.23).

- One has also a computation of primes within an \edef (subsection 6.15), with the help of \xintiloop. Also with \xintiloop an automatically generated table of factorizations (subsection 6.17).

- The code for the title page fun with Fibonacci numbers is given in subsection 6.24 with \xintFor* joining the game.

[8] Plain TeX users of xint have \xintFwOver. [9] There should be a \xintFloatFrac, but it is lacking.

- The computations of π and log 2 (subsection 13.11) using xint and the computation of the convergents of e with the further help of the xintcfrac package are among further examples.
- Also included, during the explanation of the \xintfloatexpr syntax, an expandable implementation of the Brent-Salamin algorithm for evaluating π.
- There is also an example of an interactive session, where results are output to the log or to a file.
- The functionalities of xintexpr are illustrated with various examples, and in locations such as in subsection 4.6 and subsection 4.5.

Almost all of the computational results interspersed throughout the documentation are not hard-coded in the source file of this document but are obtained via the expansion of the package macros during the TeX run.[10]

2 The xint bundle

2.1 Characteristics

The main characteristics are:

1. exact algebra on arbitrarily big numbers, integers as well as fractions,
2. floating point variants with user-chosen precision,
3. implemented via macros compatible with expansion-only context,
4. and with a parser of infix operations implementing features such as dummy variables, and coming in various incarnations depending on the kind of computation desired: purely on integers, on integers and fractions, or on floating point numbers.

`Arbitrarily big' currently means with less than about 19950 digits: the maximal number of digits for addition is at 19968 digits, and it is 19959 for multiplication.

Integers with only 10 digits and starting with a 3 already exceed the TeX bound; and TeX does not have a native processing of floating point numbers (multiplication by a decimal number of a dimension register is allowed --- this is used for example by the pgf basic math engine.)

TeX elementary operations on numbers are done via the non-expandable *\advance*, *\multiply*, and *\divide* assignments. This was changed with ε-TeX's \numexpr which does expandable computations using standard infix notations with TeX integers. But ε-TeX did not modify the TeX bound on acceptable integers, and did not add floating point support.

The bigintcalc package by Heiko Oberdiek provided expandable macros (using some of \numexpr possibilities, when available) on arbitrarily big integers, beyond the TeX bound.[11] The present package does this again, using more of \numexpr (xint requires the ε-TeX extensions) for higher speed, and also on fractions, not only integers. Arbitrary precision floating points operations are a derivative, and not the initial design goal.[12, 13]

[10] The CPU of my computer hates me for all those re-compilations after changing a single letter in the LaTeX source, which require each time to do all the zillions of evaluations contained in this document... [11] This package does not provide a parser of expressions on big integers like \xintiiexpr...\relax. One can use package bnumexpr to associate the bigintcalc macros with an expression parser. [12] currently (1.08), the only non-elementary operation implemented for floating point numbers is the square-root extraction; no signed infinities, signed zeroes, NaN's, error traps..., have been implemented, only the notion of 'scientific notation with a given number of significant figures'. [13] multiplication of two floats with P=\xinttheDigits digits is first done exactly then rounded to P digits, rather than using a specially tailored multiplication for floating point numbers which would be more efficient (it is a waste to evaluate fully the multiplication result with 2P or 2P-1 digits.)

The LaTeX3 project has implemented expandably floating-point computations with 16 significant figures (l3fp), including special functions such as exp, log, sine and cosine.[14]

More directly related to the xint bundle, there is the promising new version of the l3bigint package. It was still in development a.t.t.o.w (2015/10/09, no division yet) and is part of the experimental trunk of the LaTeX3 Project. It is devoted to expandable computations on big integers with an associated expression parser. Its author (Bruno Le Floch) succeeded brilliantly into implementing expandably the Karatsuba multiplication algorithm and he achieves *sub-quadratic growth for the computation time*. This shows up very clearly with numbers having more than one thousand digits (up to the maximum which a.t.t.o.w was at 8192 digits).

I report here briefly on a quick comparison, although as l3bigint is work in progress, the reported results could well have to be modified soon. The test was on a comparison of \bigint_eval:n {#1*#2} from the l3bigint as available in September 2015, on one hand, and on the other hand \xinttheiiexpr #1*#2\relax from xintexpr 1.2 (rather than directly \xintiiMul, to be fairer to the parsing time induced by use of \bigint_eval:n) and the computations were done with #1=#2=9999888877999988877...repeated.... I observed:

- \xintiiexpr's multiplication appears slightly faster (about 1.5x or 2x to give an average order of magnitude) up to about 900 digits,
- at 1000 digits, l3bigint runs between 15% and 25% faster,
- then its sub-quadratic growth shows up, and at 8000 digits I observed it to be about 7.5x--9x faster (I got quite varying results depending on which computer the trial was conducted). Its computation time increased from 1000 digits to 8000 digits by a factor smaller than 30, whereas for \xintiiexpr it was a factor only slightly inferior to 200 (225 on my laptop) ... Karatsuba multiplication brilliantly pays off !
- One observes the transition at the powers of two for the l3bigint algorithm, for example I observed l3bigint to be 3.5x--4x faster at 4000 digits but only 3x--3.5x faster at 5000 digits.

Once one accepts a small overhead, one can on the basis of the lengths decide for the best algorithm to use, and it is tempting viewing the above to imagine that some mixed approach could combine the best of both. But again all this is a bit premature as both packages may still evolve further.

Anyhow, all this being said, even the superior multiplication implementation from l3bigint takes of the order of seconds on my laptop for a single multiplication of two 5000-digits numbers. Hence it is not possible to do routinely such computations in a document. I have long been thinking that without the expandability constraint much higher speeds could be achieved, but perhaps I have not given enough thought to sustain that optimistic stance.[15]

I remain of the opinion that if one really wants to do computations with *thousands* of digits, one should drop the expandability requirement. Indeed, as clearly demonstrated long ago by the pi computing file by D. Roegel one can program TeX to compute with many digits at a much higher speed than what xint achieves: but, direct access to memory storage in one form or another seems a necessity for this kind of speed and one has to renounce at the complete expandability.[16] [17]

2.2 Floating point evaluations

Floating point macros are provided by package xintfrac to work with a given arbitrary precision P. The default value is P = 16 meaning that the significands of the produced (non-zero) numbers have

[14] at the time of writing (2014/10/28) the l3fp (exactly represented) floating point numbers have their exponents limited to ±9999. [15] The apnum package implements (non-expandably) arbitrary precision fixed point algebra and (v1.6) functions exp, log, sqrt, the trigonometrical direct and inverse functions. [16] 2015/09/15: as I said the latest developments on the l3bigint side do not really modify this conclusion, because the computations remain extremely slow compared to what one can do in other programming structures. Another remark one could do is that it would be tremendously easier to enhance ε-TeX than it is to embark into writing hundreds of lines of sometimes very clever TeX macro programming. [17] The LuaTeX project possibly makes endeavours such as xint appear even more insane that they are, in truth.

16 decimal digits.[18] The syntax to set the precision to P is

\xintDigits:=P;

The value is local to the group or environment (if using LaTeX). To query the current value use \xinttheDigits.

Most floating point macros accept an optional first argument [P] which then sets the target precision and replaces the \xintDigits assigned value.[19]

\xintfloatexpr...\relax also admits an optional argument [Q] but it has an altogether different meaning: the computations are always done with the prevailing \xinttheDigits precision and the optional argument says to round the final result to Q digits of precision. This makes sense only if Q<\xinttheDigits and is intended to clean up the result from dubious last digits.

Changed → with 1.2

The maximal allowed value for P in a \xintDigits:=P; assignment is 32767. It was even possible earlier to use bigger P's as optional argument to \xintFloat for a one-short conversion to a gigantic float. However since release 1.2 this can't work in complete generality: a fractional input will trigger internally the division macro which can't handle arguments of more than about 19900 digits. For example (tested 2015/11/07 with 1.2b) \message{\xintFloat [19942]{1/7}} works but not \message{\xintFloat [19943]{1/7}}. On the other hand \xintFloat [50000]{1} does work (slowly..), but there isn't much one can do afterwards with it...

More reasonably, working with significands of 24, 32, 48, 64, or even 80 digits is well within the reach of the package.

> The IEEE 754[20] requirement of *correct rounding* for addition, subtraction, multiplication, division and square root is achieved (in arbitrary precision) by the macros of xintfrac hence also by the infix operations of the \xintfloatexpr...\relax expressions: this means that for operands given with at most P significant digits (and arbitrary exponents) the output coincides exactly with the rounding of an exact evaluation (baring overflow or underflow).
>
> The rounding mode is ``round to nearest, ties away from zero''. It is not customizable.
>
> Currently xintfrac has no notion of NaNs or signed infinities or signed zeroes, but this is intended for the future.

Currently, the only non-elementary operation is the square root. The elementary transcendantal functions are not yet implemented. The power function accepts integer and half-integer exponents.[21]

The maximal theoretically allowed exponent is currently set at 2147483647 (the minimal exponent is its opposite) which is the maximal number handled by TeX. But this means that overflow or underflow are detected only via low-level \numexpr arithmetic overflows which are basically unrecoverable. Besides there are some border effects as the routines need to add or subtract lengths of numbers from exponents, possibly triggering the low-level overflows. In the future not only the Precision but also the maximal and minimal exponents Emin and Emax will be specifiable by the user.

xintfrac is confronted with the problem that its float macros have to handle inputs which not only may have much more digits than the target float precision, but may even be fractions (which in a way means infinite precision). As fractions should be handled as first class citizens, the float macros should give the same result independently of whether an argument is given as A/B or as C/D as long as A/B=C/D as rational numbers. This was briefly the case when xintfrac was first released, but very shortly thereafter the fraction-to-float parsing (via \xintFloat) was modified for speed and as a result, currently this is not the case.[22]

[18] Currently in the cases when the rounding to nearest goes to the next power of ten, the result is 10.0....0eN with P-1 zeroes after decimal mark, hence a total of P+1, not P, digits. [19] the [P] must be repeated for the macro arguments, if the latter are also xintfrac macros with arguments of their own. [20] The IEEE 754-1985 standard was for hardware implementations of binary floating-point arithmetic with a specific value for the precision (24 bits for single precision, 53 bits for double precision). The newer IEEE 754-2008 (https://en.wikipedia.org/wiki/IEEE_floating_point) normalizes five basic formats, three binaries and two decimals (16 and 34 decimal digits) and discusses extended formats with higher precision. These standards are only indirectly relevant to libraries like xint dealing with arbitrary precision. [21] Half-integer exponents work only inside \xintfloatexpr...\relax expressions, not directly via the \xintFloatPower macro. [22] The author had forgotten that the initial situation had been quickly modified, and for some time this documentation wrongly asserted that the xintfrac basic floating point operations produced a value depending solely on the inputs as abstract fractions.

Next major release of xint packages will adopt a definitive model for floats, and it will have the property above of handling fractions intrinsically. Most probably this will go via first correctly rounding the inputs (inclusive of arbitrary fractions) to the target precision.

The current situation is that for a fraction $A/B\cdot 10^N$ its numerator and denominator are each first *truncated* (not rounded, but that would not change the issue) to P+2 digits of precision if P is the asked-for precision. Imagine that the original fraction was close to a boundary value between rounding up or rounding down to a P-float. Changing even a tiny bit the numerator and denominator moves the fraction, and as a result the rounding to a P-float may be altered.

If B = 1 there is no issue because truncating to P+2 digits then rounding to nearest P-float is like rounding directly to nearest P-float (that would not have applied if we had first rounded, not truncated, to a P+2-float). For A = 1 on the contrary the issue is already there. Here is an illustration of the problem:

```
Precision: \xinttheDigits\par
\xintFloat {1/1858798855725191} ("reduced" fraction)\par
\xintFloat {5379818246175219/10000000000000000980336298241829} ("big" fraction)\par
We check that the fractions are identical:\par
\xinttheexpr 1/1858798855725191 - 5379818246175219/10000000000000000980336298241829\relax\par
And we also examine more digits of the decimal expansion:\par
\xinttheexpr trunc(1/1858798855725191, 48)\relax\dots\par
But truncating the denominator of the "big" fraction to 18 digits leads to:\par
\xinttheexpr trunc(5379818246175219/10000000000000000900000000000000, 48)\relax\dots\par
The two values rounded to 16 significant places differ.\par
```

Precision: 16
5.379818246175218e-16 ("reduced" fraction)
5.379818246175219e-16 ("big" fraction)
We check that the fractions are identical:
0
And we also examine more digits of the decimal expansion:
0.000000000000000537981824617521847259688953307375...
But truncating the denominator of the "big" fraction to 18 digits leads to:
0.000000000000000537981824617521851581635784423033...
The two values rounded to 16 significant places differ.

Here is a summary of the situation regarding floats which was prevailing from release 1.08a to release 1.2e. Next, we will explain the new situation since 1.2f.

- As explained above the \xintFloat macro which converts an input to a P-float does the following in case of fraction A/B[N]: it truncates both A and B to P+2 digits say A′ and B′ then computed the correct rounding of A′/B′ to P digits. (not changed in 1.2f and 1.2g)
- The \xintFloatAdd, \xintFloatSub, \xintFloatMul, \xintFloatDiv routines each first rounded their inputs to P+2 digits (thus applying the previous item, but with P replaced by P+2).
- The \xintFloatPow and \xintFloatPower routines rounded their inputs to Q > P+2 digits, with Q a temporary increased precision for intermediate computations, then they rounded the end result to the asked-for P digits (in the case of a negative exponent the final rounding was applied to something of the 1/C shape, up to powers of ten naturally).
- The multiplication and division produced the correctly rounded value from the (P+2)-rounded inputs. Thus, not only did these routines produce the correctly rounded result for P-digits inputs, they actually achieved it for (P+2)-digits inputs (again, for fractional inputs, the initial rounding viciates any hope of having the final result be the correctly rounded exact one).
- For addition and subtraction having now to deal with $A.10^M \pm B.10^N$, with A and B having (P+2) digits, the rule was that if $M-N \geqslant P+4$, then B was dropped altogether, if $N-M \geqslant P+4$ then A was

dropped, else the operation was done exactly then rounded to P digits. But in those cases where one of the summands was dropped, correct rounding was not guaranteed, because say A having P+2 digits could be at a mid-point between two P-floats, thus neglecting B (however small it may be) could viciate the final rounding. Also, when adding B = 0 to A, as A is already the P+2 rounded original input, the final P-digit value could end up not being the correct rounding of the original input (rounding to nearest is not transitive; this is independent of the issues of handling fractions as mentioned above).

For 1.2f I have decided that the two digits added on input to the asked-for precision by the four basic operations do not make much sense after all. Especially as since 1.2 the basic core operations are handled by blocks of eight digits, this procedure was adding a systematic overhead for inputs already of the default 16-digits precision, forcing computations as if for 24 digits. Besides, when used in \xintfloatexpr, most of the time the inputs will already be results of earlier computations hence have P digits. It appears wasteful to extend them by two zeros prior to each operation.

Changed (1.2f) New with 1.2f

Thus, 1.2f adopts the policy that the inputs will always be rounded first to P-floats, not (P+2)-floats. This does not change the behaviour for inputs being already P-floats. All four operations then and now produce the correctly rounded value. For coherence also the square root and the power macros now first round their argument to a P-float.[23] Also, 1.2f has implemented *correct rounding* (in arbitrary precision) for the square root extraction.

What is yet lacking from 1.2f (now 1.2g) is the improved handling via \xintFloat of fractions to achieve independence from the chosen representatives (which currently only applies if numerators and denominators are of lengths at most the target precision plus two -- the exponent is arbitrary).

Next major release will decide if this initial rounding of the inputs is kept; the more ambitious model would be for the basic operations to compute the correct rounding of the value corresponding to the original inputs, whatever their original lengths[24], or nature[25]. This is definitely feasible as xintfrac knows how to compute exactly, but the question is how efficiently though. I don't think that this will be the choice made.

2.3 Expansion matters

2.3.1 Generalities about expandability in TeX

TeX is a macro language, and ``expansion'' is thus a crucial aspect, which will be quite unfamiliar to most everyone with standard knowledge of the usual programming languages. The whole of xint is about *expandably* implementing arithmetic computations. What does that mean ?

LaTeX users are familiar with counters, and its command \setcounter. The first argument is the name of the counter, the second argument is a number (in the TeX sense of being acceptable in \count assignments), or more generally something which will expand to a number. For example:

```
\newcounter{Foo}
\newcommand{\Bar}{17}
\setcounter{Foo}{\Bar}
\addtocounter{Foo}{\Bar}
\arabic{Foo}
```

works and produces 34. But imagine we have some macro \Double which accepts a numerical string of digits, and outputs its double as a string of digits (or perhaps some count with this value). Can we do this:

```
\setcounter{Foo}{\Double{\Bar}}     ?
```

The answer depends on the *expandability* of \Double. Perhaps \Double will do a \newcommand internally? or perhaps it used the TeX primitives \advance or \multiply? then it is *not* expandable in

[23] The power macros do not aim at correct rounding, currently. But the change fixes the issue that there was a possible difference between the evaluation of x*x and x^2 or 1/x vs x^-1 in the \xintfloatexpr parser due to the fact that in the latter cases x was initially rounded to P+3 digits but in the former cases only to P+2 digits. [24] The MPFR library http://www.mpfr.org/ implements this ... [25] ... but it does not know fractions!

this context where TEX is seeking to do a number assignment (which is to what the LATEX \setcounter boils down in the end). This does not mean that expansion does not apply, but something completely different: expansion produces ``tokens'' which TEX does not accept in such a context of assigning a value to a \count.

Some users of LATEX know about the TEX primitive \edef. Another way to test expandability of \Double is to try \edef\test{\Double{\Bar}}: the criterion is that \Double will expand, do its stuff, and clean up everything and only leave the result of its action on \Bar. Thus typically if \Double does a \newcommand (or rather a \def), then this will not be the case. The \newcommand or \def simply does *not* get executed inside an \edef! This is not completely equivalent to the earlier context, because when TEX seeks to build a number it has additional special constructs, like " to prefix hexadecimal inputs, and thus the behavior is not exactly the same in an \edef, but I am just trying to give a gist here of what happens.

The little paradox is that TEX from the start always required expandability when doing assignments to count registers, or dimen registers, ..., but basic arithmetic was provided via \advance, \multiply, and \divide primitives which are *not* expandable.[26] For example, something like

```
\setcounter{Foo}{\count0=\Bar\relax \multiply\count0 by 2
                \advance\count0 by 15 \the\count0 }
```

although not creating errors produces only non-sense.

Since 1999, ε-TEX has extended TEX with expandable arithmetic. Thus nowadays[27] one would simply do

```
\setcounter{Foo}{\numexpr 2*\Bar+15\relax}
```

But we can not do, for example, 98765*67890 inside \numexpr, because the result 6705155850 exceeds the TEX bound of 2147483647.

2.3.2 Full expansion of the first token

The whole business of xint is to build upon \numexpr and handle arbitrarily large numbers. Each basic operation is thus done via a macro: \xintiiAdd, \xintiiSub, \xintiiMul, \xintiiDivision. In order to handle more complex operations, it must be possible to nest these macros.[28] But we saw already that an expandable macro can not do a \newcommand or \def, and it can't do either an \edef. But the macro must expand its arguments to find the digits it is supposed to manipulate. TEX provides a tool to do the job of (expandable !) repeated expansion of the first token found until hitting something non expandable, such as a digit, a \def token, a brace, a \count token, etc... is found. A space token also will stop the expansion (and be swallowed, contrarily to the non-expandable tokens).

By convention in this manual *f*-expansion (``full expansion'' or ``full first expansion'') will be this TEX process of expanding repeatedly the first token seen. For those familiar with LATEX3 (which is not used by xint) this is what is called in its documentation full expansion (whereas expansion inside \edef would be described I think as ``exhaustive'' expansion).

Most of the package macros, and all those dealing with computations[29], are expandable in the strong sense that they expand to their final result via this *f*-expansion. This will be signaled in ★ their descriptions via a star in the margin.

These macros not only have this property of *f*-expandability, they all begin by first applying *f* *f*-expansion to their arguments. Again from LATEX3's conventions this will be signaled by a margin annotation next to the description of the arguments.

---

[26] This is not to say that expandable arithmetic was not possible, it is possible and has been done via the recording of the carries of digit arithmetic in many macros and usage of some tools provided by the TEX language, but this is very cumbersome and slow (even in the TEX context). [27] I do not discuss here the calc package. It does surgery inside \setcounter to make \setcounter {Foo}{2*\Bar +15} possible, but its parser is not expandable and is functional only inside macros calc knows about. [28] Actually this would not be really needed if the goal was only to implement the parsing of expressions: as the expression is scanned from left to right, there is only at any given time one operation to be done, hence it is not really absolutely mandatory for the macros implementing the basic operations to be nestable. This is however the path followed initially by xint. [29] except \xintXTrunc.

2.3.3 Summary of important expandability aspects

1. the macros *f*-expand their arguments, this means that they expand the first token seen (for each argument), then expand, etc..., until something un-expandable such as a digit or a brace is hit against. This example
```
\def\x{98765}\def\y{43210} \xintiiAdd {\x}{\x\y}
```
is *not* a legal construct, as the \y will remain untouched by expansion and not get converted into the digits which are expected by the sub-routines of \xintiiAdd. It is a \numexpr which will expand it and an arithmetic overflow will arise as 9876543210 exceeds the TeX bounds. The same would hold for \xintAdd.

   To the contrary \xinttheiiexpr and others have no issues with things such as \xinttheiiexpr \x+\x\y\relax.

2. using \if...\fi constructs *inside* the package macro arguments requires suitably mastering TeXniques (\expandafter's and/or swapping techniques) to ensure that the *f*-expansion will indeed absorb the \else or closing \fi, else some error will arise in further processing. Therefore it is highly recommended to use the package provided conditionals such as \xintifEq, \xintifGt, \xintifSgn, \xintifOdd..., or, for LaTeX users and when dealing with short integers the etoolbox[30] expandable conditionals (for small integers only) such as \ifnumequal, \ifnumgreater, ... Use of *non-expandable* things such as \ifthenelse is impossible inside the arguments of xint macros.

   One can use naive \if..\fi things inside an \xinttheexpr-ession and cousins, as long as the test is expandable, for example
```
\xinttheiexpr\ifnum3>2 143\else 33\fi 0^2\relax→2044900=1430^2
```

3. after the definition \def\x {12}, one can not use -\x as input to one of the package macros: the *f*-expansion will act only on the minus sign, hence do nothing. The only way is to use the \xintOpp macro, or perhaps here rather \xintiOpp which does maintains integer format on output, as they replace a number with its opposite.

   Again, this is otherwise inside an \xinttheexpr-ession or \xintthefloatexpr-ession. There, the minus sign may prefix macros which will expand to numbers (or parentheses etc...)

4. With the definition
```
\def\AplusBC #1#2#3{\xintAdd {#1}{\xintMul {#2}{#3}}}
```
one obtains an expandable macro producing the expected result, not in two, but rather in three steps: a first expansion is consumed by the macro expanding to its definition. As the package macros expand their arguments until no more is possible (regarding what comes first), this \AplusBC may be used inside them: \xintAdd {\AplusBC {1}{2}{3}}{4} does work and returns 11/1[0].

   If, for some reason, it is important to create a macro expanding in two steps to its final value, one may either do:
```
\def\AplusBC #1#2#3{\romannumeral-`0\xintAdd {#1}{\xintMul {#2}{#3}}}
```
or use the *lowercase* form of \xintAdd:
```
\def\AplusBC #1#2#3{\romannumeral0\xintadd {#1}{\xintMul {#2}{#3}}}
```
and then \AplusBC will share the same properties as do the other xint `primitive' macros.

5. The \romannumeral0 and \romannumeral-`0 things above look like an invitation to hacker's territory; if it is not important that the macro expands in two steps only, there is no reason to follow these guidelines. Just chain arbitrarily the package macros, and the new ones will be completely expandable and usable one within the other.

   Since release 1.07 the \xintNewExpr command automatizes the creation of such expandable macros:

[30] http://www.ctan.org/pkg/etoolbox

```
\xintNewExpr\AplusBC[3]{#1+#2*#3}
```

creates the \AplusBC macro doing the above and expanding in two expansion steps.

6. In the expression parsers of xintexpr such as \xintexpr..\relax, \xintfloatexpr..\relax the contents are expanded completely from left to right until the ending \relax is found and swallowed, and spaces and even (to some extent) catcodes do not matter.

7. For all variants, prefixing with \xintthe allows to print the result or use it in other contexts. Shortcuts \xinttheexpr, \xintthefloatexpr, \xinttheiiexpr, ... are available.

2.4 Analogies and differences of \xintiiexpr with \numexpr

\xintiiexpr..\relax is a parser of expressions knowing only (big) integers. There are, besides the enlarged range of allowable inputs, some important differences of syntax between \numexpr and \xintiiexpr and variants:

- Contrarily to \numexpr, the \xintiiexpr parser will stop expanding only after having encountered (and swallowed) a *mandatory* \relax token.
- In particular, spaces between digits (and not only around infix operators or parentheses) do not stop \xintiiexpr, contrarily to the situation with numexpr: \the\numexpr 7 + 3 5\relax expands (in one step) to 105\relax, whereas \xintthe\xintiiexpr 7 + 3 5\relax expands (in two steps) to 42.
- Inside an \edef, expressions \xintiiexpr...\relax get fully evaluated, but to a private format which needs the prefix \xintthe to get printed or used as arguments to some macros; on the other hand expansion of \numexpr in an \edef occurs only if prefixed with \the or \number (or \romannumeral, or the expression is included in a bigger \numexpr which will be the one to have to be prefixed....)
- \the\numexpr or \number\numexpr expands in one step, but \xintthe\xintiiexpr needs two steps.
- The \xintthe prefix should not be applied to a sub \xintiiexpression, as this forces the parsers to gather again one by one the corresponding digits.
- Also worth mentioning is the fact that \numexpr -(1)\relax is illegal. But \xintiiexpr -(1)\relax is perfectly legal and gives the expected result (what else ?).

2.5 User interface

The next sections will explain the various inputs which are recognized by the package macros and the format for their outputs. Inputs have mainly five possible shapes:

1. expressions which will end up inside a \numexpr..\relax,
2. long integers in the strict format (no +, no leading zeroes, a count register or variable must be prefixed by \the or \number)
3. long integers in the general format allowing both - and + signs, then leading zeroes, and a count register or variable without prefix is allowed,
4. fractions with numerators and denominators as in the previous item, or also decimal numbers, possibly in scientific notation (with a lowercase e), and also optionally the semi-private A/B[N] format,
5. and finally expandable material understood by the \xintexpr parser.

Outputs are mostly of the following types:

1. long integers in the strict format,
2. fractions in the A/B[N] format where A and B are both strict long integers, and B is positive,
3. numbers in scientific format (with a lowercase e),
4. the private \xintexpr format which needs the \xintthe prefix in order to end up on the printed page (or get expanded in the log) or be used as argument to the package macros.

2.6 No declaration of variables for macros

There is no notion of a *declaration of a variable*, which would be needed to use the arithmetic macros.

To do a computation and assign its result to some macro \z, the user will employ the \def, \edef, or \newcommand (in LaTeX) as usual, keeping in mind that two expansion steps are needed, thus \edef is initially the main tool:

```
\def\x{1729728} \def\y{352827927} \edef\z{\xintiiMul {\x}{\y}}
\meaning\z
```

macro:->610296344513856

As an alternative to \edef the package provides \oodef which expands exactly twice the replacement text, and \fdef which applies *f*-expansion to the replacement text during the definition.

```
\def\x{1729728} \def\y{352827927} \oodef\w {\xintiiMul\x\y} \fdef\z{\xintiiMul {\x}{\y}}
\meaning\w, \meaning\z
```

macro:->610296344513856, macro:->610296344513856

☞ In practice \oodef is slower than \edef, except for computations ending in very big final replacement texts (thousands of digits). On the other hand \fdef appears to be slightly faster than \edef already in the case of expansions leading to only a few dozen digits.

xintexpr does provide an interface to declare and assign values to identifiers which can then be used in expressions: subsection 4.6.

2.7 Input formats

num *x* Some macro arguments are by nature 'short' integers, *i.e.* less than (or equal to) in absolute value 2,147,483,647. This is generally the case for arguments which serve to count or index something. They will be embedded in a \numexpr..\relax hence on input one may even use count registers or variables and expressions with infix operators. Notice though that -(..stuff..) is surprisingly not legal in the \numexpr syntax!

But xint is mainly devoted to big numbers; the allowed input formats for 'long numbers' and 'fractions' are:

f 1. the strict format is for some macros of xint which only *f*-expand their arguments. After this *f*-expansion the input should be a string of digits, optionally preceded by a unique minus sign. The first digit can be zero only if the number is zero. A plus sign is not accepted. -0 is not legal in the strict format. A count register can serve as argument of such a macro only if prefixed by \the or \number. Macros of xint such as \xintiiAdd with a double ii require this 'strict' format for the inputs. The macros such as \xintiAdd with a single i will apply the \xintNum normalizer described in the next item.

2. the macro \xintNum normalizes into strict format an input having arbitrarily many minus and plus signs, followed by a string of zeroes, then digits:

   \xintNum {+-+-+----++-++----00000000009876543210}=-9876543210

   Num *f* The extended integer format is thus for the arithmetic macros of xint which automatically parse their arguments via this \xintNum.[31]

Frac *f* 3. the fraction format is what is expected on input by the macros of xintfrac. It has two variants:

[31] A LaTeX \value {countername} is accepted as macro argument.

general: these are inputs of the shape A.BeC/D.EeF. Example:

```
\noindent\xintRaw{+--0367.8920280e17/-++278.289287e-15}\newline
\xintRaw{+--+1253.2782e++--3/---0087.123e---5}\par
```

-3678920280/278289287[31]
-12532782/87123[7]

Notice that the input process does not reduce fractions to smallest terms. Here are the rules of the format:[32]

- everything is optional, absent numbers are treated as zero, here are some extreme cases:
  `\xintRaw{}, \xintRaw{.}, \xintRaw{./1.e}, \xintRaw{-.e}, \xintRaw{e/-1}`

  0/1[0], 0/1[0], 0/1[0], 0/1[0], 0/1[0]
- AB and DE may start with pluses and minuses, then leading zeroes, then digits.
- C and F will be given to \numexpr and can be anything recognized as such and not provoking arithmetic overflow (the lengths of B and E will also intervene to build the final exponent naturally which must obey the TeX bound).
- the /, . (numerator and/or denominator) and e (numerator and/or denominator) are all optional components.
- each of A, B, C, D, E and F may arise from *f*-expansion of a macro.
- the whole thing may arise from *f*-expansion, however the /, ., and e should all come from this initial expansion. The e of scientific notation is mandatorily lowercased.

restricted: these are inputs either of the shape A[N] or A/B[N] (representing the fraction A/B times 10^N) where the whole thing or each of A, B, N (but then not / or [) may arise from *f*-expansion, A (after expansion) *must* have a unique optional minus sign and no leading zeroes, B (after expansion) if present *must* be a positive integer with no signs and no leading zeroes, [N] if present will be given to \numexpr. This format is parsed with smaller overhead than the general one, thus allowing more efficient nesting of macros as it is the one used on output (except for the floating macros). Any deviation from the rules above will result in errors.[33]

Frac *f* Notice that *, + and - contrarily to the / (which is treated simply as a kind of delimiter) are not acceptable within arguments of this type (see however subsection 2.9 for some exceptions).

4. the expression format is for inclusion in an \xintexpr...\relax, it uses infix notations, function names, complete expansion, recognizes decimal and scientific numbers, and is described in section 4 and subsection 10.2.[34]

Generally speaking, there should be no spaces among the digits in the inputs (in arguments to the package macros). Although most would be harmless in most macros, there are some cases where spaces could break havoc.[35] So the best is to avoid them entirely.

[32] Earlier releases were slightly more strict, the optional decimal parts B, E were not individually *f*-expanded. [33] With releases earlier than 1.2 the N had to be necessarily given as explicit digits, not some macro or expression expanded in \numexpr. From 1.2 to 1.2e an empty N was allowed, but 1.2f removed that. [34] Starting with release 1.2, the isolated dot "." is not legal anymore in expressions: there must be digits either before or after. [35] The \xintNum macro does not remove spaces between digits beyond the first non zero ones; however this should not really alter the subsequent functioning of the arithmetic macros, and besides, since xintcore 1.2 there is an initial parsing of the entire number, during which spaces will be gobbled. However I have not done a complete review of the legacy code to be certain of all possibilities after 1.2 release. One thing to be aware of is that \numexpr stops on spaces between digits (although it provokes an expansion to see if an infix operator follows); the exponent for \xintiiPow or the argument of the factorial \xintiFac are only subjected to such a \numexpr (there are a few other macros with such input types in xint). If the input is given as, say 1 2\x where \x is a macro, the macro \x will not be expanded by the \numexpr, and this will surely cause problems afterwards. Perhaps a later xint will force \numexpr to expand beyond spaces, but I decided that was not really worth the effort. Another immediate cause of problems is an input of the type \xintiiAdd {<space>\x }{\y }, because the space will stop the initial expansion; this will most certainly cause an arithmetic overflow later when the \x will be expanded in a \numexpr. Thus in conclusion, damages due to spaces are unlikely if only explicit digits are involved in the inputs, or arguments are single macros with no preceding space.

This is entirely otherwise inside an \xintexpr-ession, where spaces are ignored (except when they occur inside arguments to some macros, thus escaping the \xintexpr parser). See the documentation.

Num f Arithmetic macros of xint which parse their arguments automatically through \xintNum are signaled by a special symbol in the margin. This symbol also means that these arguments may contain to some extent infix algebra with count registers, see the section Use of count registers.

With xintfrac loaded the symbol Num f means that a fraction is accepted if it is a whole number in disguise; and for macros accepting the full fraction format with no restriction there is the Frac f corresponding symbol in the margin.

There are also some slighly more obscure expansion types: in particular, the \xintApplyInline and \xintFor* macros from xinttools apply a special iterated f-expansion, which gobbles spaces, to the non-braced items (braced items are submitted to no expansion because the opening brace stops $*f$ it) coming from their list argument; this is denoted by a special symbol in the margin. Some other macros such as \xintSum from xintfrac first do an f-expansion, then treat each found (braced or not) item (skipping spaces between such items) via the general fraction input parsing, this is $f \rightarrow *$ Frac f signaled as here in the margin where the signification of the * is thus a bit different from the previous case.

n, resp. o A few macros from xinttools do not expand, or expand only once their argument. This is also signaled in the margin with notations à la LaTeX3.

As the computations are done by f-expandable macros which f-expand their argument they may be chained up to arbitrary depths and still produce expandable macros.

Conversely, wherever the package expects on input a ``big'' integers, or a ``fraction'', f-expansion of the argument *must result in a complete expansion* for this argument to be acceptable.[36] The main exception is inside \xintexpr...\relax where everything will be expanded from left to right, completely.

2.8 Output formats

Package xintcore provides macros \xintiiAdd, \xintiiSub, \xintiiMul, \xintiiPow, which only f-expand their arguments and \xintiAdd, \xintiSub, \xintiMul, \xintiPow which normalize them first to strict format, thus have a bit of overhead. These macros always produce integers on output.

With xintfrac loaded \xintiiAdd, \xintiiSub, \xintiiMul, ... are not modified, and \xintiAdd, \xintiSub, \xintiMul, ... are only extended to the extent of accepting fraction inputs but they will be truncated to integers.[37] The output will be an integer.

Changed! →

The fraction handling macros from xintfrac are called \xintAdd, \xintSub, \xintMul, etc... they are *not* defined in the absence of xintfrac.

They produce on output a fractional number f=A/B[n] (which stands for (A/B)×10^n) where A and B are integers, with B positive, and n is a ``short'' integer (*i.e* less in absolute value than 2147483647.)

The output fraction is not reduced to smallest terms. The A and B may end in zeroes (*i.e*, n does not represent all powers of ten). The denominator B is always strictly positive. There is no + sign on output but only possibly a - at the numerator. The output will be expressed as a fraction even if the inputs are both integers.

- A macro \xintFrac is provided for the typesetting (math-mode only) of such a `raw' output. The command \xintFrac is not accepted as input to the package macros, it is for typesetting only (in math mode).
- \xintRaw prints the fraction directly as its internal representation A/B[n].

[36] this is not quite as stringent as claimed here, see subsection 2.9 for more details. [37] the power function does not accept a fractional exponent. Or rather, does not expect, and errors will result if one is provided.

```
$\xintRaw{273.3734e5/3395.7200e-2}=\xintFrac {273.3734e5/3395.7200e-2}$
```

2733734/33957200[7] = $\frac{2733734}{33957200}10^7$

- \xintPRaw does the same but without printing the [n] if n=0 and without printing /1 if B=1.
- \xintIrr reduces the fraction to its irreducible form C/D (without a trailing [0]), and it prints the D even if D=1.

```
$\xintIrr{273.3734e5/3395.7200e-2}$
```

2971450000/3691

- \xintNum from package xint becomes when xintfrac is loaded a synonym to its macro \xintTTrunc (same as \xintiTrunc{0}) which truncates to the nearest integer.
- See also the documentations of \xintTrunc, \xintiTrunc, \xintXTrunc, \xintRound, \xintiRound and \xintFloat.
- The \xintiAdd, \xintiSub, \xintiMul, \xintiPow macros and some others accept fractions on input which they truncate via \xintTTrunc. On output they still produce an integer with no fraction slash nor trailing [n].
- The \xintiiAdd, \xintiiSub, \xintiiMul, \xintiiPow, and others with `ii' in their names accept on input only integers in the strict format. They skip the overhead of the \xintNum parsing and naturally they output integers, with no fraction slash nor trailing [n].

Some macros return a token list of two or more numbers or fractions; they are then each enclosed in braces. Examples are \xintiiDivision which gives first the quotient and then the remainder of euclidean division, \xintBezout from the xintgcd package which outputs five numbers, \xintFtoCv from the xintcfrac package which returns the list of the convergents of a fraction, ... subsection 3.1 and subsection 3.2 mention utilities, expandable or not, to cope with such outputs.

Another type of multiple number output is when using commas inside \xintexpr..\relax:

```
\xinttheiexpr 10!,2^20,lcm(1000,725)\relax→3628800, 1048576, 29000
```

This returns a comma separated list, with a space after each comma.

2.9 Use of count registers

Inside \xintexpr..\relax and its variants, a count register or count control sequence is automatically unpacked using \number, with tacit multiplication: 1.23\counta is like 1.23*\number\counta. There is a subtle difference between count *registers* and count *variables*. In 1.23*\counta the unpacked \counta variable defines a complete operand thus 1.23*\counta 7 is a syntax error. But 1.23*\count0 just replaces \count0 by \number\count0 hence 1.23*\count0 7 is like 1.23*57 if \count0 contains the integer value 5.

Regarding now the package macros, there is first the case of arguments having to be short integers: this means that they are fed to a \numexpr...\relax, hence submitted to a *complete expansion* which must deliver an integer, and count registers and even algebraic expressions with them like \mycountA+\mycountB*17-\mycountC/12+\mycountD are admissible arguments (the slash stands here for the rounded integer division done by \numexpr). This applies in particular to the number of digits to truncate or round with, to the indices of a series partial sum, ...

The macros allowing the extended format for long numbers or dealing with fractions will *to some extent* allow the direct use of count registers and even infix algebra inside their arguments: a count register \mycountA or \count 255 is admissible as numerator or also as denominator, with no need to be prefixed by \the or \number. It is possible to have as argument an algebraic expression as would be acceptable by a \numexpr...\relax, under this condition: *each of the numerator and* *denominator is expressed with at most* eight *tokens*.[38] The slash for rounded division in a \numexpr should be written with braces {/} to not be confused with the xintfrac delimiter between numerator and denominator (braces will be removed internally). Example: \mycountA+\mycountB{/}17/1+\mycountA*\mycountB, or \count 0+\count 2{/}17/1+\count 0*\count 2, but in the latter case the numerator has the maximal allowed number of tokens (the braced slash counts for only one).

```
\cnta 10 \cntb 35 \xintRaw {\cnta+\cntb{/}17/1+\cnta*\cntb}->12/351[0]
```

For longer algebraic expressions using count registers, there are two possibilities:

1. encompass each of the numerator and denominator in \the\numexpr...\relax,
2. encompass each of the numerator and denominator in \numexpr {...}\relax.

```
\cnta 100 \cntb 10 \cntc 1
\xintPRaw {\numexpr {\cnta*\cnta+\cntb*\cntb+\cntc*\cntc+
                    2*\cnta*\cntb+2*\cnta*\cntc+2*\cntb*\cntc}\relax/%
          \numexpr {\cnta*\cnta+\cntb*\cntb+\cntc*\cntc}\relax }
12321/10101
```

The braces would not be accepted as regular \numexpr-syntax: and indeed, they are removed at some point in the processing.

2.10 Dimensions

⟨*dimen*⟩ variables can be converted into (short) integers suitable for the xint macros by prefixing them with \number. This transforms a dimension into an explicit short integer which is its value in terms of the sp unit (1/65536 pt). When \number is applied to a ⟨*glue*⟩ variable, the stretch and shrink components are lost.

For LaTeX users: a length is a ⟨*glue*⟩ variable, prefixing a length command defined by \newlength with \number will thus discard the plus and minus glue components and return the dimension component as described above, and usable in the xint bundle macros.

This conversion is done automatically inside an \xintexpr-essions, with tacit multiplication implied if prefixed by some (integral or decimal) number.

One may thus compute areas or volumes with no limitations, in units of sp^2 respectively sp^3, do arithmetic with them, compare them, etc..., and possibly express some final result back in another unit, with the suitable conversion factor and a rounding to a given number of decimal places.

A table of dimensions illustrates that the internal values used by TeX do not correspond always to the closest rounding. For example a millimeter exact value in terms of sp units is 72.27/10/2.54*65536=186467.981... and TeX uses internally 186467sp (it thus appears that TeX truncates to get an integral multiple of the sp unit).

There is something quite amusing with the Didot point. According to the TeXBook, 1157 dd=1238 pt. The actual internal value of 1 dd in TeX is 70124 sp. We can use xintcfrac to display the list of centered convergents of the fraction 70124/65536:

```
\xintListWithSep{, }{\xintFtoCCv{70124/65536}}
```

1/1, 15/14, 61/57, 107/100, 1452/1357, 17531/16384, and we don't find 1238/1157 therein, but another approximant 1452/1357!

And indeed multiplying 70124/65536 by 1157, and respectively 1357, we find the approximations (wait for more, later):

``1157 dd''=1237.998474121093...pt
``1357 dd''=1451.999938964843...pt

and we seemingly discover that 1357 dd=1452 pt is *far more accurate* than the TeXBook formula 1157 dd=1238 pt ! The formula to compute N dd was

```
\xinttheexpr trunc(N\dimexpr 1dd\relax/\dimexpr 1pt\relax,12)\relax}
```

[38] Attention! there is no problem with a LaTeX \value{countername} if if comes first, but if it comes later in the input it will not get expanded, and braces around the name will be removed and chaos will ensue inside a \numexpr. One should enclose the whole input in \the\numexpr...\relax in such cases.

Unit	definition	Exact value in sp units	TeX's value in sp units	Relative error
cm	0.01 m	236814336/127 = 1864679.811...	1864679	-0.0000%
mm	0.001 m	118407168/635 = 186467.981...	186467	-0.0005%
in	2.54 cm	118407168/25 = 4736286.720...	4736286	-0.0000%
pc	12 pt	786432 = 786432.000...	786432	0%
pt	1/72.27 in	65536 = 65536.000...	65536	0%
bp	1/72 in	1644544/25 = 65781.760...	65781	-0.0012%
3bp	1/24 in	4933632/25 = 197345.280...	197345	-0.0001%
12bp	1/6 in	19734528/25 = 789381.120...	789381	-0.0000%
72bp	1 in	118407168/25 = 4736286.720...	4736286	-0.0000%
dd	1238/1157 pt	81133568/1157 = 70124.086...	70124	-0.0001%
11dd	11*1238/1157 pt	892469248/1157 = 771364.950...	771364	-0.0001%
12dd	12*1238/1157 pt	973602816/1157 = 841489.037...	841489	-0.0000%
sp	1/65536 pt	1 = 1.000...	1	0%

TeX dimensions

What's the catch? The catch is that TeX *does not* compute 1157 dd like we just did:

```
1157 dd=\number\dimexpr 1157dd\relax/65536=1238.000000000000...pt
1357 dd=\number\dimexpr 1357dd\relax/65536=1452.001724243164...pt
```

We thus discover that TeX (or rather here, e-TeX, but one can check that this works the same in TeX82), uses indeed 1238/1157 as a conversion factor, and necessarily intermediate computations are done with more precision than is possible with only integers less than 2^{31} (or 2^{30} for dimensions). Hence the 1452/1357 ratio is irrelevant, a misleading artefact of the necessary rounding (or, as we see, truncating) for one dd as an integral number of sp's.

Let us now use \xintexpr to compute the value of the Didot point in millimeters, if the above rule is exactly verified:

```
\xinttheexpr trunc(1238/1157*25.4/72.27,12)\relax=0.376065027442...mm
```

This fits very well with the possible values of the Didot point as listed in the Wikipedia Article. The value 0.376065 mm is said to be *the traditional value in European printers' offices*. So the 1157 dd=1238 pt rule refers to this Didot point, or more precisely to the *conversion factor* to be used between this Didot and TeX points.

The actual value in millimeters of exactly one Didot point as implemented in TeX is

```
\xinttheexpr trunc(\dimexpr 1dd\relax/65536/72.27*25.4,12)\relax
=0.376064563929...mm
```

The difference of circa 5Å is arguably tiny!

By the way the *European printers' offices* (dixit Wikipedia) *Didot* is thus exactly

```
\xinttheexpr reduce(.376065/(25.4/72.27))\relax=543564351/508000000 pt
```

and the centered convergents of this fraction are 1/1, 15/14, 61/57, 107/100, 1238/1157, 11249/10513, 23736/22183, 296081/276709, 615898/575601, 11382245/10637527, 22148592/20699453, 188570981/176233151, 543564351/508000000. We do recover the 1238/1157 therein!

2.11 \ifcase, \ifnum, ... constructs

When using things such as \ifcase \xintSgn{\A} one has to make sure to leave a space after the closing brace for TeX to stop its scanning for a number: once TeX has finished expanding \xintSgn{\A} and has so far obtained either 1, 0, or -1, a space (or something `unexpandable') must stop it looking for more digits. Using \ifcase\xintSgn\A without the braces is very dangerous, because the blanks (including the end of line) following \A will be skipped and not serve to stop the number which \ifcase is looking for.

```
\begin{enumerate}[nosep]\def\A{1}
\item \ifcase \xintSgn\A 0\or OK\else ERROR\fi
\item \ifcase \xintSgn\A\space 0\or OK\else ERROR\fi
```

```
\item \ifcase \xintSgn{\A} 0\or OK\else ERROR\fi
\end{enumerate}
```

1. ERROR
2. OK
3. OK

In order to use successfully \if...\fi constructions either as arguments to the xint bundle expandable macros, or when building up a completely expandable macro of one's own, one needs some TeXnical expertise (see also item 2 on page 17).

It is thus much to be recommended to opt rather for already existing expandable branching macros, such as the ones which are provided by xint/xintfrac: among them \xintSgnFork, \xintifSgn, \xintifZero, \xintifOne, \xintifNotZero, \xintifTrueAelseB, \xintifCmp, \xintifGt, \xintifLt, \xintifEq, \xintifOdd, and \xintifInt. See their respective documentations. All these conditionals always have either two or three branches, and empty brace pairs {} for unused branches should not be forgotten.

If these tests are to be applied to standard TeX short integers, it is more efficient to use (under LaTeX) the equivalent conditional tests from the etoolbox[39] package.

2.12 Expandable implementations of mathematical algorithms

It is possible to chain \xintexpr-essions with \expandafter's, like experts do with \numexpr to compute multiple things at once. See subsection 6.24 for an example devoted to Fibonacci numbers (this section provides the code which was used on the title page for the F(1250) evaluation.) Notice that the 47th Fibonacci number is 2971215073 thus already too big for TeX and ε-TeX.

The \Fibonacci macro found in subsection 6.24 is completely expandable, (it is even f-expandable in the sense previously explained) hence can be used for example within \message to write to the log and terminal.

Also, one can thus use it as argument to the xint macros: for example if we are interested in knowing how many digits F(1250) has, it suffices to issue \xintLen {\Fibonacci {1250}} (which expands to 261). Or if we want to check the formula gcd(F(1859), F(1573)) = F(gcd(1859, 1573)) = F(143), we only need[40]

```
$\xintiiGCD{\Fibonacci{1859}}{\Fibonacci{1573}}=\Fibonacci{\xintiiGCD{1859}{1573}}$
```

which outputs:

343358302784187294870275058337 = 343358302784187294870275058337

The \Fibonacci macro expanded its \xintiiGCD{1859}{1573} argument via the services of \numexpr: this step allows only things obeying the TeX bound, naturally! (but F(2147483648) would be rather big anyhow...).

In practice, whenever one typesets things, one has left the expansion only contexts; hence there is no objection to, on the contrary it is recommended, assign the result of earlier computations to macros via an \edef (or an \fdef, see 5.1), for later use. The above could thus be coded

```
\begingroup
  \def\A {1859}  \def\B {1573}  \edef\C {\xintiiGCD\A\B}
  \edef\X {\Fibonacci\A}  \edef\Y {\Fibonacci\B}
The identity $\gcd(F(\A),F(\B))=F(\gcd(\A,\B))$ can be checked via evaluation
of both sides: $\gcd(F(\A),F(\B))=\gcd(\printnumber\X,\printnumber\Y)=
\printnumber{\xintiiGCD\X\Y} = F(\gcd(\A,\B))$.\par
          % some further computations involving \A, \B, \C, \X, \Y
\endgroup % closing the group removes assignments to \A, \B, ...
% or choose longer names less susceptible to overwriting something. Note that there
% is no LaTeX \newcommand which would be to \edef like \newcommand is to \def
```

The identity gcd(F(1859), F(1573)) = F(gcd(1859, 1573)) can be checked via evaluation of both sides: gcd(F(1859), F(1573)) = gcd(14405827913044251198771689151504042869913161495023481014222
66863670108827259757549472248243775352961945979486922735762888221630935801826408085177531997

[39] http://www.ctan.org/pkg/etoolbox [40] The \xintiiGCD macro is provided by the xintgcd package.

425695605529435028861585245173725088673642222849290822895245583889495442192655760412999290252
565979711337876105452217623490841529979811413199660087517689703410997520079993610707576019522
0876324584695551467505894985013610208598628752325727241,2443841925195118573328279459777626192
985399024815706192326053609007840133940367432124452232789599095158695811031891779769058032742
151632595307616686661013725200866754096569888951010022888016831459347310131566517721593249342
479863439947937119575876654476582795890928239007031319713554812200493864453132952484774727312
66471511289078393) = 343358302784187294870275058337 = F(143) = F(gcd(1859, 1573)).

One may thus legitimately ask the author: why expandability to such extremes, for things such as big fractions or floating point numbers (even continued fractions...) which anyhow can not be used directly within TeX's primitives such as \ifnum? the answer is that the author chose, seemingly, at some point back in his past to waste from then on his time on such useless things!

2.13 Possible syntax errors to avoid

Here is a list of imaginable input errors. Some will cause compilation errors, others are more annoying as they may pass through unsignaled.

- using - to prefix some macro: -\xintiSqr{35}/271.[41]
- using one pair of braces too many \xintIrr{{\xintiPow {3}{13}}/243} (the computation goes through with no error signaled, but the result is completely wrong).
- things like \xintiiAdd { \x}{\y} as the space will cause \x to be expanded later, most probably within a \numexpr thus provoking possibly an arithmetic overflow.
- using [] and decimal points at the same time 1.5/3.5[2], or with a sign in the denominator 3/-5[7]. The scientific notation has no such restriction, the two inputs 1.5/-3.5e-2 and -1.5e2/3.5 are equivalent: \xintRaw{1.5/-3.5e-2}=-15/35[2], \xintRaw{-1.5e2/3.5}=-15/35[2].
- generally speaking, using in a context expecting an integer (possibly restricted to the TeX bound) a macro or expression which returns a fraction: \xinttheexpr 4/2\relax outputs 4/2, not 2. Use \xintNum {\xinttheexpr 4/2\relax} or \xinttheiexpr 4/2\relax (which rounds the result to the nearest integer, here, the result is already an integer) or \xinttheiiexpr 4/2\relax. Or, divide in your head 4 by 2 and insert the result directly in the TeX source.

2.14 Error messages

In situations such as division by zero, the package will insert in the TeX processing an undefined control sequence (we copy this method from the bigintcalc package). This will trigger the writing to the log of a message signaling an undefined control sequence. The name of the control sequence is the message. The error is raised *before* the end of the expansion so as to not disturb further processing of the token stream, after completion of the operation. Generally the problematic operation will output a zero. Possible such error message control sequences:

```
\xintError:ArrayIndexIsNegative
\xintError:ArrayIndexBeyondLimit
\xintError:FactorialOfNegative
\xintError:TooBigFactorial
\xintError:DivisionByZero
\xintError:NaN
\xintError:FractionRoundedToZero
\xintError:NotAnInteger
\xintError:ExponentTooBig
```

```
\xintError:RootOfNegative
\xintError:NoBezoutForZeros
\xintError:ignored
\xintError:removed
\xintError:inserted
\xintError:unknownfunction
\xintError:we_are_doomed
\xintError:missing_xintthe!
```

[41] to the contrary, this *is* allowed inside an \xintexpr-ession.

There are now a few more if for example one attempts to use \xintAdd without having loaded xint-
Changed! → frac (with only xint loaded, only \xintiAdd and \xintiiAdd are legal).

```
\Did_you_mean_iiAbs?or_load_xintfrac
\Did_you_mean_iiOpp?or_load_xintfrac
\Did_you_mean_iiAdd?or_load_xintfrac
\Did_you_mean_iiSub?or_load_xintfrac
\Did_you_mean_iiMul?or_load_xintfrac
\Did_you_mean_iiPow?or_load_xintfrac
\Did_you_mean_iiSqr?or_load_xintfrac
\Did_you_mean_iiMax?or_load_xintfrac
\Did_you_mean_iiMin?or_load_xintfrac
\Did_you_mean_iMaxof?or_load_xintfrac
\Did_you_mean_iMinof?or_load_xintfrac
\Did_you_mean_iiSum?or_load_xintfrac
\Did_you_mean_iiPrd?or_load_xintfrac
\Did_you_mean_iiPrdExpr?or_load_xintfrac
\Did_you_mean_iiSumExpr?or_load_xintfrac
```

Don't forget to set \errorcontextlines to at least 2 to get from LaTeX more meaningful error messages. Errors occuring during the parsing of \xintexpr-essions try to provide helpful information about the offending token.

Release 1.1 employs in some situations delimited macros and there is the possibility in case of an ill-formed expression to end up beyond the \relax end-marker. The errors inevitably arising could then lead to very cryptic messages; but nothing unusual or especially traumatizing for the daring experienced TeX/LaTeX user.

2.15 Package namespace, catcodes

The bundle packages needs that the \space and \empty control sequences are pre-defined with the identical meanings as in Plain TeX or LaTeX2e.

Private macros of xintkernel, xintcore, xinttools, xint, xintfrac, xintexpr, xintbinhex, xintgcd, xintseries, and xintcfrac use one or more underscores _ as private letter, to reduce the risk of getting overwritten. They almost all begin either with \XINT_ or with \xint_, a handful of these private macros such as \XINTsetupcatcodes, \XINTdigits and those with names such as \XINTinFloat... or \XINTinfloat... do not have any underscore in their names (for obscure legacy reasons).

xinttools provides \odef, \oodef, \fdef (if macros with these names already exist xinttools will not overwrite them but provide \xintodef etc...) but all other public macros from the xint bundle packages start with \xint.

For the good functioning of the macros, standard catcodes are assumed for the minus sign, the forward slash, the square brackets, the letter `e'. These requirements are dropped inside an \xintexpr-ession: spaces are gobbled, catcodes mostly do not matter, the e of scientific notation may be E (on input) . . .

If a character used in the \xintexpr syntax is made active, this will surely cause problems; prefixing it with \string is one option. There is \xintexprSafeCatcodes and \xintexprRestoreCatcodes to temporarily turn off potentially active characters (but setting catcodes is an un-expandable action).

> For advanced TeX users. At loading time of the packages the catcode configuration may be arbitrary as long as it satisfies the following requirements: the percent is of category code comment character, the backslash is of category code escape character, digits have category code other and letters have category code letter. Nothing else is assumed.

2.16 Origins of the package

2013/03/28. Package bigintcalc by Heiko Oberdiek already provides expandable arithmetic operations on ``big integers'', exceeding the TeX limits (of 2^{31} - 1), so why another[42] one?

[42] this section was written before the xintfrac package; the author is not aware of another package allowing expandable computations with arbitrarily big fractions.

I got started on this in early March 2013, via a thread on the c.t.tex usenet group, where Ulrich Diez used the previously cited package together with a macro (\ReverseOrder) which I had contributed to another thread.[43] What I had learned in this other thread thanks to interaction with Ulrich Diez and GL on expandable manipulations of tokens motivated me to try my hands at addition and multiplication.

I wrote macros \bigMul and \bigAdd which I posted to the newsgroup; they appeared to work comparatively fast. These first versions did not use the ε-TeX \numexpr primitive, they worked one digit at a time, having previously stored carry-arithmetic in 1200 macros.

I noticed that the bigintcalc package used \numexpr if available, but (as far as I could tell) not to do computations many digits at a time. Using \numexpr for one digit at a time for \bigAdd and \bigMul slowed them a tiny bit but avoided cluttering TeX memory with the 1200 macros storing precomputed digit arithmetic. I wondered if some speed could be gained by using \numexpr to do four digits at a time for elementary multiplications (as the maximal admissible number for \numexpr has ten digits).

2013/04/14. This initial xint was followed by xintfrac which handled exactly fractions and decimal numbers.

2013/05/25. Later came xintexpr and at the same time xintfrac got extended to handle floating point numbers.

2013/11/22. Later, xinttools was detached.

2014/10/28. Release 1.1 significantly extended the xintexpr parsers.

2015/10/10. Release 1.2 rewrote the core integer routines which had remained essentially unmodified, apart from a slight improvement of division early 2014.

This 1.2 release also got its impulse from a fast ``reversing'' macro, which I wrote after my interest got awakened again as a result of correspondance with Bruno Le Floch during September 2015: this new reverse uses a TeXnique which *requires* the tokens to be digits. I wrote a routine which works (expandably) in quasi-linear time, but a less fancy O(N^2) variant which I developed concurrently proved to be faster all the way up to perhaps 7000 digits, thus I dropped the quasi-linear one. The less fancy variant has the advantage that xint can handle numbers with more than 19900 digits (but not much more than 19950). This is with the current common values of the input save stack and maximal expansion depth: 5000 and 10000 respectively.

2.17 Installation instructions

xint is made available under the LaTeX Project Public License 1.3c. It is included in the major TeX distributions, thus there is probably no need for a custom install: just use the package manager to update if necessary xint to the latest version available.

After installation, issuing in terminal texdoc --list xint, on installations with a "texdoc" or similar utility, will offer the choice to display one of the documentation files: xint.pdf (this file), sourcexint.pdf (source code), README, README.pdf, README.html, CHANGES.pdf, and CHANGES.html.

For manual installation, follow the instructions from the README file which is to be found on CTAN; it is also available there in PDF and HTML formats. The simplest method proposed is to use the archive file xint.tds.zip, downloadable from the same location.

The next simplest one is to make use of the Makefile, which is also downloadable from CTAN. This is for GNU/Linux systems and Mac OS X, and necessitates use of the command line. If for some reason you have xint.dtx but no internet access, you can recreate Makefile as a file with this name and the following contents:

```
include Makefile.mk
Makefile.mk: xint.dtx ; etex xint.dtx
```

Then run make in a working repertory where there is xint.dtx and the file named Makefile and having only the two lines above. The make will extract the package files from xint.dtx and display some further instructions.

[43] the \ReverseOrder could be avoided in that circumstance, but it does play a crucial rôle here.

If you have xint.dtx, no internet access and can not use the Makefile method: etex xint.dtx extracts all files and among them the README as a file with name README.md. Further help and options will be found therein.

2.18 Changes

The detailed cumulative change log since the initial release is in files CHANGES.html (also available on CTAN via this link) and CHANGES.pdf.

In a command line console, issue texdoc --list xint to access them.

It is also possible to extract them from the source xint.dtx: etex xint.dtx in a working repertory which will extract a CHANGES.md file with Markdown syntax. On a unix-like system you can then run make -f Makefile.mk CHANGES.html to get the html version (requires pandoc).

The last major release is 1.2 (2015/10/10). It came with a complete rewrite of the core arithmetic routines. The efficiency for numbers with less than 20 or 30 digits was slightly compromised (for addition/subtraction) but it got increased for bigger numbers. For multiplication and division the gains were there for almost all sizes, and became quite noticeable for numbers with hundreds of digits. The allowable inputs are constrained to have less than about 19950 digits (19968 for addition, 19959 for multiplication).

1.2a (2015/10/19) to 1.2g (2016/03/19): unfortunately, release 1.2 had at least four bad bugs. Subsequent releases were supposed to fix them but sometimes they themselves introduced new bugs which had to get fixed in a later one. For example 1.2f fixes a bug in subtraction which was introduced by 1.2c's fix of 1.2's subtraction.

We managed to add some new features:

1. list selectors [L][a:b] or [L][n] in expressions are faster.
2. faster \xintiiSquareRoot (and allied square root macros) and \xintFloatSqrt. The latter achieves *correct rounding* in arbitrary precision. Changed (1.2f)
3. faster \xintFloatPow and \xintFloatPower. The ^ in \xintthefloatexpr...\relax admits an half-integer exponent.
4. New with 1.2f: functions factorial, binomial, pfactorial and associated macros \xintiiBinomial, \xintFloatBinomial, \xintiiPFactorial, \xintFloatPFactorial.
5. \xintdeffunc, \xintdefiifunc, \xintdeffloatfunc macros to define functions recognized as such by the xintexpr parsers.
6. new \xintunassignvar, new \xintverbosetrue.
7. declarations as per \xintdeffunc *et al.* and macro definitions as per \xintNewExpr *et al.* now with only local scope.
8. tacit multiplication extended to more cases and now always ``ties more'' than the regular division and multiplication.
9. \xintTrimUnbraced and \xintKeepUnbraced.
10. New with 1.2f: more precise discussion of floating point issues in subsection 2.2, but the coverage by xintfrac of floating point operations is yet to be substantially extended.
11. Changed (1.2f): the float macros for addition, subtraction, multiplication, division now first round their two operands to P significant places (not P+2 as earlier) before doing the actual computation (P is the asked for precision or \xinttheDigits). The same applies to the float power and square root operations.
12. some float operations are faster a handling small operands in the context of a large float precision.
13. extensive update to ``Commands of the xintexpr package'' (section 10.)
14. complete re-write of the Quick Sort algorithm section.
15. documentation fixes (including random shuffling around of various sections), and more code comments, particularly in the xintcore.sty section of sourcexint.pdf. Trimming of irrelevant old comments.

Notice that 1.2f and 1.2g had a few backwards incompatible changes: no \xintFac but \xintiFac, list item numbering in expressions starts from zero not from one, iter keyword has a new meaning and the former one is now associated with iterr.

3 Some utilities from the xinttools package

This is a first overview. Many examples combining these utilities with the arithmetic macros of xint are to be found in section 6.

3.1 Assignments

It might not be necessary to maintain at all times complete expandability. A devoted syntax is provided to make these things more efficient, for example when using the \xintiDivision macro which computes both quotient and remainder at the same time:

```
    \xintAssign \xintiiDivision{\xintiiPow {2}{1000}}{\xintiiFac{100}}\to\A\B
```

give: \meaning\A: macro:->114813249641507505482278393872551066259805517784186172883663478065826541894704737970419535798876630484358265060061503749531707793118627774829601 and \meaning\B: macro:->5493629452133983225138128786223912807341050049847605059532189961231327664902288381328787024445820751296031520410548049646250831385676526243868372056680693 76. Another example (which uses \xintBezout from the xintgcd package):

```
    \xintAssign \xintBezout{357}{323}\to\A\B\U\V\D
```

is equivalent to setting \A to 357, \B to 323, \U to -9, \V to -10, and \D to 17. And indeed (-9)×357-(-10)×323=17 is a Bezout Identity.

Thus, what \xintAssign does is to first apply an *f*-expansion to what comes next; it then defines one after the other (using \def; an optional argument allows to modify the expansion type, see subsection 6.26 for details), the macros found after \to to correspond to the successive braced contents (or single tokens) located prior to \to. In case the first token (after the optional parameter within brackets, *cf.* the \xintAssign detailed document) is not an opening brace {, \xintAssign consider that there is only one macro to define, and that its replacement text should be all that follows until the \to.

```
    \xintAssign\xintBezout{3570902836026}{200467139463}\to\A\B\U\V\D
```

gives then \U: 5812117166, \V: 103530711951 and \D=3.

In situations when one does not know in advance the number of items, one has \xintAssignArray or its synonym \xintDigitsOf:

```
    \xintDigitsOf\xintiPow{2}{100}\to\DIGITS
```

This defines \DIGITS to be macro with one parameter, \DIGITS{0} gives the size N of the array and \DIGITS{n}, for n from 1 to N then gives the nth element of the array, here the nth digit of 2^{100}, from the most significant to the least significant. As usual, the generated macro \DIGITS is completely expandable (in two steps). As it wouldn't make much sense to allow indices exceeding the TeX bounds, the macros created by \xintAssignArray put their argument inside a \numexpr, so it is completely expanded and may be a count register, not necessarily prefixed by \the or \number. Consider the following code snippet:

```
    % \newcount\cnta
    % \newcount\cntb
    \begingroup
    \xintDigitsOf\xintiPow{2}{100}\to\DIGITS
    \cnta = 1
    \cntb = 0
    \loop
    \advance \cntb \xintiSqr{\DIGITS{\cnta}}
    \ifnum \cnta < \DIGITS{0}
    \advance\cnta 1
    \repeat
```

```
|2^{100}| (=\xintiPow {2}{100}) has \DIGITS{0} digits and the sum of their squares is \the\cntb.
These digits are, from the least to the most significant: \cnta = \DIGITS{0} \loop
\DIGITS{\cnta}\ifnum \cnta > 1 \advance\cnta -1 , \repeat.\endgroup
```

2^{100} (=1267650600228229401496703205376) has 31 digits and the sum of their squares is 679. These digits are, from the least to the most significant: 6, 7, 3, 5, 0, 2, 3, 0, 7, 6, 9, 4, 1, 0, 4, 9, 2, 2, 8, 2, 2, 0, 0, 6, 0, 5, 6, 7, 6, 2, 1.

Warning: \xintAssign, \xintAssignArray and \xintDigitsOf *do not do any check* on whether the macros they define are already defined.

3.2 Utilities for expandable manipulations

The package now has more utilities to deal expandably with `lists of things', which were treated un-expandably in the previous section with \xintAssign and \xintAssignArray: \xintReverseOrder and \xintLength since the first release, \xintApply and \xintListWithSep since 1.04, \xintRevWithBraces, \xintCSVtoList, \xintNthElt since 1.06, \xintApplyUnbraced, since 1.06b, \xintloop and \xintiloop since 1.09g.[44]

As an example the following code uses only expandable operations:

```
$2^{100}$ (=\xintiPow {2}{100}) has \xintLen{\xintiPow {2}{100}} digits and the sum of their
  squares is \xintiiSum{\xintApply {\xintiSqr}{\xintiPow {2}{100}}}. These digits are, from the
  least to the most significant: \xintListWithSep {, }{\xintRev{\xintiPow {2}{100}}}. The thirteenth
  most significant digit is \xintNthElt{13}{\xintiPow {2}{100}}. The seventh least significant one
  is \xintNthElt{7}{\xintRev{\xintiPow {2}{100}}}.
```

2^{100} (=1267650600228229401496703205376) has 31 digits and the sum of their squares is 679. These digits are, from the least to the most significant: 6, 7, 3, 5, 0, 2, 3, 0, 7, 6, 9, 4, 1, 0, 4, 9, 2, 2, 8, 2, 2, 0, 0, 6, 0, 5, 6, 7, 6, 2, 1. The thirteenth most significant digit is 8. The seventh least significant one is 3.

It would be more efficient to do once and for all \edef\z{\xintiPow {2}{100}}, and then use \z in place of \xintiPow {2}{100} everywhere as this would spare the CPU some repetitions.

Expandably computing primes is done in subsection 6.12.

3.3 A new kind of for loop

As part of the utilities coming with the xinttools package, there is a new kind of for loop, \xintFor. Check it out (subsection 6.19).

3.4 A new kind of expandable loop

Also included in xinttools, \xintiloop is an expandable loop giving access to an iteration index, without using count registers which would break expandability. Check it out (subsection 6.15).

4 The syntax of xintexpr expressions

[44] All these utilities, as well as \xintAssign, \xintAssignArray and the \xintFor loops are now available from the xinttools package, independently of the big integers facilities of xint.

4.1 Infix and other operators and their precedence levels

We go through the various syntax elements from highest to lowest precedence.

- Functions share the highest precedence.
- The `.` as decimal mark; the number scanner treats it as an inherent, optional and unique component of a being formed number. One can do things such as

```
\xinttheexpr 0.^2+2^.0\relax
```

which is 0^2+2^0 and produces 1.

However a single dot "." as in `\xinttheexpr .^2\relax` is now illegal input.
- The e and E for scientific notation. They are parsed like the decimal mark is. 1e3^2 is 1[6]
- The " for hexadecimal numbers: it is treated with highest priority, allowed only at locations where the parser expects to start forming a numeric operand, once encountered it triggers the hexadecimal scanner which looks for successive hexadecimal digits (as usual skipping spaces and expanding forward everything; letters A, ..., F, but not a, ..., f) possibly a unique optional dot (allowed directly in front) and then an optional (possibly empty) fractional part. The dot and fractional part are not allowed in `\xintiiexpr..\relax`. The " functionality requires package xintbinhex (there is no warning, but an ``undefined control sequence'' error will naturally results if the package has not been loaded). "A*"A^"A is 100000000000.
- The postfix operators ! and the branching conditionals ?, ??.

  ! computes the factorial of an integer.

  ? is used as (cond)?{yes}{no}. It evaluates the (numerical) condition (any non-zero value counts as true, zero counts as false). It then acts as a macro with two mandatory arguments within braces (hence this escapes from the parser scope, the braces can not be hidden in a macro), chooses the correct branch *without evaluating the wrong one*. Once the braces are removed, the parser scans and expands the uncovered material so for example

```
\xinttheiexpr (3>2)?{5+6}{7-1}2^3\relax
```

  is legal and computes 5+62^3=238333. Note though that it would be better practice to include here the 2^3 inside the branches. The contents of the branches may be arbitrary as long as once glued to what is next the syntax is respected: `\xintexpr (3>2)?{5+(6}{7-(1}2^3)\relax` also works. Differs thus from the if conditional in two ways: the false branch is not at all computed, and the number scanner is still active on exit, more digits may follow.

  ?? is used as (cond)??{<0}{=0}{>0}. cond is anything, its sign is evaluated and depending on the sign the correct branch is un-braced, the two others are swallowed. The un-braced branch will then be parsed as usual. Differs from the ifsgn conditional as the two false branches are not evaluated and furthermore the number scanner is still active on exit.

```
\def\x{0.33}\def\y{1/3}
\xinttheexpr (\x-\y)??{sqrt}{0}{1/}(\y-\x)\relax=5773502691896257[-17]
```

- The minus sign - as prefix unary operator inherits the precedence of the infix operator it follows. With things such as 5+------2*3, the xintexpr parsers don't try to be efficient: once 2*3 is evaluated the opposite function will be applied the necessary number of times. On the other hand the plus sign + as prefix unary operator as in, for example 5-++++++2*3, is immediately gobbled.
- The power operator ^, or **. It is left associative: `\xinttheiexpr 2^2^3\relax` evaluates to 64, not 256. Note that if the float precision is too low, iterated powers within `\xintfloatexpr..\relax` may fail: for example with the default setting (1+1e-8)^(12^16) will be computed with 12^16 approximated from its 16 most significant digits but it has 18 digits (=184884258895036416), hence the result is wrong:

  $1.879,985,375,897,266 \times 10^{802,942,130}$

  One should code

```
\xintthe\xintfloatexpr (1+1e-8)^\xintiiexpr 12^16\relax \relax
```

to obtain the correct floating point evaluation

$1.000,000,01^{12^{16}} \approx 1.879,985,676,694,948 \times 10^{802,942,130}$

- Multiplication and division `*`, `/`. The division is left associative, too: `\xinttheiexpr 100/50/2\relax` evaluates to 1, not 4. Inside `\xintiiexpr`, `/` does *rounded* division.
- Truncated division `//` and modulo `/:` (equivalently `'mod'`, quotes mandatory) are at the same level of priority than multiplication and division, thus left-associative with them. Apply parentheses for disambiguation.

```
\xinttheexpr 100000//13, 100000/:13, 100000 'mod' 13, trunc(100000/13,10),
             trunc(100000/:13/13,10)\relax
```

7692, 4, 4, 7692.3076923076, 0.3076923076

- The list itemwise operators `*[`, `/[`, `^[`, `**[`, `]*`, `]/`, `]^`, `]**` are at the same precedence level as, respectively, `*` and `/` or `^` and `**`.
- Addition and subtraction `+`, `-`. Again, `-` is left associative: `\xinttheiexpr 100-50-2\relax` evaluates to 48, not 52.
- The list itemwise operators `+[`, `-[`, `]+`, `]-`, are at the same precedence level as `+` and `-`,
- Comparison operators `<`, `>`, `=` (same as `==`), `<=`, `>=`, `!=` all at the same level of precedence, use parentheses for disambiguation.
- Conjunction (logical and): `&&` or equivalently `'and'` (quotes mandatory).
- Inclusive disjunction (logical or): `||` and equivalently `'or'` (quotes mandatory).
- XOR: `'xor'` with mandatory quotes is at the same level of precedence as `||`.
- The list generation operators `..`, `..[`, `]..` are at the same (low) precedence level as the `||` operator of logical disjunction.
- The comma: with `\xinttheexpr 2^3,3^4,5^6\relax` one obtains as output 8, 81, 15625.
- The parentheses. The list outer brackets `[`, `]` share the same functional precedence as parentheses. The semi-colon `;` in an `iter` or `rseq` has the same precedence as a closing parenthesis.

4.2 Tacit multiplication

Tacit multiplication (insertion of a `*`) applies when the parser is currently either scanning the digits of a number (or its decimal part or scientific part, or hexadecimal input), or is looking for an infix operator, and: (1.) *encounters a count or dimen or skip register or variable or an ε-TeX expression*, or (2.) *encounters a sub-\xintexpression*, or (3.) *encounters an opening parenthesis*, or (4.) *encounters a letter (which is interpreted as signaling the start of either a variable or a function name)*.

For example, if `x`, `y`, `z` are variables all three of `(x+y)z`, `x(y+z)`, `(x+y)(x+z)` will create a tacit multiplication.

Changed → Furthermore starting with release 1.2e, whenever tacit multiplication is applied, in all cases it *always* ``ties'' more than normal multiplication or division, but still less than power. Thus `x/2y` is interpreted as `x/(2y)` and similarly for `x/2max(3,5)` but `x^2y` is still interpreted as `(x^2)*y` and `2n!` as `2*n!`.

```
\xintdefvar x:=30;\xintdefvar y:=5;%
\xinttheexpr (x+y)x, x/2y, x^2y, x!, 2x!, x/2max(x,y)\relax
```

1050, 30/10, 4500, 265252859812191058636308480000000, 530505719624382117272616960000000, 30/60

The ``tie more'' rule applies to all cases of tacit multiplication. It impacts only situations when a division was the last seen operator, as the normal rule for the xintexpr parsers is left-associativity in case of equal precedence.

```
\xinttheexpr (1+2)/(3+4)(5+6), 2/x(10), 2/10x, 3/y\xintiiexpr 5+6\relax, 1/x(y)\relax
```
3/77, 2/300, 2/300, 3/55, 1/150

Note that y\xinttheiiexpr 5+6\relax would have tried to use a variable with name y11 rather than doing y*11: tacit multiplication works only in front of sub-\xintexpressions, not in front of \xinttheexpressions which are unlocked into explicit digits.

These examples above redeclared the x and y which thus can not be used as dummy variables anymore (but see \xintunassignvar); as this happened inside a LaTeX environment (for the frame), they will have recovered their meanings after the frame. In the meantime we use letter z for the next example. Here is an expression whose meaning is completely modified by the ``tie more'' property of tacit multiplication:

```
\xintdeffunc e(z):=1+z(1+z/2(1+z/3(1+z/4)));
```
will be parsed as 1+z*(1+z/(2*(1+z/(3*(1+z/4))))) which is not at all like the presumably hoped:

```
\xintdeffunc e(z):=1+z(1+z/2*(1+z/3*(1+z/4)));
```
This form can also be used, alternatively:

```
\xintdeffunc e(z):=(((z/4+1)z/3+1)z/2+1)z+1;
```

Attention! tacit multiplication before an opening parenthesis applies always, but tacit multiplication after a closing parenthesis *does not* apply in front of digits: (1+1)5 is not legal. But subs((1+1)x,x=5) is, because in that case a variable is following the closing parenthesis.

4.3 List operations

By *list* we hereby mean simply comma-separated values, for example 3, -7, 1e5. This section describes some syntax which allows to manipulate such lists, for example [3, -7, 1e5][1] extracts -7 (we follow the Python convention of enumerating starting at zero; see the frame next).

In the context of dummy variables, lists can be used in substitutions:

```
\xinttheiiexpr subs(`+`(L), L = 1, 3, 5, 7, 9)\relax\newline
```
25

and also the rseq and iter constructs allow @ to refer to a list:

```
\xinttheiiexpr iter(0, 1; ([@][1], [@][0]+[@][1]), i=1..10)\relax\newline
```
55, 89

where each step constructs a new list with two entries.

However, despite appearances there is not really internally a notion of a *list type* and it is currently impossible to create, manipulate, or return on output a *list of lists*. There is a special reserved variable nil which stands for the empty list: for example len() is not legal but len(nil) works.

The syntax which is explained next includes in particular what are called *list itemwise operators* such as:

```
\xinttheiiexpr 37+[13,100,1000]\relax\newline
```
50, 137, 1037

This part of the syntax is considered provisory, for the reason that its presence might make more difficult some extensions in the future. On the other hand the Python-like slicing syntax should not change.

A backwards incompatible change.

Up to release 1.2f inclusive, the item accessor [list][n] returned the nth element of a list. The Python-like slices [list][a:b] on the other hand act exactly as in Python where list items are enumerated starting at zero. For example [list][:5] or equivalently [list][0:5] have the effect to keep only the first five elements. Thus [list][n] which returned the nth element was akin to [list][n-1:n] whereas in Python which enumerates from zero it would be [list][n:n+1].

Changed (1.2g)

One reason for that choice was that [list][0] allowed access to the length, and there was thus no need to add a new function to the list of recognized keywords.

1.2g does the backwards incompatible change to adhere more fully to Python conventions and now [list][1] picks the *second* element of the list and [list][0] the *first*. There is len(list) for the length.

The reason for the change is that the author has become more accustomed to Python than he was when he first introduced list operations to xintexpr, and the difference was becoming distracting.

- a..b constructs the **small** integers from the ceil ⌈a⌉ to the floor ⌊b⌋ (possibly a decreasing sequence): one has to be careful if using this for algorithms that 1..0 for example is not empty or 1 but expands to 1, 0. Again, a..b *can not* be used with a and b greater than $2^{31} - 1$. Also, only about at most 5000 integers can be generated (this depends upon some TeX memory settings).

  The .. has lower precedence than the arithmetic operations.

  ```
  \xinttheexpr 1.5+0.4..2.3+1.1\relax; \xinttheexpr 1.9..3.4\relax; \xinttheexpr 2..3\relax
  ```

  2, 3; 2, 3; 2, 3

- a..[d]..b allows to generate big integers, or also fractions, it proceeds with step (non necessarily integral nor positive) d. It does *not* replace a by its ceil, nor b by its floor. The generated list is empty if b-a and d are of opposite signs; if d=0 or if a=b the list expands to single element a.

  ```
  \xinttheexpr 1.5..[1.01]..11.23\relax
  ```

  15[-1], 251[-2], 352[-2], 453[-2], 554[-2], 655[-2], 756[-2], 857[-2], 958[-2], 1059[-2]

Changed (1.2g)

- [list][n] extracts the n+1th element if n>=0. If n<0 it extracts from the tail. List items are numbered as in Python. len(list) computes the number of items of the list.

  ```
  \xinttheiexpr \empty[0..10][6], len(0..10), [0..10][-1], [0..10][23*18-22*19]\relax\
  (and 23*18-22*19 has value \the\numexpr 23*18-22*19\relax).
  ```

  6, 11, 10, 7 (and 23*18-22*19 has value -4).

  See the next frame for why the example above has \empty token at start.

  As shown, it is perfectly legal to do operations in the index parameter, which will be handled by the parser as everything else. The same remark applies to the next items.

- [list][:n] extracts the first n elements if n>0, or suppresses the last |n| elements if n<0.

  ```
  \xinttheiiexpr [0..10][:6]\relax\ and \xinttheiiexpr [0..10][:-6]\relax
  ```

  0, 1, 2, 3, 4, 5 and 0, 1, 2, 3, 4

- [list][n:] suppresses the first n elements if n>0, or extracts the last |n| elements if n<0.

  ```
  \xinttheiiexpr [0..10][6:]\relax\ and \xinttheiiexpr [0..10][-6:]\relax
  ```

  6, 7, 8, 9, 10 and 5, 6, 7, 8, 9, 10

- More generally, [list][a:b] works according to the Python ``slicing'' rules (inclusive of negative indices). Notice though that there is no optional third argument for the step, which always defaults to +1.

  ```
  \xinttheiiexpr [1..20][6:13]\relax\ = \xinttheiiexpr [1..20][6-20:13-20]\relax
  ```

  7, 8, 9, 10, 11, 12, 13 = 7, 8, 9, 10, 11, 12, 13

- It is naturally possible to nest these things:

```
\xinttheexpr [[1..50][13:37]][10:-10]\relax
```

24, 25, 26, 27

- itemwise operations either on the left or the right are possible:

```
\xinttheiiexpr 123*[1..10]^2\relax
```

123, 492, 1107, 1968, 3075, 4428, 6027, 7872, 9963, 12300

List operations are implemented using square brackets, but the \xintiexpr and \xintfloatexpr parsers also check to see if an optional parameter within brackets is specified before the start of the expression. To avoid the resulting confusion if this [actually serves to delimit comma separated values for list operations, one can either:

- insert something before the bracket such as \empty token,

```
\xinttheiexpr \empty [1,3,6,99,100,200][2:4]\relax
```

6, 99

- use parentheses:

```
\xinttheiexpr ([1,3,6,99,100,200][2:4])\relax
```

6, 99

Notice though that ([1,3,6,99,100,200])[2:4] would not work: it is mandatory for][and][: not to be interspersed with parentheses. Spaces are perfectly legal:

```
\xinttheiexpr \empty[1..10 ]   [  :  7  ]\relax
```

1, 2, 3, 4, 5, 6, 7

Similarly all the +[, *[, ...and]**,]/, ...operators admit spaces but nothing else between their constituent characters.

```
\xinttheiexpr \empty [ 1 . . 1 0 ]  * *  1 1 \relax
```

1, 2048, 177147, 4194304, 48828125, 362797056, 1977326743, 8589934592, 31381059609, 100000000000

In an other vein, the parser will be confused by 1..[a,b,c][1], and one must write 1..([a,b,c][1]). And things such as [100,300,500,700][2]//11 or [100,300,500,700][2]/11 are syntax errors and one must use parentheses, as in ([100,300,500,700][2])/11.

4.4 Predefined functions

Functions are at the same top level of priority. All functions even ? and ! (as prefix) require parentheses around their arguments.

num, qint, qfrac, qfloat, reduce, abs, sgn, frac, floor, ceil, sqr, sqrt, sqrtr, factorial, binomial, pfactorial, float, round, trunc, mod, quo, rem, gcd, lcm, max, min, `+`, `*`, ?, !, not, all, any, xor, if, ifsgn, even, odd, first, last, reversed, len, bool, togl

factorial, binomial, pfactorial, quo, rem, even, odd, gcd and lcm will first truncate their arguments to integers; the latter two require package xintgcd; togl requires the etoolbox package; all, any, xor, `+`, `*`, max, min, first, last, reversed and len are functions with arbitrarily many comma separated arguments.

bool, togl use delimited macros to fetch their argument and the closing parenthesis which thus must be explicit, not arising from expansion.

New with 1.2

The same holds for qint, qfrac, qfloat.

functions with a single (numeric) argument

num truncates to the nearest integer (truncation towards zero).

```
\xinttheexpr num(3.1415^20)\relax
```

8764785276

qint skips the token by token parsing of the input. The ending parenthesis must be physically present rather than arising from expansion. The q stands for ``quick''. This ``function'' handles the input exactly like do the i macros of xintcore, via \xintiNum. Hence leading signs and the leading zeroes (coming next) will be handled appropriately but spaces will not be systematically stripped. They should cause no harm and will be removed as soon as the number is used with one of the basic operators. This input form *does not accept decimal part or scientific part*.

New with 1.2

```
\def\x{....many many many ... digits}\def\y{....also many many many digits...}
\xinttheiiexpr qint(\x)*qint(\y)+qint(\y)^2\relax
```

qfrac does the same as qint excepts that it accepts fractions, decimal numbers, scientific numbers as they are understood by the macros of package xintfrac. Not to be used within an \xintiiexpr-ession, except if hidden inside functions such as round or trunc which produce integers from fractions.

New with 1.2

qfloat does the same as qfrac and converts to a float with the precision given by the setting of \xintDigits.

reduce reduces a fraction to smallest terms

```
\xinttheexpr reduce(50!/20!/20!/10!)\relax
```

1415997888807961859400

Recall that this is NOT done automatically, for example when adding fractions.

abs absolute value

sgn sign

frac fractional part

```
\xinttheexpr frac(-355/113), frac(-1129.218921791279)\relax
```

-16/113, -218921791279[-12]

floor floor function.

ceil ceil function.

sqr square.

sqrt in \xintiiexpr, truncated square root; in \xintexpr or \xintfloatexpr this is the floating point square root, and there is an optional second argument for the precision.

sqrtr in \xintiiexpr only, rounded square root.

factorial factorial function, same as previously available post-fix ! operator. When used in \xintexpr or \xintfloatexpr there is an optional second argument. See discussion later.

? ?(x) is the truth value, 1 if non zero, 0 if zero. Must use parentheses.

! !(x) is logical not, 0 if non zero, 1 if zero. Must use parentheses.

not logical not.

even evenness of the truncation.

```
\xinttheexpr seq((x,even(x)), x=-5/2..[1/3]..+5/2)\relax
```

-5/2, 1, -13/6, 1, -11/6, 0, -9/6, 0, -7/6, 0, -5/6, 1, -3/6, 1, -1/6, 1, 1/6, 1, 3/6, 1, 5/6, 1, 7/6, 0, 9/6, 0, 11/6, 0, 13/6, 1, 15/6, 1

odd oddness of the truncation.

```
\xinttheexpr seq((x,odd(x)), x=-5/2..[1/3]..+5/2)\relax
```

-5/2, 0, -13/6, 0, -11/6, 1, -9/6, 1, -7/6, 1, -5/6, 0, -3/6, 0, -1/6, 0, 1/6, 0, 3/6, 0, 5/6, 0, 7/6, 1, 9/6, 1, 11/6, 1, 13/6, 0, 15/6, 0

functions with an alphabetical argument

bool,togl. bool(name) returns 1 if the TeX conditional \ifname would act as \iftrue and 0 otherwise. This works with conditionals defined by \newif (in TeX or LaTeX) or with primitive conditionals such as \ifmmode. For example:

```
\xintifboolexpr{25*4-if(bool(mmode),100,75)}{YES}{NO}
```

will return NO if executed in math mode (the computation is then 100 - 100 = 0) and YES if not (the if conditional is described below; the \xintifboolexpr test automatically encapsulates its first argument in an \xintexpr and follows the first branch if the result is non-zero (see subsection 10.16)).

The alternative syntax 25*4-\ifmmode100\else75\fi could have been used here, the usefulness of bool(name) lies in the availability in the \xintexpr syntax of the logic operators of conjunction &&, inclusive disjunction ||, negation ! (or not), of the multi-operands functions all, any, xor, of the two branching operators if and ifsgn (see also ? and ??), which allow arbitrarily complicated combinations of various bool(name).

Similarly togl(name) returns 1 if the LaTeX package etoolbox[45] has been used to define a toggle named name, and this toggle is currently set to true. Using togl in an \xintexpr..\relax without having loaded etoolbox will result in an error from \iftoggle being a non-defined macro. If etoolbox is loaded but togl is used on a name not recognized by etoolbox the error message will be of the type ``ERROR: Missing \endcsname inserted.'', with further information saying that \protect should have not been encountered (this \protect comes from the expansion of the non-expandable etoolbox error message).

When bool or togl is encountered by the \xintexpr parser, the argument enclosed in a parenthesis pair is expanded as usual from left to right, token by token, until the closing parenthesis is found, but everything is taken literally, no computations are performed. For example togl(2+3) will test the value of a toggle declared to etoolbox with name 2+3, and not 5. Spaces are gobbled in this process. It is impossible to use togl on such names containing spaces, but \iftoggle{name with spaces}{1}{0} will work, naturally, as its expansion will pre-empt the \xintexpr scanner.

There isn't in \xintexpr... a test function available analogous to the test{\ifsometest} construct from the etoolbox package; but any *expandable* \ifsometest can be inserted directly in an \xintexpr-ession as \ifsometest10 (or \ifsometest{1}{0}), for example if(\ifsometest{1}{0},YES,NO) (see the if operator below) works.

A straight \ifsometest{YES}{NO} would do the same more efficiently, the point of \ifsometest10 is to allow arbitrary boolean combinations using the (described later) & and | logic operators: \ifsometest10 & \ifsomeothertest10 | \ifsomethirdtest10, etc... YES or NO above stand for material compatible with the \xintexpr parser syntax.

See also \xintifboolexpr, in this context.

functions with one mandatory and a second but optional argument

round For example round(-2^9/3^5,12)=-2.106995884774.

trunc For example trunc(-2^9/3^5,12)=-2.106995884773.

float For example float(-2^9/3^5,12)=-210699588477[-11].

[45] http://www.ctan.org/pkg/etoolbox

sqrt in \xintexpr...\relax and \xintfloatexpr...\relax it achieves the precision given by the optional second argument.

```
\xinttheexpr sqrt(2,31)\relax\ and \xinttheiiexpr sqrt(num(2e60))\relax
```

1414213562373095048801688724210[-30] and 1414213562373095048801688724209

New with 1.2f

factorial when the second optional argument is made use of inside \xintexpr...\relax, this switches to the use of the float version, rather than the exact one.

```
\xinttheexpr factorial (100,32)\relax, {\xintDigits:=32;\xintthefloatexpr
                                          factorial (100)\relax}\newline
\xinttheexpr factorial (50)\relax\newline
\xinttheexpr factorial (50, 32)\relax
```

93326215443944152681699238856267[126], 9.3326215443944152681699238856267e157
30414093201713378043612608166064768844377641568960512000000000000
30414093201713378043612608166065[33]

functions with two arguments

quo first truncates the arguments then computes the Euclidean quotient.

rem first truncates the arguments then computes the Euclidean remainder.

mod computes the modulo associated to the truncated division, same as /: infix operator.

```
\xinttheexpr mod(11/7,1/13), reduce(((11/7)//(1/13))*1/13+mod(11/7,1/13)),
mod(11/7,1/13)- (11/7)/:(1/13), (11/7)//(1/13)\relax
```

3/91, 11/7, 0, 20

New with 1.2f

binomial computes binomial coefficients.

```
\xinttheexpr seq(binomial(20, i), i=0..20)\relax
```

1, 20, 190, 1140, 4845, 15504, 38760, 77520, 125970, 167960, 184756, 167960, 125970, 77520, 38760, 15504, 4845, 1140, 190, 20, 1

```
\xintthefloatexpr seq(binomial(100, 50+i), i=-5..+5)\relax
```

6.144847121413618e28, 7.347099819081500e28, 8.441348728306404e28, 9.320655887504988e28, 9.891308288780803e28, 1.008913445455642e29, 9.891308288780803e28, 9.320655887504988e28, 8.441348728306404e28, 7.347099819081500e28, 6.144847121413618e28 The arguments must be (expand to) short integers.

New with 1.2f

pfactorial computes partial factorials.

```
\xinttheexpr seq(pfactorial(20, i), i=20..30)\relax
```

1, 21, 462, 10626, 255024, 6375600, 165765600, 4475671200, 125318793600, 3634245014400, 109027350432000 The arguments must be (expand to) short integers.

the if conditional (twofold way)

if(cond,yes,no) checks if cond is true or false and takes the corresponding branch. Any non zero number or fraction is logical true. The zero value is logical false. Both ``branches'' are evaluated (they are not really branches but just numbers). See also the ? operator.

the ifsgn conditional (threefold way)

ifsgn(cond,<0,=0,>0) checks the sign of cond and proceeds correspondingly. All three are evaluated. See also the ?? operator.

functions with an arbitrary number of arguments

This argument may well be generated by one or many a..b or a..[d]..b constructs, separated by commas.

all inserts a logical AND in between arguments and evaluates,

any inserts a logical OR in between all arguments and evaluates,

xor inserts a logical XOR in between all arguments and evaluates,

`+` adds (left ticks mandatory):

```
\xinttheexpr `+`(1,3,19), `+`(1*2,3*4,19*20)\relax
```

23, 394

`*` multiplies (left ticks mandatory):

```
\xinttheexpr `*`(1,3,19), `*`(1^2,3^2,19^2), `*`(1*2,3*4,19*20)\relax
```

57, 3249, 9120

max maximum,

min minimum,

gcd first truncates to integers then computes the GCD, requires xintgcd,

lcm first truncates to integers then computes the LCM, requires xintgcd,

first first among comma separated items, first(list) is like [list][:1].

```
\xinttheiiexpr first(-7..3), [-7..3][:1]\relax
```

-7, -7

last last among comma separated items, last(list) is like [list][-1:].

```
\xinttheiiexpr last(-7..3), [-7..3][-1:]\relax
```

3, 3

reversed reverses the order

```
\xinttheiiexpr reversed(123..150)\relax
```

150, 149, 148, 147, 146, 145, 144, 143, 142, 141, 140, 139, 138, 137, 136, 135, 134, 133, 132, 131, 130, 129, 128, 127, 126, 125, 124, 123

New with 1.2g

len computes the number of items in a comma separated list. Earlier syntax was [a,b,...,z][0] but since 1.2g this now returns the first element of the list.

```
\xinttheiiexpr len(1..50, 101..150, 1001..1050)\relax
```

150

4.5 Constructs with dummy variables

The ``functions'' add, mul, seq, subs, rseq, iter, rrseq, iterr use some delimited macros to fetch the ,<letter> part, checking the correct balance of parentheses, and allowing nesting. The <letter> (lowercase or uppercase) will be followed by a = sign and then a comma separated list of values or a list expression such as a..b, or both, which will be treated in the normal manner to provide the values which will be in succession assigned to the <letter>. This <letter> must not have been assigned a value before via \xintdefvar.

The functions rseq, iter, rrseq and iterr have an additional mandatory initial comma separated list which is separated by a semi-colon from the expression to evaluate iteratively. This portion up to and including the semi-colon may arise entirely from expansion (contrarily to the ,<letter>= part discussed above).

seq, rseq, iter, rrseq, iterr but not add, mul, subs admit the omit, abort, and break(..) keywords, possibly but not necessarily in combination with a potentially infinite list generated by a n++ expression.

They may be nested.

Dummy variables are necessarily single-character letters, and all lowercase and uppercase Latin letters are pre-configured for that usage.

subs for variable substitution

```
\xinttheexpr subs(subs(seq(x*z,x=1..10),z=y^2),y=10)\relax\newline
```

100, 200, 300, 400, 500, 600, 700, 800, 900, 1000

Attention that xz generates an error, one must use explicitely x*z, else the parser expects a variable with name xz.

This is useful for example when defining macros for which some argument #1 will be used more than once but may itself be a complicated expression or macro, and should be evaluated only once, for matters of efficiency.

The substituted variable may be a comma separated list (this is impossible with seq which will always pick one item after the other from a list).

```
\xinttheexpr subs([x]^2,x=-123,17,32)\relax
```

15129, 289, 1024

See the examples related to the 3x3 determinant in the subsection 10.10 for an illustration of list substitution.

add addition

```
\xinttheiiexpr add(x^3,x=1..50), add(x(x+1), x=1,3,19)\relax\newline
```

1625625, 394

See `+` for syntax without a dummy variable.

mul multiplication

```
\xinttheiiexpr mul(x^2, x=1,3,19), mul(2n+1,n=1..10)\relax\newline
```

3249, 13749310575

See `*` for syntax without a dummy variable.

seq comma separated values generated according to a formula

```
\xinttheiiexpr seq(x(x+1)(x+2)(x+3),x=1..10), `*`(seq(3x+2,x=1..10))\relax
```

24, 120, 360, 840, 1680, 3024, 5040, 7920, 11880, 17160, 1162274713600

```
\xinttheiiexpr seq(seq(i^2+j^2, i=0..j), j=0..10)\relax
```

0, 1, 2, 4, 5, 8, 9, 10, 13, 18, 16, 17, 20, 25, 32, 25, 26, 29, 34, 41, 50, 36, 37, 40, 45, 52, 61, 72, 49, 50, 53, 58, 65, 74, 85, 98, 64, 65, 68, 73, 80, 89, 100, 113, 128, 81, 82, 85, 90, 97, 106, 117, 130, 145, 162, 100, 101, 104, 109, 116, 125, 136, 149, 164, 181, 200

rseq recursive sequence, @ for the previous value.

```
\printnumber {\xintthefloatexpr subs(rseq (1; @/2+y/2@, i=1..10),y=1000)\relax }\newline
```

1.000000000000000, 500.5000000000000, 251.2490009990010, 127.6145581634591, 67.72532736082604, 41.24542607499115, 32.74526934448864, 31.64201586865079, 31.62278245070105, 31.62277660168434, 31.62277660168379

☞ Attention: in the example above y/2@ is interpreted as y/(2*@). With versions 1.2c or earlier it would have been interpreted as (y/2)*@.

In case the initial stretch is a comma separated list, @ refers at the first iteration to the whole list. Use parentheses at each iteration to maintain this ``nuple''. For example:

```
\printnumber{\xintthefloatexpr rseq(1,10^6;
                 (sqrt([@][0]*[@][1]),([@][0]+[@][1])/2), i=1..7)\relax }
```

1.000000000000000, 1.000000000000000e6, 1000.000000000000, 500000.5000000000, 22360.69095533499, 250500.2500000000, 74842.22521066670, 136430.4704776675, 101048.3052657827, 105636.3478441671, 103316.8617608946, 103342.3265549749, 103329.5933734841, 103329.5941579348, 103329.5937657094, 103329.5937657095

Changed (1.2g)

iter is exactly like rseq, except that it only prints the last iteration. Strangely it was lacking from 1.1 release, or rather what was available from 1.1 to 1.2f is what is called now iterr (described below).

The new iter is convenient to handle compactly higher order iterations. We can illustrate its use with an expandable (!) implementation of the Brent-Salamin algorithm for the computation of π:

```
\xintDigits:= 91;
\xintdeffloatfunc BS(a, b, t, p):= (a+b)/2, sqrt(a*b), t-p(a-b)^2, \xintiiexpr 2p\relax;
\xintthefloatexpr [88] % use 3 guard digits (output value is *rounded*)
  iter(1, 1/sqrt(2), 1, 1; % initial values
      ([@][0]-[@][1]<2[-45])? % if a-b is small enough stop iterating and ...
      {break(([@][0]+[@][1])^2/[@][2])}     % ... do final computation,
      {BS(@)}, % else do iteration via pre-defined (for convenience) function BS.
       i=1++)  % This generates infinite iteration. The i is not used.
\relax
\xintDigits:=16;
```

3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628035

You can try with \xintDigits:=1001; and 2[-501], but don't round to 88 digits of course ... and better wrap the whole thing in \message because it will run in the right margin (about 7.4s on my laptop last time I tried).

rrseq recursive sequence with multiple initial terms. Say, there are K of them. Then @1, ..., @4 and then @@(n) up to n=K refer to the last K values. Notice the difference with rseq for which @ refers to the complete list of all initial terms if there are more than one and may thus be a ``list'' object. This is impossible with rrseq. This construct is effective for scalar finite order recursions, and may be perhaps a bit more efficient than using the rseq syntax with a ``list'' value.

```
\xinttheiiexpr rrseq(0,1; @1+@2, i=2..30)\relax
```

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040

```
\xinttheiiexpr rseq(1; 2@, i=1..10)\relax
```

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

```
\xinttheiiexpr rseq(1; 2@+1, i=1..10)\relax
```

1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047

```
\xinttheiiexpr rseq(2; @(@+1)/2, i=1..5)\relax
```

2, 3, 6, 21, 231, 26796

```
\xinttheiiexpr rrseq(0,1,2,3,4,5; @1+@2+@3+@4+@@(5)+@@(6), i=1..20)\relax
```

0, 1, 2, 3, 4, 5, 15, 30, 59, 116, 229, 454, 903, 1791, 3552, 7045, 13974, 27719, 54984, 109065, 216339, 429126, 851207, 1688440, 3349161, 6643338

I implemented an Rseq which at all times keeps the memory of *all* previous items, but decided to drop it as the package was becoming big.

iterr same as rrseq but does not print any value until the last K.

```
\xinttheiiexpr iterr(0,1; @1+@2, i=2..5, 6..10)\relax
% the iterated over list is allowed to have disjoint defining parts.
```

34, 55

Recursions may be nested, with @@@(n) giving access to the values of the outer recursion... and there is even @@@@(n) to access the outer outer recursion but I never tried it!

With seq, rseq, iter, rrseq, iterr, **but not** with subs, add, mul, one has:

abort stop here and now.

omit omit this value.

break break(stuff) to abort and have stuff as last value.

n++ serves to generate a potentially infinite list. The n++ construct in conjunction with an abort or break is often more efficient, because in other cases the list to iterate over is first completely constructed.

```
\xinttheiiexpr iter(1;(@>10^40)?{break(@)}{2@},i=1++)\relax
```

10889035741470030830827987437816582766592 is the smallest power of 2 with at least fourty one digits.

Note that n++ can not work in the format i=10,17,30++, only <start>++ nothing before.

```
First Fibonacci number at least |2^31| and its index
% we use iterr to refer via @1 and @2 to the previous and previous to previous.
\xinttheiiexpr iterr(0,1; (@1>=2^31)?{break(i)}{@2+@1}, i=1++)\relax
```

First Fibonacci number at least 2^31 and its index 2971215073, 47

Some additional examples are to be found in subsection 10.3.

4.6 User definable variables and functions

Since release 1.1 it is possible to assign a variable name to let it be known to the parsers of xintexpr.

```
\xintdefvar Pi:=3.141592653589793238462643;
\xintthefloatexpr Pi^100\relax
\xintdefvar x_1 := 10;\xintdefvar x_2 := 20;\xintdefvar y@3 := 30;
\quad $x_1\cdot x_2\cdot y@3+1=\xinttheiiexpr x_1*x_2*y@3+1\relax$.
```

5.187848314319574e49 $x_1 \cdot x_2 \cdot y@3 + 1 = 6001$.

As x_1x is a licit variable name, as well as x_1x_ and x_1x_2 and x_1x_2y etc... we could not count on tacit multiplication being applied to something like x_1x_2; the parser goes not go to the effort of tracing back its steps. Hence we had to insert explicit * infix operators (one often falls into this trap when playing with variables and counting too much on the divinatory talents of xintexpr...).

The variable definition is done with \xintdefvar, \xintdefiivar, or with \xintdeffloatvar, the variable will be computed using respectively \xintexpr, \xintiiexpr or \xintfloatexpr. The variable once defined can be used in the other parsers, except naturally that in \xintiiexpr only integers are accepted.

When defining a variable with \xintdeffloatvar, it is important that reduction to \xinttheDigits digits of precision happens inside \xintfloatexpr only if an operation is executed. Thus, for a variable declaration with no operations, the value is registered with all its digits.

```
\xintdeffloatvar e:=2.71828182845904523536028747135266249775724709369995957496696;%
\xinttheexpr         e\relax\newline       % shows the recorded value
\xintthefloatexpr    e\relax\newline       % output rounds
\xintthefloatexpr 1+e\relax\newline % the rounding was done by addition (trust me...)
\xintdeffloatvar e:=float(2.71828182845904523536028747135266249775724709369995957496696);%
\xinttheexpr e\relax\par % use of float forced immediate rounding
```

271828182845904523536028747135266249775724709369995957496696[-61]
2.718281828459045
3.718281828459045
2718281828459045[-15]

In the next examples we examine the effect of cumulated float operations on rounding errors:

```
\xintdefvar       e_1:=add(1/i!, i=0..10);% exact sum
\xintdeffloatvar  e_2:=add(1/i!, i=0..10);% float sum
\xintthefloatexpr e_1, e_2\relax\newline
\xintdefvar       e_3:=e_1+add(1/i!, i=11..20);% exact sum
\xintdeffloatvar  e_4:=e_2+add(1/i!, i=11..20);% float sum
\xintthefloatexpr e_3, e_4\relax\newline
\xintdeffloatvar e:=2.71828182845904523536028747135266249775724709369995957496696;%
\xintDigits:=24;
\xintthefloatexpr[16] e, e^1000, e^1000000\relax (e rounded to 24 digits first)\newline
\xintDigits:=16;
\xintthefloatexpr     e, e^1000, e^1000000\relax (e rounded to 16 digits first)\par
```

2.718281801146384, 2.718281801146385
2.718281828459045, 2.718281828459046
2.718281828459045, 1.970071114017047e434, 3.033215396802088e434294(e rounded to 24 digits first)
2.718281828459045, 1.970071114016876e434, 3.033215396539459e434294(e rounded to 16 digits first)

Legal variable names are composed of letters, digits, @ and _ signs. They can not start with a digit. They may start with @ or _. Currently @, @1, @2, @3, and @4 are reserved because they have special meanings for use in iterations. The @@, @@@, @@@@ are also reserved but are technically functions, not variables. Thus a user may possibly use @@ as a variable name, but if it is followed by parentheses, the function interpretation will be applied, rather than doing a tacit multiplication.

Single letter names a..z and A..Z are pre-declared by the package for use as special type of variables called ``dummy variables''. They can be assigned values via \xintdefvar et al., and be later unassigned using \xintunassign (see later in this section).

Since release 1.2c it is possible to also declare functions:

```
\xintdeffunc
    Rump(x,y):=1335 y^6/4 + x^2 (11 x^2 y^2 - y^6 - 121 y^4 - 2) + 11 y^8/2 + x/2y;
```

(notice the numerous tacit multiplications in this expression; and that x/2y is interpreted as x/(2y).)

> The (dummy) variables used in the function declaration are necessarily single letters (lowercase or uppercase) which have *not* been re-declared via \xintdefvar as assigned variables. The choice of the letters is entirely up to the user and has nil influence on the actual function, naturally.
>
> A function can have at most nine variables.
> 46

Let's try the famous Rump test:

```
\xinttheexpr Rump(77617,33096)\relax.
```

-54767/66192. Nothing problematic for an *exact* evaluation, naturally !

A function may be declared either via \xintdeffunc, \xintdefiifunc, \xintdeffloatfunc. It will then be known *only* to the parser which was used for its definition.

Thus to test the Rump polynomial (it is not quite a polynomial with its x/2y final term) with floats, we *must* also declare Rump as a function to be used there:

```
\xintdeffloatfunc
    Rump(x,y):=333.75 y^6 + x^2 (11 x^2 y^2 - y^6 - 121 y^4 - 2) + 5.5 y^8 + x/2y;
```

(I used coefficients 333.75 and 5.5 rather than fractions only because this is how I saw the polynomial defined in one computer class reference found on internet; and for float operations this may matter on the rounding).

The numbers are scanned with the current precision, hence as here it is 16, they are scanned exactly in this case. We can then vary the precision for the evaluation.

```
\def\CR{\cr}
\halign
{\tabskip1ex
\hfil\bfseries#&\xintDigits:=\xintiloopindex;\xintthefloatexpr Rump(77617,33096)#\cr
\xintiloop [8+1]
\xintiloopindex &\relax\CR
\ifnum\xintiloopindex<40 \repeat
}
```

8 7.0000000e29
9 -1.00000000e28
10 5.000000000e27
11 -3.0000000000e26

[46] with the current syntax, the ; as used for iterr, rseq, rrseq must be hidden as {;} to not be confused with the ; ending the declaration.

```
12 4.00000000000e25
13 3.000000000000e24
14 3.0000000000000e23
15 -2.00000000000000e22
16 1.000000000000000e21
17 -5.0000000000000000e20
18 1.17260394005317863
19 1.000000000000000001e18
20 -9.99999999999999998827e16
21 1.00000000000000011726e16
22 3.000000000000001172604e15
23 -9.999999999999998827396060e13
24 -1.999999999999988273960599e13
25 -1.9999999999998827396059947e12
26 1.1726039400531786318588349
27 -5.99999999988273960599468214e10
28 -9.99999998827396059946821368 1e8
29 2.00000001172603940053178631 86e8
30 1.0000001172603940053178631858 8e7
31 -999998.82739605994682136814 11651
32 200001.17260394005317863185883490
33 -9998.82739605994682136814116509548
34 -1998.827396059946821368141165095480
35 -198.8273960599468213681411650954798 2
36 21.17260394005317863185883490452018 37
37 -0.827396059946821368141165095479816 2920
38 -0.8273960599468213681411650954798162 9200
39 -0.82739605994682136814116509547981629 2000
40 -0.827396059946821368141165095479816291 9990
```

It is licit to overload a variable name (all Latin letters are predefined as dummy variables) with a function name and vice versa. The parsers will decide from the context if the function or variable interpretation must be used (dropping various cases of tacit multiplication as normally applied).

```
\xintdefiifunc f(x):=x^3;
\xinttheiiexpr add(f(f),f=100..120)\relax\newline
\xintdeffunc f(x,y):=x^2+y^2;
\xinttheexpr mul(f(f(f,f),f(f,f)),f=1..10)\relax
```

28205100
1861881348675788854278488064000000000

The mechanism for functions is identical with the one underlying the \xintNewExpr command. A function once declared is a first class citizen, its expression is entirely parsed and converted into a big nested *f*-expandable macro. When used its action is via this defined macro. For example

```
\xintdeffunc
    e(z):=(((((((((z/10+1)z/9+1)z/8+1)z/7+1)z/6+1)z/5+1)z/4+1)z/3+1)z/2+1)z+1;
```

creates a macro whose meaning one can find in the log file, after \xintverbosetrue. Here it is:

```
    Function e for \xintexpr parser associated to \XINT_expr_userfunc_e with me
aning macro:#1,->\xintAdd {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\x
intDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\
xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {
\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {#1}{10}}{1}}{#1}}{9
}}{1}}{#1}}{8}}{1}}{#1}}{7}}{1}}{#1}}{6}}{1}}{#1}}{5}}{1}}{#1}}{4}}{1}}{#1}}{3}
}{1}}{#1}}{2}}{1}}{#1}}{1}
```

See subsubsection 10.10.3 for some limitations of the syntax, shared with those of the \xintNewExpr command. Some constructs with dummy variables will not work, when the iterated-over values depend upon the function arguments. For example `\xintdeffunc f(x):=add(i^2,i=1..x);` leads to an unusable `f`. But in this case one can use the alternative syntax with list operations:[47]

```
\xintdeffunc f(x):=`+`([1..x]^2);\xinttheexpr seq(f(x), x=1..20)\relax
```

1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870

Side remark: as the `seq(f(x), x=1..10)` does many times the same computations, an `rseq` here would be more efficient:[48]

```
\xinttheexpr rseq(1; (x>20)?{abort}{@+x^2}, x=2++)\relax
```

1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870

On the other hand a construct like the following has no issue, as the values iterated over do not depend upon the function parameters:

```
\xintdeffunc f(x):=iter(1{;} @*x/i+1, i=10..1);%  one must hide the first semi-colon !
\xinttheexpr e(1), f(1)\relax
```

9864101/3628800, 9864101/3628800

It is somewhat frustrating not to be able to use the whole xintexpr syntax in `\xintdeffunc` and `\xintNewExpr`. The explanation is simply that the implementation of `seq`, `iter`, etc... relies on exhaustive expansion inside `\csname ... \endcsname` whereas `\xintdeffunc` tries to construct an *f*-expandable macro. Furthermore the `omit` and `abort` keywords as well as the `break()` function are discovered ``dynamically'' when an expression is parsed from left to right; if they were to be used with an abstract value list, the information of their presence would have to be coded especially. This could end up being not that different from storing the whole `seq`, `iter`, etc.. thing ``as is'' into a macro definition:

```
\def\macro #1#2{\xinttheexpr iter(1{;} @*#2/i+1, i=#1..1)\relax}
```

which does not all achieve what a function declaration, if possible, would. Side remark: beware that using it with `#2=1+1` will cause unexpected result, the definition of `\macro` should have employed `(#2)` rather than `#2`.

With `\xintverbosetrue` the values of the variables and the meanings of the functions (or rather their associated macros) will be written to the log. For example the first `Rump` declaration above generates this in the log file:

```
    Function Rump for \xintexpr parser associated to \XINT_expr_userfunc_Rump w
ith meaning macro:#1,#2,->\xintAdd {\xintAdd {\xintAdd {\xintDiv {\xintMul {133
5}{\xintPow {#2}{6}}}{4}}{\xintMul {\xintPow {#1}{2}}{\xintSub {\xintSub {\xint
Sub {\xintMul {11}{\xintMul {\xintPow {#1}{2}}{\xintPow {#2}{2}}}}{\xintPow {#2
}{6}}}{\xintMul {121}{\xintPow {#2}{4}}}}{2}}}}{\xintDiv {\xintMul {11}{\xintPo
w {#2}{8}}}{2}}}{\xintDiv {#1}{\xintMul {2}{#2}}}
```

and the declaration `\xintdeffunc f(x):=iter(1{;} @*x/i+1, i=10..1);` generates:

```
    Function f for \xintexpr parser associated to \XINT_expr_userfunc_f with me
aning macro:#1,->\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\x
intAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\
xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {
\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul
{1}{#1}}{10/1[0]}}{1}}{#1}}{9/1[0]}}{1}}{#1}}{8/1[0]}}{1}}{#1}}{7/1[0]}}{1}}{#1
}}{6/1[0]}}{1}}{#1}}{5/1[0]}}{1}}{#1}}{4/1[0]}}{1}}{#1}}{3/1[0]}}{1}}{#1}}{2/1[
0]}}{1}}{#1}}{1/1[0]}}{1}
```

☞ Starting with 1.2d the definitions made by `\xintNewExpr` have local scope, hence this is also the case with the definitions made by `\xintdeffunc`. One can not ``undeclare'' a function, but naturally one can provide a new definition for it.

☞ Variable declarations also are local. One can not really ``unassign'' a declared variable, but macro `\xintunassignvar` will let it insert a zero and provoke a TeX ``undefined macro'' error. Also, using `\xintunassignvar` on a letter will let it recover fully its original meaning as dummy variable. This may even be used for other characters, if they are used in expressions with catcode 11. As most every character in the ascii range already has some meaning for xintexpr, this is not really recommended, though.

```
\xintFor #1 in {e_1, e_2, e_3, e_4, e} \do {\xintunassignvar {#1}}
```

It is possible to define functions which expand to comma-separated values, for example the declarations:

---

[47] It turns out `` `+`(seq(i^2, i=1..x)) `` would work here, but this isn't always the case with `seq` constructs. [48] Note that `omit` and `abort` are not usable in `add` or `mul` (currently).

```
\xintdeffunc f(x):= x, x^2, x^3, x^x;
\xintdeffunc g(x):= x^[0..x];% x^[1, 2, 3, x] would be like f above.
```

will generate

```
    Function f for \xintexpr parser associated to \XINT_expr_userfunc_f with me
aning macro:#1,->#1,\xintPow {#1}{2},\xintPow {#1}{3},\xintPow {#1}{#1}
    Function g for \xintexpr parser associated to \XINT_expr_userfunc_g with me
aning macro:#1,->\xintApply::csv {\xintPow {#1}}{\xintSeq::csv {0}{#1}}
```

and we can check that they work:

```
\xinttheexpr f(10)\relax; \xinttheexpr g(10)\relax
```

10, 100, 1000, 10000000000; 1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 1000000000, 10000000000

N.B.: we declared in this section e, f, g as functions. Except naturally if the function declarations are done in a group or a LaTeX environment whose scope has ended, they can not be completely undone, and if e, f, or g are used as dummy variables the tacit multiplication in front of parentheses will not be applied, it is their function interpretation which will prevail. However, with an explicit * in front of the opening parenthesis, it does work:

```
\xinttheexpr add(f*(f+f), f= 1..10)\relax % f is used as variable, not function.
```

770

5 Commands of the xintkernel package

The xintkernel package contains mainly the common code base for handling the load-order of the bundle packages, the management of catcodes at loading time, definition of common constants and macro utilities which are used throughout the code etc ... it is automatically loaded by all packages of the bundle.

It provides a few macros possibly useful in other contexts.

5.1 \odef, \oodef, \fdef

\oodef\controlsequence {<stuff>} does

```
        \expandafter\expandafter\expandafter\def
        \expandafter\expandafter\expandafter\controlsequence
        \expandafter\expandafter\expandafter{<stuff>}
```

This works only for a single \controlsequence, with no parameter text, even without parameters. An alternative would be:

```
    \def\oodef #1#{\def\oodefparametertext{#1}%
                   \expandafter\expandafter\expandafter\expandafter
                   \expandafter\expandafter\expandafter\def
                   \expandafter\expandafter\expandafter\oodefparametertext
                   \expandafter\expandafter\expandafter }
```

but it does not allow \global as prefix, and, besides, would have anyhow its use (almost) limited to parameter texts without macro parameter tokens (except if the expanded thing does not see them, or is designed to deal with them).

There is a similar macro \odef with only one expansion of the replacement text <stuff>, and \fdef which expands fully <stuff> using \romannumeral-`0.

They can be prefixed with \global. It appears than \fdef is generally a bit faster than \edef when expanding macros from the xint bundle, when the result has a few dozens of digits. \oodef needs thousands of digits it seems to become competitive.

5.2 \xintReverseOrder

n ★ \xintReverseOrder{⟨*list*⟩} does not do any expansion of its argument and just reverses the order of the tokens in the ⟨*list*⟩. Braces are removed once and the enclosed material, now unbraced, does not get reversed. Unprotected spaces (of any character code) are gobbled.

```
    \xintReverseOrder{\xintDigitsOf\xintiPow {2}{100}\to\Stuff}
    gives: \Stuff\to1002\xintiPow\xintDigitsOf
```

5.3 \xintLength

n ★ \xintLength{⟨*list*⟩} does not do *any* expansion of its argument and just counts how many tokens there are (possibly none). So to use it to count things in the replacement text of a macro one should do \expandafter\xintLength\expandafter{\x}. One may also use it inside macros as \xintLength{#1}. Things enclosed in braces count as one. Blanks between tokens are not counted. See \xintNthElt{0} (from xinttools) for a variant which first *f*-expands its argument.

```
    \xintLength {\xintiPow {2}{100}}=3
     ≠ \xintLen {\xintiPow {2}{100}}=31
```

6 Commands of the xinttools package

These utilities used to be provided within the xint package; since 1.09g (2013/11/22) they have been moved to an independently usable package xinttools, which has none of the xint facilities regarding big numbers. Whenever relevant release 1.09h has made the macros \long so they accept \par tokens on input.

First the completely expandable utilities up to \xintiloop, then the non expandable utilities.

This section contains various concrete examples and ends with a completely expandable implementation of the Quick Sort algorithm together with a graphical illustration of its action.

See also 5.2 and 5.3 which come with package xintkernel, automatically loaded by xinttools.

6.1 \xintRevWithBraces

f ★ \xintRevWithBraces{⟨*list*⟩} first does the *f*-expansion of its argument then it reverses the order of the tokens, or braced material, it encounters, maintaining existing braces and adding a brace pair around each naked token encountered. Space tokens (in-between top level braces or naked tokens) are gobbled. This macro is mainly thought out for use on a ⟨*list*⟩ of such braced material; with such a list as argument the *f*-expansion will only hit against the first opening brace, hence do nothing, and the braced stuff may thus be macros one does not want to expand.

```
\edef\x{\xintRevWithBraces{12345}}
\meaning\x:macro:->{5}{4}{3}{2}{1}
\edef\y{\xintRevWithBraces\x}
\meaning\y:macro:->{1}{2}{3}{4}{5}
```

The examples above could be defined with \edef's because the braced material did not contain macros. Alternatively:

```
\expandafter\def\expandafter\w\expandafter
{\romannumeral0\xintrevwithbraces{{\A}{\B}{\C}{\D}{\E}}}
\meaning\w:macro:->{\E }{\D }{\C }{\B }{\A }
```

n ★ The macro \xintReverseWithBracesNoExpand does the same job without the initial expansion of its argument.

6.2 \xintZapFirstSpaces, \xintZapLastSpaces, \xintZapSpaces, \xintZapSpacesB

n ★ \xintZapFirstSpaces{⟨*stuff*⟩} does not do *any* expansion of its argument, nor brace removal of any sort, nor does it alter ⟨*stuff*⟩ in anyway apart from stripping away all *leading* spaces.

This macro will be mostly of interest to programmers who will know what I will now be talking about. *The essential points, naturally, are the complete expandability and the fact that no brace removal nor any other alteration is done to the input.*

TeX's input scanner already converts consecutive blanks into single space tokens, but \xintZapFirstSpaces handles successfully also inputs with consecutive multiple space tokens. However, it is assumed that ⟨*stuff*⟩ does not contain (except inside braced sub-material) space tokens of character code distinct from 32.

It expands in two steps, and if the goal is to apply it to the expansion text of \x to define \y, then one should do: \expandafter\def\expandafter\y\expandafter {\romannumeral0\expandafter\xintzapfirstspaces\expandafter{\x}}.

Other use case: inside a macro as \edef\x{\xintZapFirstSpaces {#1}} assuming naturally that #1 is compatible with such an \edef once the leading spaces have been stripped.

```
\xintZapFirstSpaces { \a { \X } { \b \Y } }->\a { \X } { \b \Y } +++
```

n ★ \xintZapLastSpaces{⟨*stuff*⟩} does not do *any* expansion of its argument, nor brace removal of any sort, nor does it alter ⟨*stuff*⟩ in anyway apart from stripping away all *ending* spaces. The same remarks as for \xintZapFirstSpaces apply.

```
\xintZapLastSpaces { \a { \X } { \b \Y } }-> \a { \X } { \b \Y }+++
```

n ★ \xintZapSpaces{⟨*stuff*⟩} does not do *any* expansion of its argument, nor brace removal of any sort, nor does it alter ⟨*stuff*⟩ in anyway apart from stripping away all *leading* and all *ending* spaces. The same remarks as for \xintZapFirstSpaces apply.

```
\xintZapSpaces { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
```

n ★ \xintZapSpacesB{⟨*stuff*⟩} does not do *any* expansion of its argument, nor does it alter ⟨*stuff*⟩ in anyway apart from stripping away all leading and all ending spaces and possibly removing one level of braces if ⟨*stuff*⟩ had the shape <spaces>{braced}<spaces>. The same remarks as for \xintZapFirstSpaces apply.

```
\xintZapSpacesB { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
\xintZapSpacesB { { \a { \X } { \b \Y } } }-> \a { \X } { \b \Y } +++
```

The spaces here at the start and end of the output come from the braced material, and are not removed (one would need a second application for that; recall though that the xint zapping macros do not expand their argument).

6.3 \xintCSVtoList

f ★ \xintCSVtoList{a,b,c...,z} returns {a}{b}{c}...{z}. A *list* is by convention in this manual simply a succession of tokens, where each braced thing will count as one item (``items'' are defined according to the rules of TeX for fetching undelimited parameters of a macro, which are exactly the same rules as for LaTeX and command arguments [they are the same things]). The word `list' in `comma separated list of items' has its usual linguistic meaning, and then an ``item'' is what is delimited by commas.

So \xintCSVtoList takes on input a `comma separated list of items' and converts it into a `TeX list of braced items'. The argument to \xintCSVtoList may be a macro: it will first be *f*-expanded. Hence the item before the first comma, if it is itself a macro, will be expanded which may or may not be a good thing. A space inserted at the start of the first item serves to stop that expansion (and
n ★ disappears). The macro \xintCSVtoListNoExpand does the same job without the initial expansion of the list argument.

Apart from that no expansion of the items is done and the list items may thus be completely arbitrary (and even contain perilous stuff such as unmatched \if and \fi tokens).

Contiguous spaces and tab characters, are collapsed by TeX into single spaces. All such spaces around commas[49] are removed, as well as the spaces at the start and the spaces at the end of the list.[50] The items may contain explicit \par's or empty lines (converted by the TeX input parsing into \par tokens).

```
\xintCSVtoList { 1 ,{ 2 , 3 , 4 , 5 }, a , {b,T} U , { c , d } , { {x , y} } }
->{1}{2 , 3 , 4 , 5}{a}{{b,T} U}{ c , d }{ {x , y} }
```

One sees on this example how braces protect commas from sub-lists to be perceived as delimiters of the top list. Braces around an entire item are removed, even when surrounded by spaces before and/or after. Braces for sub-parts of an item are not removed.

We observe also that there is a slight difference regarding the brace stripping of an item: if the braces were not surrounded by spaces, also the initial and final (but no other) spaces of the *enclosed* material are removed. This is the only situation where spaces protected by braces are nevertheless removed.

From the rules above: for an empty argument (only spaces, no braces, no comma) the output is {} (a list with one empty item), for ``<opt. spaces>{}<opt. spaces>'' the output is {} (again a list with one empty item, the braces were removed), for ``{ }'' the output is {} (again a list with one empty item, the braces were removed and then the inner space was removed), for `` { }'' the output is {} (again a list with one empty item, the initial space served only to stop the expansion, so this was like ``{ }'' as input, the braces were removed and the inner space was stripped), for `` { } '' the output is { } (this time the ending space of the first item meant that after brace removal the inner spaces were kept; recall though that TeX collapses on input consecutive blanks into one space token), for ``,'' the output consists of two consecutive empty items {}{}. Recall that on output everything is braced, a {} is an ``empty'' item. Most of the above is mainly irrelevant for every day use, apart perhaps from the fact to be noted that an empty input does not give an empty output but a one-empty-item list (it is as if an ending comma was always added at the end of the input).

```
\def\y{ \a,\b,\c,\d,\e} \xintCSVtoList\y->{\a }{\b }{\c }{\d }{\e }
\def\t {{\if},\ifnum,\ifx,\ifdim,\ifcat,\ifmmode}
\xintCSVtoList\t->{\if }{\ifnum }{\ifx }{\ifdim }{\ifcat }{\ifmmode }
```

The results above were automatically displayed using TeX's primitive \meaning, which adds a space after each control sequence name. These spaces are not in the actual braced items of the produced lists. The first items \a and \if were either preceded by a space or braced to prevent expansion. The macro \xintCSVtoListNoExpand would have done the same job without the initial expansion of the list argument, hence no need for such protection but if \y is defined as \def\y{\a,\b,\c,\d,\e} we then must do:

```
\expandafter\xintCSVtoListNoExpand\expandafter {\y}
```

Else, we may have direct use:

```
\xintCSVtoListNoExpand {\if,\ifnum,\ifx,\ifdim,\ifcat,\ifmmode}
->{\if }{\ifnum }{\ifx }{\ifdim }{\ifcat }{\ifmmode }
```

Again these spaces are an artefact from the use in the source of the document of \meaning (or rather here, \detokenize) to display the result of using \xintCSVtoListNoExpand (which is done for real in this document source).

For the similar conversion from comma separated list to braced items list, but without removal of spaces around the commas, there is \xintCSVtoListNonStripped and \xintCSVtoListNonStrippedNoExpand. nf ★

6.4 \xintNthElt

num x f ★ \xintNthElt{x}{⟨list⟩} gets (expandably) the xth braced item of the ⟨list⟩. An unbraced item token will be returned as is. The list itself may be a macro which is first *f*-expanded.

[49] and multiple space tokens are not a problem; but those at the top level (not hidden inside braces) *must* be of character code 32. [50] let us recall that this is all done completely expandably... There is absolutely no alteration of any sort of the item apart from the stripping of initial and final space tokens (of character code 32) and brace removal if and only if the item apart from intial and final spaces (or more generally multiple char 32 space tokens) is braced.

```
\xintNthElt {3}{{agh}\u{zzz}\v{Z}} is zzz
\xintNthElt {3}{{agh}\u{{zzz}}\v{Z}} is {zzz}
\xintNthElt {2}{{agh}\u{{zzz}}\v{Z}} is \u
\xintNthElt {37}{\xintiiFac {100}}=9 is the thirty-seventh digit of 100!.
\xintNthElt {10}{\xintFtoCv {566827/208524}}=1457/536
is the tenth convergent of 566827/208524 (uses xintcfrac package).
\xintNthElt {7}{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}=7
\xintNthElt {0}{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}=9
\xintNthElt {-3}{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}=7
```

If `x=0`, the macro returns the *length* of the expanded list: this is not equivalent to `\xintLength` which does no pre-expansion. And it is different from `\xintLen` which is to be used only on integers or fractions.

If `x<0`, the macro returns the `|x|`th element from the end of the list.

```
\xintNthElt {-5}{{{agh}}\u{zzz}\v{Z}} is {agh}
```

num x n ★ The macro `\xintNthEltNoExpand` does the same job but without first expanding the list argument: `\xintNthEltNoExpand {-4}{\u\v\w T\x\y\z}` is T.

In cases where `x` is larger (in absolute value) than the length of the list then `\xintNthElt` returns nothing.

6.5 \xintKeep

num x f ★ `\xintKeep{x}{⟨list⟩}` expands the token list argument and returns a new list containing only the first `x` items. If `x<0` the macro returns the last `|x|` elements (in the same order as in the initial list). If `|x|` equals or exceeds the length of the list, the list (as arising from expansion of the second argument) is returned. For `x=0` the empty list is returned.

If `x>0` the (non space) items from the original end up braced in the output: if one later wants to remove all brace pairs (either added to a naked token, or initially present), one may use `\xintListWithSep` with an empty separator.

On the other hand, if `x<0` the macro acts by suppressing items from the head of the list, and no brace pairs are added to the kept elements from the tail (originally present ones are not removed).

Description corrected in 1.2a doc →

`\xintKeepNoExpand` does the same without first *f*-expanding its list argument.

```
\fdef\test {\xintKeep {17}{\xintKeep {-69}{\xintSeq {1}{100}}}}\meaning\test\par
\noindent\fdef\test {\xintKeep {7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintKeep {-7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintKeep {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintKeep {-7}{123456789}}\meaning\test\par
```

macro:->{32}{33}{34}{35}{36}{37}{38}{39}{40}{41}{42}{43}{44}{45}{46}{47}{48}
macro:->{1}{2}{3}{4}{5}{6}{7}
macro:->{3}{4}{5}{6}{7}{8}{9}
macro:->{1}{2}{3}{4}{5}{6}{7}
macro:->3456789

6.6 \xintKeepUnbraced

Sames as `\xintKeep` but no brace pairs are added around the kept items from the head of the list. Each item will lose one level of brace pairs. For `x<0` is not different from `\xintKeep`.

New with 1.2a

`\xintKeepUnbracedNoExpand` does the same without first *f*-expanding its list argument.

```
\fdef\test {\xintKeepUnbraced {10}{\xintSeq {1}{100}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {-7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {-7}{123456789}}\meaning\test\par
```

macro:->12345678910
macro:->1234567

macro:->{3}{4}{5}{6}{7}{8}{9}
macro:->1234567
macro:->3456789

6.7 \xintTrim

num x f ★ \xintTrim{x}{⟨list⟩} expands the list argument and gobbles its first x elements. The remaining ones are left as they are (no brace pairs added). If x<0 the macro gobbles the last |x| elements, and the kept elements from the head of the list end up braced in the output. If |x| equals or exceeds the length of the list, the empty list is returned. For x=0 the full list is returned.

\xintTrimNoExpand does the same without first *f*-expanding its list argument.

```
\fdef\test {\xintTrim {17}{\xintTrim {-69}{\xintSeq {1}{100}}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {-7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintTrim {-7}{123456789}}\meaning\test\par
```

macro:->{18}{19}{20}{21}{22}{23}{24}{25}{26}{27}{28}{29}{30}{31}
macro:->{8}{9}
macro:->{1}{2}
macro:->89
macro:->{1}{2}

6.8 \xintTrimUnbraced

Same as \xintTrim but in case of a negative x (cutting items from the tail), the kept items from the head are not enclosed in brace pairs. They will lose one level of braces.

New with 1.2a

\xintTrimUnbracedNoExpand does the same without first *f*-expanding its list argument.

```
\fdef\test {\xintTrimUnbraced {-90}{\xintSeq {1}{100}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {-7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {-7}{123456789}}\meaning\test\par
```

macro:->12345678910
macro:->{8}{9}
macro:->12
macro:->89
macro:->12

6.9 \xintListWithSep

n f ★ \xintListWithSep{sep}{⟨list⟩} inserts the given separator sep in-between all items of the given list of braced items: this separator may be a macro (or multiple tokens) but will not be expanded. The second argument also may be itself a macro: it is *f*-expanded. Applying \xintListWithSep removes the braces from the list items (for example {1}{2}{3} turns into 1,2,3 via \xintListWithSep{,}{{1}{2}{3}}). An empty input gives an empty output, a singleton gives a singleton, the separator is used starting with at least two elements. Using an empty separator has the net effect of unbracing the braced items constituting the ⟨list⟩ (in such cases the new list may thus be longer than the original).

```
\xintListWithSep{:}{\xintiiFac {20}}=2:4:3:2:9:0:2:0:0:8:1:7:6:6:4:0:0:0:0
```

n n ★ The macro \xintListWithSepNoExpand does the same job without the initial expansion.

6.10 \xintApply

f f ★ \xintApply{\macro}{⟨list⟩} expandably applies the one parameter command \macro to each item in the ⟨*list*⟩ given as second argument and returns a new list with these outputs: each item is given one after the other as parameter to \macro which is expanded at that time (as usual, *i.e.* fully for what comes first), the results are braced and output together as a succession of braced items (if \macro is defined to start with a space, the space will be gobbled and the \macro will not be expanded; it is allowed to have its own arguments, the list items serve as last arguments to \macro). Hence \xintApply{\macro}{{1}{2}{3}} returns {\macro{1}}{\macro{2}}{\macro{3}} where all instances of \macro have been already *f*-expanded.

Being expandable, \xintApply is useful for example inside alignments where implicit groups make standard loops constructs usually fail. In such situation it is often not wished that the new list elements be braced, see \xintApplyUnbraced. The \macro does not have to be expandable: \xintApply will try to expand it, the expansion may remain partial.

The ⟨*list*⟩ may itself be some macro expanding (in the previously described way) to the list of tokens to which the command \macro will be applied. For example, if the ⟨*list*⟩ expands to some positive number, then each digit will be replaced by the result of applying \macro on it.

```
\def\macro #1{\the\numexpr 9-#1\relax}
\xintApply\macro{\xintiiFac {20}}=7567097991823359999
```

fn ★ The macro \xintApplyNoExpand does the same job without the first initial expansion which gave the ⟨*list*⟩ of braced tokens to which \macro is applied.

6.11 \xintApplyUnbraced

ff ★ \xintApplyUnbraced{\macro}{⟨*list*⟩} is like \xintApply. The difference is that after having expanded its list argument, and applied \macro in turn to each item from the list, it reassembles the outputs without enclosing them in braces. The net effect is the same as doing

```
\xintListWithSep {}{\xintApply {\macro}{⟨list⟩}}
```

This is useful for preparing a macro which will itself define some other macros or make assignments, as the scope will not be limited by brace pairs.

```
\def\macro #1{\expandafter\def\csname myself#1\endcsname {#1}}
\xintApplyUnbraced\macro{{elta}{eltb}{eltc}}
\begin{enumerate}[nosep,label=(\arabic{*})]
\item \meaning\myselfelta
\item \meaning\myselfeltb
\item \meaning\myselfeltc
\end{enumerate}
```

(1) macro:->elta
(2) macro:->eltb
(3) macro:->eltc

fn ★ The macro \xintApplyUnbracedNoExpand does the same job without the first initial expansion which gave the ⟨*list*⟩ of braced tokens to which \macro is applied.

6.12 \xintSeq

[num x] num x num x ★ \xintSeq[d]{x}{y} generates expandably {x}{x+d}... up to and possibly including {y} if d>0 or down to and including {y} if d<0. Naturally {y} is omitted if y-x is not a multiple of d. If d=0 the macro returns {x}. If y-x and d have opposite signs, the macro returns nothing. If the optional argument d is omitted it is taken to be the sign of y-x. Hence \xintSeq {1}{0} is not empty but {1}{0}. But \xintSeq [1]{1}{0} is empty.

The arguments x and y are expanded inside a \numexpr so they may be count registers or a LaTeX \value{countername}, or arithmetic with such things.

```
\xintListWithSep{,\hskip2pt plus 1pt minus 1pt }{\xintSeq {12}{-25}}
```

12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11, -12, -13, -14, -15, -16, -17, -18, -19, -20, -21, -22, -23, -24, -25

```
\xintiiSum{\xintSeq [3]{1}{1000}}
```

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When the macro is used without the optional argument d, it can only generate up to about 5000 numbers, the precise value depends upon some TeX memory parameter (input save stack).
With the optional argument d the macro proceeds differently (but less efficiently) and does not stress the input save stack.

6.13 Completely expandable prime test

Let us now construct a completely expandable macro which returns 1 if its given input is prime and 0 if not:

```
\def\remainder #1#2{\the\numexpr #1-(#1/#2)*#2\relax }
\def\IsPrime #1%
 {\xintANDof {\xintApply {\remainder {#1}}{\xintSeq {2}{\xintiSqrt{#1}}}}}
```

This uses \xintiSqrt and assumes its input is at least 5. Rather than xint's own \xintiRem we used a quicker \numexpr expression as we are dealing with short integers. Also we used \xintANDof which will return 1 only if all the items are non-zero. The macro is a bit silly with an even input, ok, let's enhance it to detect an even input:

```
\def\IsPrime #1%
   {\xintifOdd {#1}
        {\xintANDof % odd case
            {\xintApply {\remainder {#1}}
                        {\xintSeq [2]{3}{\xintiSqrt{#1}}}%
            }%
        }
        {\xintifEq {#1}{2}{1}{0}}%
   }
```

We used the xint provided expandable tests (on big integers or fractions) in oder for \IsPrime to be f-expandable.

Our integers are short, but without \expandafter's with \@firstoftwo, or some other related techniques, direct use of \ifnum..\fi tests is dangerous. So to make the macro more efficient we are going to use the expandable tests provided by the package etoolbox[51]. The macro becomes:

```
\def\IsPrime #1%
   {\ifnumodd {#1}
    {\xintANDof % odd case
     {\xintApply {\remainder {#1}}{\xintSeq [2]{3}{\xintiSqrt{#1}}}}}
    {\ifnumequal {#1}{2}{1}{0}}}
```

In the odd case however we have to assume the integer is at least 7, as \xintSeq generates an empty list if #1=3 or 5, and \xintANDof returns 1 when supplied an empty list. Let us ease up a bit \xintANDof's work by letting it work on only 0's and 1's. We could use:

```
\def\IsNotDivisibleBy #1#2%
  {\ifnum\numexpr #1-(#1/#2)*#2=0 \expandafter 0\else \expandafter1\fi}
```

where the \expandafter's are crucial for this macro to be f-expandable and hence work within the applied \xintANDof. Anyhow, now that we have loaded etoolbox, we might as well use:

```
\newcommand{\IsNotDivisibleBy}[2]{\ifnumequal{#1-(#1/#2)*#2}{0}{0}{1}}
```

Let us enhance our prime macro to work also on the small primes:

```
\newcommand{\IsPrime}[1] % returns 1 if #1 is prime, and 0 if not
  {\ifnumodd {#1}
    {\ifnumless {#1}{8}
      {\ifnumequal{#1}{1}{0}{1}}% 3,5,7 are primes
      {\xintANDof
         {\xintApply
       { \IsNotDivisibleBy {#1}}{\xintSeq [2]{3}{\xintiSqrt{#1}}}}%
       }}% END OF THE ODD BRANCH
    {\ifnumequal {#1}{2}{1}{0}}% EVEN BRANCH
}
```

The input is still assumed positive. There is a deliberate blank before \IsNotDivisibleBy to use this feature of \xintApply: a space stops the expansion of the applied macro (and disappears).

[51] http://ctan.org/pkg/etoolbox

This expansion will be done by \xintANDof, which has been designed to skip everything as soon as it finds a false (i.e. zero) input. This way, the efficiency is considerably improved.

We did generate via the \xintSeq too many potential divisors though. Later sections give two variants: one with \xintiloop (subsection 6.16) which is still expandable and another one (subsection 6.23) which is a close variant of the \IsPrime code above but with the \xintFor loop, thus breaking expandability. The xintiloop variant does not first evaluate the integer square root, the xintFor variant still does. I did not compare their efficiencies.

Let us construct with this expandable primality test a table of the prime numbers up to 1000. We need to count how many we have in order to know how many tab stops one shoud add in the last row.[52] There is some subtlety for this last row. Turns out to be better to insert a \\ only when we know for sure we are starting a new row; this is how we have designed the \OneCell macro. And for the last row, there are many ways, we use again \xintApplyUnbraced but with a macro which gobbles its argument and replaces it with a tabulation character. The \xintFor* macro would be more elegant here.

```
\newcounter{primecount}
\newcounter{cellcount}
\newcommand{\NbOfColumns}{13}
\newcommand{\OneCell}[1]{%
    \ifnumequal{\IsPrime{#1}}{1}
     {\stepcounter{primecount}
      \ifnumequal{\value{cellcount}}{\NbOfColumns}
       {\\\setcounter{cellcount}{1}#1}
       {&\stepcounter{cellcount}#1}%
     } % was prime
  {}% not a prime, nothing to do
}
\newcommand{\OneTab}[1]{&}
\begin{tabular}{|*{\NbOfColumns}{r}|}
\hline
2  \setcounter{cellcount}{1}\setcounter{primecount}{1}%
   \xintApplyUnbraced \OneCell {\xintSeq [2]{3}{999}}%
   \xintApplyUnbraced \OneTab
      {\xintSeq [1]{1}{\the\numexpr\NbOfColumns-\value{cellcount}\relax}}%
    \\
\hline
\end{tabular}
There are \arabic{primecount} prime numbers up to 1000.
```

The table has been put in float which appears on the following page. We had to be careful to use in the last row \xintSeq with its optional argument [1] so as to not generate a decreasing sequence from 1 to 0, but really an empty sequence in case the row turns out to already have all its cells (which doesn't happen here but would with a number of columns dividing 168).

6.14 \xintloop, \xintbreakloop, \xintbreakloopanddo, \xintloopskiptonext

☆ \xintloop⟨*stuff*⟩\if<test>...\repeat is an expandable loop compatible with nesting. However to break out of the loop one almost always need some un-expandable step. The cousin \xintiloop is \xintloop with an embedded expandable mechanism allowing to exit from the loop. The iterated commands may contain \par tokens or empty lines.

If a sub-loop is to be used all the material from the start of the main loop and up to the end of the entire subloop should be braced; these braces will be removed and do not create a group. The simplest to allow the nesting of one or more sub-loops is to brace everything between \xintloop and \repeat, being careful not to leave a space between the closing brace and \repeat.

As this loop and \xintiloop will primarily be of interest to experienced TeX macro programmers, my description will assume that the user is knowledgeable enough. Some examples in this document

[52] although a tabular row may have less tabs than in the preamble, there is a problem with the | vertical rule, if one does that.

2	3	5	7	11	13	17	19	23	29	31	37	41
43	47	53	59	61	67	71	73	79	83	89	97	101
103	107	109	113	127	131	137	139	149	151	157	163	167
173	179	181	191	193	197	199	211	223	227	229	233	239
241	251	257	263	269	271	277	281	283	293	307	311	313
317	331	337	347	349	353	359	367	373	379	383	389	397
401	409	419	421	431	433	439	443	449	457	461	463	467
479	487	491	499	503	509	521	523	541	547	557	563	569
571	577	587	593	599	601	607	613	617	619	631	641	643
647	653	659	661	673	677	683	691	701	709	719	727	733
739	743	751	757	761	769	773	787	797	809	811	821	823
827	829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997	

There are 168 prime numbers up to 1000.

will be perhaps more illustrative than my attemps at explanation of use.

One can abort the loop with \xintbreakloop; this should not be used inside the final test, and one should expand the \fi from the corresponding test before. One has also \xintbreakloopanddo whose first argument will be inserted in the token stream after the loop; one may need a macro such as \xint_afterfi to move the whole thing after the \fi, as a simple \expandafter will not be enough.

One will usually employ some count registers to manage the exit test from the loop; this breaks expandability, see \xintiloop for an expandable integer indexed loop. Use in alignments will be complicated by the fact that cells create groups, and also from the fact that any encountered unexpandable material will cause the TeX input scanner to insert \endtemplate on each encountered & or \cr; thus \xintbreakloop may not work as expected, but the situation can be resolved via \xint_firstofone{&} or use of \TAB with \def\TAB{&}. It is thus simpler for alignments to use rather than \xintloop either the expandable \xintApplyUnbraced or the non-expandable but alignment compatible \xintApplyInline, \xintFor or \xintFor*.

As an example, let us suppose we have two macros \A{⟨i⟩}{⟨j⟩} and \B{⟨i⟩}{⟨j⟩} behaving like (small) integer valued matrix entries, and we want to define a macro \C{⟨i⟩}{⟨j⟩} giving the matrix product (i and j may be count registers). We will assume that \A[I] expands to the number of rows, \A[J] to the number of columns and want the produced \C to act in the same manner. The code is very dispendious in use of \count registers, not optimized in any way, not made very robust (the defined macro can not have the same name as the first two matrices for example), we just wanted to quickly illustrate use of the nesting capabilities of \xintloop.[53]

```
\newcount\rowmax   \newcount\colmax   \newcount\summax
\newcount\rowindex \newcount\colindex \newcount\sumindex
\newcount\tmpcount
\makeatletter
\def\MatrixMultiplication #1#2#3{%
    \rowmax #1[I]\relax
    \colmax #2[J]\relax
    \summax #1[J]\relax
    \rowindex 1
    \xintloop % loop over row index i
    {\colindex 1
     \xintloop % loop over col index k
     {\tmpcount 0
      \sumindex 1
```

[53] for a more sophisticated implementation of matrix multiplication, inclusive of determinants, inverses, and display utilities, with entries big integers or decimal numbers or even fractions see http://tex.stackexchange.com/a/143035/4686 from November 11, 2013.

```
      \xintloop % loop over intermediate index j
      \advance\tmpcount \numexpr #1\rowindex\sumindex*#2\sumindex\colindex\relax
      \ifnum\sumindex<\summax
         \advance\sumindex 1
      \repeat }%
     \expandafter\edef\csname\string#3{\the\rowindex.\the\colindex}\endcsname
      {\the\tmpcount}%
     \ifnum\colindex<\colmax
         \advance\colindex 1
     \repeat }%
    \ifnum\rowindex<\rowmax
    \advance\rowindex 1
    \repeat
    \expandafter\edef\csname\string#3{I}\endcsname{\the\rowmax}%
    \expandafter\edef\csname\string#3{J}\endcsname{\the\colmax}%
    \def #3##1{\ifx[##1\expandafter\Matrix@helper@size
                    \else\expandafter\Matrix@helper@entry\fi #3{##1}}%
}%
\def\Matrix@helper@size #1#2#3]{\csname\string#1{#3}\endcsname }%
\def\Matrix@helper@entry #1#2#3%
   {\csname\string#1{\the\numexpr#2.\the\numexpr#3}\endcsname }%
\def\A #1{\ifx[#1\expandafter\A@size
            \else\expandafter\A@entry\fi {#1}}%
\def\A@size #1#2]{\ifx I#23\else4\fi}% 3rows, 4columns
\def\A@entry #1#2{\the\numexpr #1+#2-1\relax}% not pre-computed...
\def\B #1{\ifx[#1\expandafter\B@size
            \else\expandafter\B@entry\fi {#1}}%
\def\B@size #1#2]{\ifx I#24\else3\fi}% 4rows, 3columns
\def\B@entry #1#2{\the\numexpr #1-#2\relax}% not pre-computed...
\makeatother
\MatrixMultiplication\A\B\C \MatrixMultiplication\C\C\D
\MatrixMultiplication\C\D\E \MatrixMultiplication\C\E\F
\begin{multicols}2
  \[\begin{pmatrix}
    \A11&\A12&\A13&\A14\\
    \A21&\A22&\A23&\A24\\
    \A31&\A32&\A33&\A34
  \end{pmatrix}
  \times
  \begin{pmatrix}
    \B11&\B12&\B13\\
    \B21&\B22&\B23\\
    \B31&\B32&\B33\\
    \B41&\B42&\B43
  \end{pmatrix}
  =
  \begin{pmatrix}
    \C11&\C12&\C13\\
    \C21&\C22&\C23\\
    \C31&\C32&\C33
  \end{pmatrix}\]
  \[\begin{pmatrix}
    \C11&\C12&\C13\\
    \C21&\C22&\C23\\
    \C31&\C32&\C33
  \end{pmatrix}^2 = \begin{pmatrix}
    \D11&\D12&\D13\\
```

```
      \D21&\D22&\D23\\
      \D31&\D32&\D33
    \end{pmatrix}\]
    \[\begin{pmatrix}
      \C11&\C12&\C13\\
      \C21&\C22&\C23\\
      \C31&\C32&\C33
    \end{pmatrix}^3 = \begin{pmatrix}
      \E11&\E12&\E13\\
      \E21&\E22&\E23\\
      \E31&\E32&\E33
    \end{pmatrix}\]
    \[\begin{pmatrix}
      \C11&\C12&\C13\\
      \C21&\C22&\C23\\
      \C31&\C32&\C33
    \end{pmatrix}^4 = \begin{pmatrix}
      \F11&\F12&\F13\\
      \F21&\F22&\F23\\
      \F31&\F32&\F33
    \end{pmatrix}\]
  \end{multicols}
```

$$\begin{pmatrix}1&2&3&4\\2&3&4&5\\3&4&5&6\end{pmatrix}\times\begin{pmatrix}0&-1&-2\\1&0&-1\\2&1&0\\3&2&1\end{pmatrix}=\begin{pmatrix}20&10&0\\26&12&-2\\32&14&-4\end{pmatrix}$$

$$\begin{pmatrix}20&10&0\\26&12&-2\\32&14&-4\end{pmatrix}^2=\begin{pmatrix}660&320&-20\\768&376&-16\\876&432&-12\end{pmatrix}$$

$$\begin{pmatrix}20&10&0\\26&12&-2\\32&14&-4\end{pmatrix}^3=\begin{pmatrix}20880&10160&-560\\24624&11968&-688\\28368&13776&-816\end{pmatrix}$$

$$\begin{pmatrix}20&10&0\\26&12&-2\\32&14&-4\end{pmatrix}^4=\begin{pmatrix}663840&322880&-18080\\781632&380224&-21184\\899424&437568&-24288\end{pmatrix}$$

6.15 `\xintiloop`, `\xintiloopindex`, `\xintouteriloopindex`, `\xintbreakiloop`, `\xintbreakiloopanddo`, `\xintiloopskiptonext`, `\xintiloopskipandredo`

☆ `\xintiloop[start+delta]`⟨*stuff*⟩`\if<test> ... \repeat` is a completely expandable nestable loop. complete expandability depends naturally on the actual iterated contents, and complete expansion will not be achievable under a sole *f*-expansion, as is indicated by the hollow star in the margin; thus the loop can be used inside an `\edef` but not inside arguments to the package macros. It can be used inside an `\xintexpr..\relax`. The `[start+delta]` is mandatory, not optional.

This loop benefits via `\xintiloopindex` to (a limited access to) the integer index of the iteration. The starting value `start` (which may be a `\count`) and increment `delta` (*id.*) are mandatory arguments. A space after the closing square bracket is not significant, it will be ignored. Spaces inside the square brackets will also be ignored as the two arguments are first given to a `\numexpr...\relax`. Empty lines and explicit `\par` tokens are accepted.

As with `\xintloop`, this tool will mostly be of interest to advanced users. For nesting, one puts inside braces all the material from the start (immediately after `[start+delta]`) and up to and inclusive of the inner loop, these braces will be removed and do not create a loop. In case of nesting, `\xintouteriloopindex` gives access to the index of the outer loop. If needed one could write on its model a macro giving access to the index of the outer outer loop (or even to the nth outer loop).

The `\xintiloopindex` and `\xintouteriloopindex` can not be used inside braces, and generally speaking this means they should be expanded first when given as argument to a macro, and that this macro receives them as delimited arguments, not braced ones. Or, but naturally this will break expandability, one can assign the value of \xintiloopindex to some \count. Both \xintiloopindex and \xintouteriloopindex extend to the litteral representation of the index, thus in \ifnum tests, if it comes last one has to correctly end the macro with a \space, or encapsulate it in a \numexpr..\relax.

When the repeat-test of the loop is, for example, \ifnum\xintiloopindex<10 \repeat, this means that the last iteration will be with \xintiloopindex=10 (assuming delta=1). There is also \ifnum\xintiloopindex=10 \else\repeat to get the last iteration to be the one with \xintiloopindex=10.

One has \xintbreakiloop and \xintbreakiloopanddo to abort the loop. The syntax of \xintbreakiloopanddo is a bit surprising, the sequence of tokens to be executed after breaking the loop is not within braces but is delimited by a dot as in:

```
\xintbreakiloopanddo <afterloop>.etc.. etc... \repeat
```

The reason is that one may wish to use the then current value of \xintiloopindex in <afterloop> but it can't be within braces at the time it is evaluated. However, it is not that easy as \xintiloopindex must be expanded before, so one ends up with code like this:

```
\expandafter\xintbreakiloopanddo\expandafter\macro\xintiloopindex.%
etc.. etc.. \repeat
```

As moreover the \fi from the test leading to the decision of breaking out of the loop must be cleared out of the way, the above should be a branch of an expandable conditional test, else one needs something such as:

```
\xint_afterfi{\expandafter\xintbreakiloopanddo\expandafter\macro\xintiloopindex.}%
\fi etc..etc.. \repeat
```

There is \xintiloopskiptonext to abort the current iteration and skip to the next, \xintiloopskipandredo to skip to the end of the current iteration and redo it with the same value of the index (something else will have to change for this not to become an eternal loop...).

Inside alignments, if the looped-over text contains a & or a \cr, any un-expandable material before a \xintiloopindex will make it fail because of \endtemplate; in such cases one can always either replace & by a macro expanding to it or replace it by a suitable \firstofone{&}, and similarly for \cr.

As an example, let us construct an \edef\z{...} which will define \z to be a list of prime numbers:

```
\begingroup
\edef\z
{\xintiloop [10001+2]
  {\xintiloop [3+2]
   \ifnum\xintouteriloopindex<\numexpr\xintiloopindex*\xintiloopindex\relax
          \xintouteriloopindex,
          \expandafter\xintbreakiloop
   \fi
   \ifnum\xintouteriloopindex=\numexpr
        (\xintouteriloopindex/\xintiloopindex)*\xintiloopindex\relax
   \else
   \repeat
  }% no space here
 \ifnum \xintiloopindex < 10999 \repeat }%
\meaning\z\endgroup
```

macro:->10007, 10009, 10037, 10039, 10061, 10067, 10069, 10079, 10091, 10093, 10099, 10103, 10111, 10133, 10139, 10141, 10151, 10159, 10163, 10169, 10177, 10181, 10193, 10211, 10223, 10243, 10247, 10253, 10259, 10267, 10271, 10273, 10289, 10301, 10303, 10313, 10321, 10331, 10333, 10337, 10343, 10357, 10369, 10391, 10399, 10427, 10429, 10433, 10453, 10457, 10459, 10463, 10477, 10487, 10499, 10501, 10513, 10529, 10531, 10559, 10567, 10589, 10597, 10601, 10607, 10613, 10627, 10631, 10639, 10651, 10657, 10663, 10667, 10687, 10691, 10709, 10711, 10723, 10729, 10733, 10739, 10753, 10771, 10781, 10789, 10799, 10831, 10837, 10847, 10853, 10859, 10861, 10867, 10883, 10889, 10891, 10903, 10909, 10937, 10939, 10949, 10957, 10973, 10979, 10987, 10993, and we should have taken some steps to not have a trailing comma, but the point was to show that one can do that in an \edef! See also subsection 6.16 which extracts from this code its way of testing primality.

Let us create an alignment where each row will contain all divisors of its first entry. Here is the output, thus obtained without any count register:

```
\begin{multicols}2
\tabskip1ex \normalcolor
\halign{&\hfil#\hfil\cr
    \xintiloop [1+1]
    {\expandafter\bfseries\xintiloopindex &
     \xintiloop [1+1]
     \ifnum\xintouteriloopindex=\numexpr
           (\xintouteriloopindex/\xintiloopindex)*\xintiloopindex\relax
     \xintiloopindex&\fi
     \ifnum\xintiloopindex<\xintouteriloopindex\space % CRUCIAL \space HERE
     \repeat \cr }%
    \ifnum\xintiloopindex<30
    \repeat
}
\end{multicols}
```

1	1							
2	1	2						
3	1	3						
4	1	2	4					
5	1	5						
6	1	2	3	6				
7	1	7						
8	1	2	4	8				
9	1	3	9					
10	1	2	5	10				
11	1	11						
12	1	2	3	4	6	12		
13	1	13						
14	1	2	7	14				
15	1	3	5	15				
16	1	2	4	8	16			
17	1	17						
18	1	2	3	6	9	18		
19	1	19						
20	1	2	4	5	10	20		
21	1	3	7	21				
22	1	2	11	22				
23	1	23						
24	1	2	3	4	6	8	12	24
25	1	5	25					
26	1	2	13	26				
27	1	3	9	27				
28	1	2	4	7	14	28		
29	1	29						
30	1	2	3	5	6	10	15	30

We wanted this first entry in bold face, but \bfseries leads to unexpandable tokens, so the \expandafter was necessary for \xintiloopindex and \xintouteriloopindex not to be confronted with a hard to digest \endtemplate. An alternative way of coding:

```
\tabskip1ex
\def\firstofone #1{#1}%
\halign{&\hfil#\hfil\cr
  \xintiloop [1+1]
    {\bfseries\xintiloopindex\firstofone{&}%
    \xintiloop [1+1] \ifnum\xintouteriloopindex=\numexpr
    (\xintouteriloopindex/\xintiloopindex)*\xintiloopindex\relax
    \xintiloopindex\firstofone{&}\fi
    \ifnum\xintiloopindex<\xintouteriloopindex\space % \space is CRUCIAL
    \repeat \firstofone{\cr}}%
  \ifnum\xintiloopindex<30 \repeat }
```

6.16 Another completely expandable prime test

The \IsPrime macro from subsection 6.13 checked expandably if a (short) integer was prime, here is a partial rewrite using \xintiloop. We use the etoolbox expandable conditionals for convenience, but not everywhere as \xintiloopindex can not be evaluated while being braced. This is also the reason why \xintbreakiloopanddo is delimited, and the next macro \SmallestFactor which returns the smallest prime factor examplifies that. One could write more efficient completely expandable routines, the aim here was only to illustrate use of the general purpose \xintiloop. A little table giving the first values of \SmallestFactor follows, its coding uses \xintFor, which is described later; none of this uses count registers.

```
\let\IsPrime\undefined \let\SmallestFactor\undefined % clean up possible previous mess
\newcommand{\IsPrime}[1] % returns 1 if #1 is prime, and 0 if not
  {\ifnumodd {#1}
    {\ifnumless {#1}{8}
      {\ifnumequal{#1}{1}{0}{1}}% 3,5,7 are primes
      {\if
       \xintiloop [3+2]
       \ifnum#1<\numexpr\xintiloopindex*\xintiloopindex\relax
           \expandafter\xintbreakiloopanddo\expandafter1\expandafter.%
       \fi
       \ifnum#1=\numexpr (#1/\xintiloopindex)*\xintiloopindex\relax
       \else
       \repeat 00\expandafter0\else\expandafter1\fi
      }%
    }% END OF THE ODD BRANCH
    {\ifnumequal {#1}{2}{1}{0}}% EVEN BRANCH
}%
\catcode`_ 11
\newcommand{\SmallestFactor}[1] % returns the smallest prime factor of #1>1
  {\ifnumodd {#1}
    {\ifnumless {#1}{8}
      {#1}% 3,5,7 are primes
      {\xintiloop [3+2]
       \ifnum#1<\numexpr\xintiloopindex*\xintiloopindex\relax
           \xint_afterfi{\xintbreakiloopanddo#1.}%
       \fi
       \ifnum#1=\numexpr (#1/\xintiloopindex)*\xintiloopindex\relax
           \xint_afterfi{\expandafter\xintbreakiloopanddo\xintiloopindex.}%
       \fi
       \iftrue\repeat
      }%
     }% END OF THE ODD BRANCH
   {2}% EVEN BRANCH
}%
\catcode`_ 8
{\centering
  \begin{tabular}{|c|*{10}c|}
    \hline
    \xintFor #1 in {0,1,2,3,4,5,6,7,8,9}\do {&\bfseries #1}\\
    \hline
    \bfseries 0&--&--&2&3&2&5&2&7&2&3\\
    \xintFor #1 in {1,2,3,4,5,6,7,8,9}\do
    {\bfseries #1%
      \xintFor #2 in {0,1,2,3,4,5,6,7,8,9}\do
      {&\SmallestFactor{#1#2}}\\}%
    \hline
  \end{tabular}\par
}
```

	0	1	2	3	4	5	6	7	8	9
0	--	--	2	3	2	5	2	7	2	3
1	2	11	2	13	2	3	2	17	2	19
2	2	3	2	23	2	5	2	3	2	29
3	2	31	2	3	2	5	2	37	2	3
4	2	41	2	43	2	3	2	47	2	7
5	2	3	2	53	2	5	2	3	2	59
6	2	61	2	3	2	5	2	67	2	3
7	2	71	2	73	2	3	2	7	2	79
8	2	3	2	83	2	5	2	3	2	89
9	2	7	2	3	2	5	2	97	2	3

6.17 A table of factorizations

As one more example with \xintiloop let us use an alignment to display the factorization of some numbers. The loop will actually only play a minor rôle here, just handling the row index, the row contents being almost entirely produced via a macro \factorize. The factorizing macro does not use \xintiloop as it didn't appear to be the convenient tool. As \factorize will have to be used on \xintiloopindex, it has been defined as a delimited macro.

To spare some fractions of a second in the compilation time of this document (which has many many other things to do), 2147483629 and 2147483647, which turn out to be prime numbers, are not given to factorize but just typeset directly; this illustrates use of \xintiloopskiptonext.

The code next generates a table which has been made into a float appearing on the next page. Here is now the code for factorization; the conditionals use the package provided \xint_firstoftwo and \xint_secondoftwo, one could have employed rather LaTeX's own \@firstoftwo and \@secondoftwo, or, simpler still in LaTeX context, the \ifnumequal, \ifnumless . . . , utilities from the package etoolbox which do exactly that under the hood. Only TeX acceptable numbers are treated here, but it would be easy to make a translation and use the xint macros, thus extending the scope to big numbers; naturally up to a cost in speed.

The reason for some strange looking expressions is to avoid arithmetic overflow.

```
\catcode`_ 11
\def\abortfactorize #1\xint_secondoftwo\fi #2#3{\fi}

\def\factorize #1.{\ifnum#1=1 \abortfactorize\fi
          \ifnum\numexpr #1-2=\numexpr ((#1/2)-1)*2\relax
               \expandafter\xint_firstoftwo
          \else\expandafter\xint_secondoftwo
          \fi
         {2&\expandafter\factorize\the\numexpr#1/2.}%
         {\factorize_b #1.3.}}%

\def\factorize_b #1.#2.{\ifnum#1=1 \abortfactorize\fi
         \ifnum\numexpr #1-(#2-1)*#2<#2
                 #1\abortfactorize
         \fi
         \ifnum \numexpr #1-#2=\numexpr ((#1/#2)-1)*#2\relax
              \expandafter\xint_firstoftwo
         \else\expandafter\xint_secondoftwo
         \fi
         {#2&\expandafter\factorize_b\the\numexpr#1/#2.#2.}%
         {\expandafter\factorize_b\the\numexpr #1\expandafter.%
                                   \the\numexpr #2+2.}}%
\catcode`_ 8
\begin{figure*}[ht!]
\centering\phantomsection\label{floatfactorize}\normalcolor
```

```
\tabskip1ex
\centeredline{\vbox{\halign {\hfil\strut#\hfil&&\hfil#\hfil\cr\noalign{\hrule}
          \xintiloop ["7FFFFFE0+1]
          \expandafter\bfseries\xintiloopindex &
          \ifnum\xintiloopindex="7FFFFFED
               \number"7FFFFFED\cr\noalign{\hrule}
          \expandafter\xintiloopskiptonext
          \fi
          \expandafter\factorize\xintiloopindex.\cr\noalign{\hrule}
          \ifnum\xintiloopindex<"7FFFFFFE
          \repeat
          \bfseries \number"7FFFFFFF&\number "7FFFFFFF\cr\noalign{\hrule}
}}}
\centeredline{A table of factorizations}
\end{figure*}
```

2147483616	2	2	2	2	2	3	2731	8191	
2147483617	6733	318949							
2147483618	2	7	367	417961					
2147483619	3	3	23	353	29389				
2147483620	2	2	5	4603	23327				
2147483621	14741	145681							
2147483622	2	3	17	467	45083				
2147483623	79	967	28111						
2147483624	2	2	2	11	13	1877171			
2147483625	3	5	5	5	7	199	4111		
2147483626	2	19	37	1527371					
2147483627	47	53	862097						
2147483628	2	2	3	3	59652323				
2147483629	2147483629								
2147483630	2	5	6553	32771					
2147483631	3	137	263	19867					
2147483632	2	2	2	2	7	73	262657		
2147483633	5843	367531							
2147483634	2	3	12097	29587					
2147483635	5	11	337	115861					
2147483636	2	2	536870909						
2147483637	3	3	3	13	6118187				
2147483638	2	2969	361651						
2147483639	7	17	18046081						
2147483640	2	2	2	3	5	29	43	113	127
2147483641	2699	795659							
2147483642	2	23	46684427						
2147483643	3	715827881							
2147483644	2	2	233	1103	2089				
2147483645	5	19	22605091						
2147483646	2	3	3	7	11	31	151	331	
2147483647	2147483647								

A table of factorizations

The next utilities are not compatible with expansion-only context.

6.18 \xintApplyInline

*o *f* \xintApplyInline{\macro}{⟨*list*⟩} works non expandably. It applies the one-parameter \macro to the first element of the expanded list (\macro may have itself some arguments, the list item will be appended as last argument), and is then re-inserted in the input stream after the tokens resulting from this first expansion of \macro. The next item is then handled.

This is to be used in situations where one needs to do some repetitive things. It is not expandable and can not be completely expanded inside a macro definition, to prepare material for later execution, contrarily to what \xintApply or \xintApplyUnbraced achieve.

```
\def\Macro #1{\advance\cnta #1 , \the\cnta}
\cnta 0
0\xintApplyInline\Macro {3141592653}.
```

0, 3, 4, 8, 9, 14, 23, 25, 31, 36, 39. The first argument \macro does not have to be an expandable macro.

\xintApplyInline submits its second, token list parameter to an *f*-expansion. Then, each *unbraced* item will also be *f*-expanded. This provides an easy way to insert one list inside another. *Braced* items are not expanded. Spaces in-between items are gobbled (as well as those at the start or the end of the list), but not the spaces *inside* the braced items.

\xintApplyInline, despite being non-expandable, does survive to contexts where the executed \macro closes groups, as happens inside alignments with the tabulation character &. This tabular provides an example:

```
\centerline{\normalcolor\begin{tabular}{ccc}
     $N$ & $N^2$ & $N^3$ \\ \hline
     \def\Row #1{ #1 & \xintiiSqr {#1} & \xintiiPow {#1}{3} \\ \hline }%
     \xintApplyInline \Row {\xintCSVtoList{17,28,39,50,61}}
\end{tabular}}\medskip
```

N	N^2	N^3
17	289	4913
28	784	21952
39	1521	59319
50	2500	125000
61	3721	226981

We see that despite the fact that the first encountered tabulation character in the first row close a group and thus erases \Row from TEX's memory, \xintApplyInline knows how to deal with this.

Using \xintApplyUnbraced is an alternative: the difference is that this would have prepared all rows first and only put them back into the token stream once they are all assembled, whereas with \xintApplyInline each row is constructed and immediately fed back into the token stream: when one does things with numbers having hundreds of digits, one learns that keeping on hold and shuffling around hundreds of tokens has an impact on TEX's speed (make this ``thousands of tokens'' for the impact to be noticeable).

One may nest various \xintApplyInline's. For example (see the table on the following page):

```
\begin{figure*}[ht!]
  \centering\phantomsection\label{float}
  \def\Row #1{#1:\xintApplyInline {\Item {#1}}{0123456789}\\ }%
  \def\Item #1#2{&\xintiPow {#1}{#2}}%
  \centeredline {\begin{tabular}{ccccccccccc} &0&1&2&3&4&5&6&7&8&9\\ \hline
      \xintApplyInline \Row {0123456789}
    \end{tabular}}
\end{figure*}
```

	0	1	2	3	4	5	6	7	8	9
0:	1	0	0	0	0	0	0	0	0	0
1:	1	1	1	1	1	1	1	1	1	1
2:	1	2	4	8	16	32	64	128	256	512
3:	1	3	9	27	81	243	729	2187	6561	19683
4:	1	4	16	64	256	1024	4096	16384	65536	262144
5:	1	5	25	125	625	3125	15625	78125	390625	1953125
6:	1	6	36	216	1296	7776	46656	279936	1679616	10077696
7:	1	7	49	343	2401	16807	117649	823543	5764801	40353607
8:	1	8	64	512	4096	32768	262144	2097152	16777216	134217728
9:	1	9	81	729	6561	59049	531441	4782969	43046721	387420489

One could not move the definition of \Item inside the tabular, as it would get lost after the first &. But this works:

```
\begin{tabular}{ccccccccccc}
    &0&1&2&3&4&5&6&7&8&9\\ \hline
    \def\Row #1{#1:\xintApplyInline {&\xintiPow {#1}}{0123456789}\\ }%
    \xintApplyInline \Row {0123456789}
\end{tabular}
```

A limitation is that, contrarily to what one may have expected, the \macro for an \xintApplyInline can not be used to define the \macro for a nested sub-\xintApplyInline. For example, this does not work:

```
  \def\Row #1{#1:\def\Item ##1{&\xintiPow {#1}{##1}}%
                \xintApplyInline \Item {0123456789}\\ }%
  \xintApplyInline \Row {0123456789} % does not work
```

But see \xintFor.

6.19 \xintFor, \xintFor*

on \xintFor is a new kind of for loop. Rather than using macros for encapsulating list items, its behavior is more like a macro with parameters: #1, #2, . . . , #9 are used to represent the items for up to nine levels of nested loops. Here is an example:

```
\xintFor #9 in {1,2,3} \do {%
  \xintFor #1 in {4,5,6} \do {%
    \xintFor #3 in {7,8,9} \do {%
      \xintFor #2 in {10,11,12} \do {%
      $$#9\times#1\times#3\times#2=\xintiiPrd{{#1}{#2}{#3}{#9}}$$}}}}
```

This example illustrates that one does not have to use #1 as the first one: the order is arbitrary. But each level of nesting should have its specific macro parameter. Nine levels of nesting is presumably overkill, but I did not know where it was reasonable to stop. \par tokens are accepted in both the comma separated list and the replacement text.

> A macro \macro whose definition uses internally an \xintFor loop may be used inside another \xintFor loop even if the two loops both use the same macro parameter. Note: the loop definition inside \macro must double the character # as is the general rule in TeX with definitions done inside macros.
>
> The macros \xintFor and \xintFor* are not expandable, one can not use them inside an \edef. But they may be used inside alignments (such as a LaTeX tabular), as will be shown in examples.

The spaces between the various declarative elements are all optional; furthermore spaces around the commas or at the start and end of the list argument are allowed, they will be removed. If an item must contain itself commas, it should be braced to prevent these commas from being misinterpreted as list separator. These braces will be removed during processing. The list argument may be a macro

\MyList expanding in one step to the comma separated list (if it has no arguments, it does not have to be braced). It will be expanded (only once) to reveal its comma separated items for processing, comma separated items will not be expanded before being fed into the replacement text as #1, or #2, etc..., only leading and trailing spaces are removed.

fn A starred variant \xintFor deals with lists of braced items, rather than comma separated items. It has also a distinct expansion policy, which is detailed below.

Contrarily to what happens in loops where the item is represented by a macro, here it is truly exactly as when defining (in LaTeX) a ``command'' with parameters #1, etc... This may avoid the user quite a few troubles with \expandafters or other \edef/\noexpands which one encounters at times when trying to do things with LaTeX's \@for or other loops which encapsulate the item in a macro expanding to that item.

The non-starred variant \xintFor deals with comma separated values (*spaces before and after the commas are removed*) and the comma separated list may be a macro which is only expanded once (to prevent expansion of the first item \x in a list directly input as \x,\y,... it should be input as {\x},\y,.. or <space>\x,\y,.., naturally all of that within the mandatory braces of the \xintFor #n in {list} syntax). The items are not expanded, if the input is <stuff>,\x,<stuff> then #1 will be at some point \x not its expansion (and not either a macro with \x as replacement text, just the token \x). Input such as <stuff>,,<stuff> creates an empty #1, the iteration is not skipped. An empty list does lead to the use of the replacement text, once, with an empty #1 (or #n). Except if the entire list is represented as a single macro with no parameters, it must be braced.

The starred variant \xintFor* deals with token lists (*spaces between braced items or single tokens are not significant*) and *f*-expands each *unbraced* list item. This makes it easy to simulate concatenation of various list macros \x, \y, ... If \x expands to {1}{2}{3} and \y expands to {4}{5}{6} then {\x\y} as argument to \xintFor* has the same effect as {{1}{2}{3}{4}{5}{6}}[54]. Spaces at the start, end, or in-between items are gobbled (but naturally not the spaces which may be inside *braced* items). Except if the list argument is a single macro with no parameters, it must be braced. Each item which is not braced will be fully expanded (as the \x and \y in the example above). An empty list leads to an empty result.

The macro \xintSeq which generates arithmetic sequences may only be used with \xintFor* (numbers from output of \xintSeq are braced, not separated by commas).

```
\xintFor* #1 in {\xintSeq [+2]{-7}{+2}}\do {stuff with #1}
```

will have #1=-7,-5,-3,-1, and 1. The #1 as issued from the list produced by \xintSeq is the literal representation as would be produced by \arabic on a LaTeX counter, it is not a count register. When used in \ifnum tests or other contexts where TeX looks for a number it should thus be postfixed with \relax or \space.

When nesting \xintFor* loops, using \xintSeq in the inner loops is inefficient, as the arithmetic sequence will be re-created each time. A more efficient style is:

```
\edef\innersequence {\xintSeq[+2]{-50}{50}}%
\xintFor* #1 in {\xintSeq {13}{27}} \do
    {\xintFor* #2 in \innersequence \do {stuff with #1 and #2}%
     .. some other macros .. }
```

This is a general remark applying for any nesting of loops, one should avoid recreating the inner lists of arguments at each iteration of the outer loop. However, in the example above, if the .. some other macros .. part closes a group which was opened before the \edef\innersequence, then this definition will be lost. An alternative to \edef, also efficient, exists when dealing with

[53] braces around single token items are optional so this is the same as {123456}.

arithmetic sequences: it is to use the \xintintegers keyword (described later) which simulates infinite arithmetic sequences; the loops will then be terminated via a test #1 (or #2 etc...) and subsequent use of \xintBreakFor.

The \xintFor loops are not completely expandable; but they may be nested and used inside alignments or other contexts where the replacement text closes groups. Here is an example (still using LaTeX's tabular):

```
\begin{tabular}{rccccc}
    \xintFor #7 in {A,B,C} \do {%
      #7:\xintFor* #3 in {abcde} \do {&($ #3 \to #7 $)}\\ }%
\end{tabular}
```

A:	(a → A)	(b → A)	(c → A)	(d → A)	(e → A)
B:	(a → B)	(b → B)	(c → B)	(d → B)	(e → B)
C:	(a → C)	(b → C)	(c → C)	(d → C)	(e → C)

When inserted inside a macro for later execution the # characters must be doubled.[54] For example:

```
\def\T{\def\z {}%
  \xintFor* ##1 in {{u}{v}{w}} \do {%
    \xintFor ##2 in {x,y,z} \do {%
      \expandafter\def\expandafter\z\expandafter {\z\sep (##1,##2)} }%
  }%
}%
\T\def\sep {\def\sep{, }}\z
```

(u,x), (u,y), (u,z), (v,x), (v,y), (v,z), (w,x), (w,y), (w,z)

Similarly when the replacement text of \xintFor defines a macro with parameters, the macro character # must be doubled.

It is licit to use inside an \xintFor a \macro which itself has been defined to use internally some other \xintFor. The same macro parameter #1 can be used with no conflict (as mentioned above, in the definition of \macro the # used in the \xintFor declaration must be doubled, as is the general rule in TeX with things defined inside other things).

The iterated commands as well as the list items are allowed to contain explicit \par tokens. Neither \xintFor nor \xintFor* create groups. The effect is like piling up the iterated commands with each time #1 (or #2 ...) replaced by an item of the list. However, contrarily to the completely expandable \xintApplyUnbraced, but similarly to the non completely expandable \xintApplyInline each iteration is executed first before looking at the next #1[55] (and the starred variant \xintFor* keeps on expanding each unbraced item it finds, gobbling spaces).

6.20 \xintifForFirst, \xintifForLast

nn ★ \xintifForFirst{YES branch}{NO branch} and \xintifForLast{YES branch}{NO branch} execute the YES or NO branch if the \xintFor or \xintFor* loop is currently in its first, respectively last, iteration.

Designed to work as expected under nesting. Don't forget an empty brace pair {} if a branch is to do nothing. May be used multiple times in the replacement text of the loop.

There is no such thing as an iteration counter provided by the \xintFor loops; the user is invited to define if needed his own count register or LaTeX counter, for example with a suitable \stepcounter inside the replacement text of the loop to update it.

It is a known feature of these conditionals that they cease to function if put at a location of the \xintFor replacement text which has closed a group, for example in the last cell of an alignment created by the loop, assuming the replacement text of the \xintFor loop creates a

[54] sometimes what seems to be a macro argument isn't really; in \raisebox{1cm}{\xintFor #1 in {a,b,c}\do {#1}} no doubling should be done. [55] to be completely honest, both \xintFor and \xintFor* initially scoop up both the list and the iterated commands; \xintFor scoops up a second time the entire comma separated list in order to feed it to \xintCSVtoList. The starred variant \xintFor* which does not need this step will thus be a bit faster on equivalent inputs.

row. The conditional must be used before the first cell is closed. This is not likely to change in future versions. It is not an intrinsic limitation as the branches of the conditional can be the complete rows, inclusive of all &'s and the tabular newline \\.

6.21 \xintBreakFor, \xintBreakForAndDo

One may immediately terminate an \xintFor or \xintFor* loop with \xintBreakFor. As the criterion for breaking will be decided on a basis of some test, it is recommended to use for this test the syntax of ifthen[56] or etoolbox[57] or the xint own conditionals, rather than one of the various \if...\fi of TeX. Else (and this is without even mentioning all the various pecularities of the \if...\fi constructs), one has to carefully move the break after the closing of the conditional, typically with \expandafter\xintBreakFor\fi.[58]

There is also \xintBreakForAndDo. Both are illustrated by various examples in the next section which is devoted to ``forever'' loops.

6.22 \xintintegers, \xintdimensions, \xintrationals

If the list argument to \xintFor (or \xintFor*, both are equivalent in this context) is \xintintegers (equivalently \xintegers) or more generally \xintintegers[start+delta] (*the whole within braces*!)[59], then \xintFor does an infinite iteration where #1 (or #2, ..., #9) will run through the arithmetic sequence of (short) integers with initial value start and increment delta (default values: start=1, delta=1; if the optional argument is present it must contains both of them, and they may be explicit integers, or macros or count registers). The #1 (or #2, ..., #9) will stand for \numexpr <opt sign><digits>\relax, and the litteral representation as a string of digits can thus be obtained as \the#1 or \number#1. Such a #1 can be used in an \ifnum test with no need to be postfixed with a space or a \relax and one should *not* add them.

If the list argument is \xintdimensions or more generally \xintdimensions[start+delta] (*within braces*!), then \xintFor does an infinite iteration where #1 (or #2, ..., #9) will run through the arithmetic sequence of dimensions with initial value start and increment delta. Default values: start=0pt, delta=1pt; if the optional argument is present it must contain both of them, and they may be explicit specifications, or macros, or dimen registers, or length commands in LaTeX (the stretch and shrink components will be discarded). The #1 will be \dimexpr <opt sign><digits>sp\relax, from which one can get the litteral (approximate) representation in points via \the#1. So #1 can be used anywhere TeX expects a dimension (and there is no need in conditionals to insert a \relax, and one should *not* do it), and to print its value one uses \the#1. The chosen representation guarantees exact incrementation with no rounding errors accumulating from converting into points at each step.

If the list argument to \xintFor (or \xintFor*) is \xintrationals or more generally \xintrationals[start+delta] (*within braces*!), then \xintFor does an infinite iteration where #1 (or #2, ..., #9) will run through the arithmetic sequence of xintfrac fractions with initial value start and increment delta (default values: start=1/1, delta=1/1). This loop works *only with xintfrac loaded*. if the optional argument is present it must contain both of them, and they may be given in any of the formats recognized by xintfrac (fractions, decimal numbers, numbers in scientific notations, numerators and denominators in scientific notation, etc...) , or as macros or count registers (if they are short integers). The #1 (or #2, ..., #9) will be an a/b fraction (without a [n] part), where the denominator b is the product of the denominators of start and delta (for reasons of speed #1 is not reduced to irreducible form, and for another reason explained later start and delta are not put either into irreducible form; the input may use explicitely \xintIrr to achieve that).

[56] http://ctan.org/pkg/ifthen [57] http://ctan.org/pkg/etoolbox [58] the difficulties here are similar to those mentioned in subsection 2.11, although less severe, as complete expandability is not to be maintained; hence the allowed use of ifthen. [59] the start+delta optional specification may have extra spaces around the plus sign of near the square brackets, such spaces are removed. The same applies with \xintdimensions and \xintrationals.

```
\begingroup\small
\noindent\parbox{\dimexpr\linewidth-3em}{\color[named]{OrangeRed}%
\xintFor #1 in {\xintrationals [10/21+1/21]} \do
{#1=\xintifInt {#1}
    {\textcolor{blue}{\xintTrunc{10}{#1}}}
    {\xintTrunc{10}{#1}}% display in blue if an integer
    \xintifGt {#1}{1.123}{\xintBreakFor}{, }%
  }}
\endgroup\smallskip
```

10/21=0.4761904761, 11/21=0.5238095238, 12/21=0.5714285714, 13/21=0.6190476190, 14/21=0.6666666666, 15/21=0.7142857142, 16/21=0.7619047619, 17/21=0.8095238095, 18/21=0.8571428571, 19/21=0.9047619047, 20/21=0.9523809523, 21/21=1.0000000000, 22/21=1.0476190476, 23/21=1.0952380952, 24/21=1.1428571428

The example above confirms that computations are done exactly, and illustrates that the two initial (reduced) denominators are not multiplied when they are found to be equal. It is thus recommended to input start and delta with a common smallest possible denominator, or as fixed point numbers with the same numbers of digits after the decimal mark; and this is also the reason why start and delta are not by default made irreducible. As internally the computations are done with numerators and denominators completely expanded, one should be careful not to input numbers in scientific notation with exponents in the hundreds, as they will get converted into as many zeroes.

```
\noindent\parbox{\dimexpr.7\linewidth}{\raggedright
\xintFor #1 in {\xintrationals [0.000+0.125]} \do
{\edef\tmp{\xintTrunc{3}{#1}}%
 \xintifInt {#1}
    {\textcolor{blue}{\tmp}}
    {\tmp}%
    \xintifGt {#1}{2}{\xintBreakFor}{, }%
  }}\smallskip
```

0, 0.125, 0.250, 0.375, 0.500, 0.625, 0.750, 0.875, 1.000, 1.125, 1.250, 1.375, 1.500, 1.625, 1.750, 1.875, 2.000, 2.125

We see here that \xintTrunc outputs (deliberately) zero as 0, not (here) 0.000, the idea being not to lose the information that the truncated thing was truly zero. Perhaps this behavior should be changed? or made optional? Anyhow printing of fixed points numbers should be dealt with via dedicated packages such as numprint or siunitx.

6.23 Another table of primes

As a further example, let us dynamically generate a tabular with the first 50 prime numbers after 12345. First we need a macro to test if a (short) number is prime. Such a completely expandable macro was given in subsection 6.12, here we consider a variant which will be slightly more efficient. This new \IsPrime has two parameters. The first one is a macro which it redefines to expand to the result of the primality test applied to the second argument. For convenience we use the etoolbox wrappers to various \ifnum tests, although here there isn't anymore the constraint of complete expandability (but using explicit \if..\fi in tabulars has its quirks); equivalent tests are provided by xint, but they have some overhead as they are able to deal with arbitrarily big integers.

```
\def\IsPrime #1#2% #1=\Result, #2=tested number (assumed >0).
{\edef\TheNumber {\the\numexpr #2}% hence #2 may be a count or \numexpr.
 \ifnumodd {\TheNumber}
 {\ifnumgreater {\TheNumber}{1}
  {\edef\ItsSquareRoot{\xintiSqrt \TheNumber}%
    \xintFor ##1 in {\xintintegers [3+2]}\do
    {\ifnumgreater {##1}{\ItsSquareRoot} % ##1 is a \numexpr.
               {\def#1{1}\xintBreakFor}
```

```
                  {}%
         \ifnumequal {\TheNumber}{(\TheNumber/##1)*##1}
                     {\def#1{0}\xintBreakFor }
                     {}%
        }}
      {\def#1{0}}}% 1 is not prime
     {\ifnumequal {\TheNumber}{2}{\def#1{1}}{\def#1{0}}}%
    }
```

As we used \xintFor inside a macro we had to double the # in its #1 parameter. Here is now the code which creates the prime table (the table has been put in a float, which should be found on page 71):

```
\newcounter{primecount}
\newcounter{cellcount}
\begin{figure*}[ht!]
  \centering
  \begin{tabular}{|*{7}c|}
  \hline
  \setcounter{primecount}{0}\setcounter{cellcount}{0}%
  \xintFor #1 in {\xintintegers [12345+2]} \do
% #1 is a \numexpr.
  {\IsPrime\Result{#1}%
   \ifnumgreater{\Result}{0}
   {\stepcounter{primecount}%
    \stepcounter{cellcount}%
    \ifnumequal {\value{cellcount}}{7}
       {\the#1 \\\setcounter{cellcount}{0}}
       {\the#1 &}}
   {}%
    \ifnumequal {\value{primecount}}{50}
     {\xintBreakForAndDo
      {\multicolumn {6}{l|}{These are the first 50 primes after 12345.}\\}}
     {}%
  }\hline
\end{tabular}
\end{figure*}
```

12347	12373	12377	12379	12391	12401	12409
12413	12421	12433	12437	12451	12457	12473
12479	12487	12491	12497	12503	12511	12517
12527	12539	12541	12547	12553	12569	12577
12583	12589	12601	12611	12613	12619	12637
12641	12647	12653	12659	12671	12689	12697
12703	12713	12721	12739	12743	12757	12763
12781	These are the first 50 primes after 12345.					

6.24 Some arithmetic with Fibonacci numbers

Here is the code employed on the title page to compute (expandably, of course!) the 1250th Fibonacci number:

```
\catcode`_ 11
\def\Fibonacci #1{%  \Fibonacci{N} computes F(N) with F(0)=0, F(1)=1.
    \expandafter\Fibonacci_a\expandafter
        {\the\numexpr #1\expandafter}\expandafter
        {\romannumeral0\xintiieval 1\expandafter\relax\expandafter}\expandafter
        {\romannumeral0\xintiieval 1\expandafter\relax\expandafter}\expandafter
```

```
          {\romannumeral0\xintiieval 1\expandafter\relax\expandafter}\expandafter
          {\romannumeral0\xintiieval 0\relax}}
%
\def\Fibonacci_a #1{%
    \ifcase #1
          \expandafter\Fibonacci_end_i
    \or
          \expandafter\Fibonacci_end_ii
    \else
          \ifodd #1
              \expandafter\expandafter\expandafter\Fibonacci_b_ii
          \else
              \expandafter\expandafter\expandafter\Fibonacci_b_i
          \fi
    \fi {#1}%
}% * signs are omitted from the next macros, tacit multiplications
\def\Fibonacci_b_i #1#2#3{\expandafter\Fibonacci_a\expandafter
  {\the\numexpr #1/2\expandafter}\expandafter
  {\romannumeral0\xintiieval sqr(#2)+sqr(#3)\expandafter\relax\expandafter}\expandafter
  {\romannumeral0\xintiieval (2#2-#3)#3\relax}%
}% end of Fibonacci_b_i
\def\Fibonacci_b_ii #1#2#3#4#5{\expandafter\Fibonacci_a\expandafter
  {\the\numexpr (#1-1)/2\expandafter}\expandafter
  {\romannumeral0\xintiieval sqr(#2)+sqr(#3)\expandafter\relax\expandafter}\expandafter
  {\romannumeral0\xintiieval (2#2-#3)#3\expandafter\relax\expandafter}\expandafter
  {\romannumeral0\xintiieval #2#4+#3#5\expandafter\relax\expandafter}\expandafter
  {\romannumeral0\xintiieval #2#5+#3(#4-#5)\relax}%
}% end of Fibonacci_b_ii
%         code as used on title page:
%\def\Fibonacci_end_i  #1#2#3#4#5{\xintthe#5}
%\def\Fibonacci_end_ii #1#2#3#4#5{\xinttheiiexpr #2#5+#3(#4-#5)\relax}
%         new definitions:
\def\Fibonacci_end_i  #1#2#3#4#5{{#4}{#5}}% {F(N+1)}{F(N)} in \xintexpr format
\def\Fibonacci_end_ii #1#2#3#4#5%
    {\expandafter
     {\romannumeral0\xintiieval #2#4+#3#5\expandafter\relax
      \expandafter}\expandafter
     {\romannumeral0\xintiieval #2#5+#3(#4-#5)\relax}}% idem.
% \FibonacciN returns F(N) (in encapsulated format: needs \xintthe for printing)
\def\FibonacciN {\expandafter\xint_secondoftwo\romannumeral-`0\Fibonacci }%
\catcode`_ 8
```

I have modified the ending: we want not only one specific value F(N) but a pair of successive values which can serve as starting point of another routine devoted to compute a whole sequence F(N), F(N+1), F(N+2),..... This pair is, for efficiency, kept in the encapsulated internal xintexpr format. \FibonacciN outputs the single F(N), also as an \xintexpr-ession, and printing it will thus need the \xintthe prefix.

Here a code snippet which checks the routine via a \message of the first 51 Fibonacci numbers (this is not an efficient way to generate a sequence of such numbers, it is only for validating \FibonacciN).

```
\def\Fibo #1.{\xintthe\FibonacciN {#1}}%
\message{\xintiloop [0+1] \expandafter\Fibo\xintiloopindex.,
                          \ifnum\xintiloopindex<49 \repeat \xintthe\FibonacciN{50}.}
```

The various \romannumeral0\xintiieval could very well all have been \xintiiexpr's but then we would have needed more \expandafter's. Indeed the order of expansion must be controlled for the whole thing to work, and \romannumeral0\xintiieval is the first expanded form of \xintiiexpr.

The way we use \expandafter's to chain successive \xintexpr evaluations is exactly analogous to well-known expandable techniques made possible by \numexpr.

There is a difference though: \numexpr is *NOT* expandable, and to force its expansion we must prefix it with \the or \number. On the other hand \xintexpr, \xintiexpr, ..., (or \xinteval, \xintieval, ...) expand fully when prefixed by \romannumeral-`0: the computation is fully executed and its result encapsulated in a private format.

Using \xintthe as prefix is necessary to print the result (this is like \the for \numexpr), but it is not necessary to get the computation done (contrarily to the situation with \numexpr).

And, starting with release 1.09j, it is also allowed to expand a non \xintthe prefixed \xintexpr-ession inside an \edef: the private format is now protected, hence the error message complaining about a missing \xintthe will not be executed, and the integrity of the format will be preserved.

This new possibility brings some efficiency gain, when one writes non-expandable algorithms using xintexpr. If \xintthe is employed inside \edef the number or fraction will be un-locked into its possibly hundreds of digits and all these tokens will possibly weigh on the upcoming shuffling of (braced) tokens. The private encapsulated format has only a few tokens, hence expansion will proceed a bit faster.

see footnote[60]

Our \Fibonacci expands completely under f-expansion, so we can use \fdef rather than \edef in a situation such as

```
\fdef \X {\FibonacciN {100}}
```

but for the reasons explained above, it is as efficient to employ \edef. And if we want

```
\edef \Y {(\FibonacciN{100},\FibonacciN{200})},
```

then \edef is necessary.

Allright, so let's now give the code to generate a sequence of braced Fibonacci numbers {F(N)}{F(N+1)}{F(N+2)}..., using \Fibonacci for the first two and then using the standard recursion F(N+2)=F(N+1)+F(N):

```
\catcode`_ 11
\def\FibonacciSeq #1#2{%#1=starting index, #2>#1=ending index
    \expandafter\Fibonacci_Seq\expandafter
    {\the\numexpr #1\expandafter}\expandafter{\the\numexpr #2-1}%
}%
\def\Fibonacci_Seq #1#2{%
     \expandafter\Fibonacci_Seq_loop\expandafter
                {\the\numexpr #1\expandafter}\romannumeral0\Fibonacci {#1}{#2}%
}%
\def\Fibonacci_Seq_loop #1#2#3#4{% standard Fibonacci recursion
    {#3}\unless\ifnum #1<#4  \Fibonacci_Seq_end\fi
        \expandafter\Fibonacci_Seq_loop\expandafter
        {\the\numexpr #1+1\expandafter}\expandafter
        {\romannumeral0\xintiieval #2+#3\relax}{#2}{#4}%
}%
\def\Fibonacci_Seq_end\fi\expandafter\Fibonacci_Seq_loop\expandafter
    #1\expandafter #2#3#4{\fi {#3}}%
\catcode`_ 8
```

Deliberately and for optimization, this \FibonacciSeq macro is completely expandable but not f-expandable. It would be easy to modify it to be so. But I wanted to check that the \xintFor* does apply full expansion to what comes next each time it fetches an item from its list argument. Thus, there is no need to generate lists of braced Fibonacci numbers beforehand, as \xintFor*, without using any \edef, still manages to generate the list via iterated full expansion.

60 To be completely honest the examination by TeX of all successive digits was not avoided, as it occurs already in the locking-up of the result, what is avoided is to spend time un-locking, and then have the macros shuffle around possibly hundreds of digit tokens rather than a few control words.

I initially used only one `\halign` in a three-column `multicols` environment, but `multicols` only knows to divide the page horizontally evenly, thus I employed in the end one `\halign` for each column (I could have then used a `tabular` as no column break was then needed).

30.	832040	0	**60.**	1548008755920	0	**90.**	2880067194370816120	0
31.	1346269	514229	**61.**	2504730781961	1	**91.**	4660046610375530309	514229
32.	2178309	514229	**62.**	4052739537881	1	**92.**	7540113804746346429	514229
33.	3524578	196418	**63.**	6557470319842	2	**93.**	12200160415121876738	196418
34.	5702887	710647	**64.**	10610209857723	3	**94.**	19740274219868223167	710647
35.	9227465	75025	**65.**	17167680177565	5	**95.**	31940434634990099905	75025
36.	14930352	785672	**66.**	27777890035288	8	**96.**	51680708854858323072	785672
37.	24157817	28657	**67.**	44945570212853	13	**97.**	83621143489848422977	28657
38.	39088169	814329	**68.**	72723460248141	21	**98.**	135301852344706746049	814329
39.	63245986	10946	**69.**	117669030460994	34	**99.**	218922995834555169026	10946
40.	102334155	825275	**70.**	190392490709135	55	**100.**	354224848179261915075	825275
41.	165580141	4181	**71.**	308061521170129	89	**101.**	573147844013817084101	4181
42.	267914296	829456	**72.**	498454011879264	144	**102.**	927372692193078999176	829456
43.	433494437	1597	**73.**	806515533049393	233	**103.**	1500520536206896083277	1597
44.	701408733	831053	**74.**	1304969544928657	377	**104.**	2427893228399975082453	831053
45.	1134903170	610	**75.**	2111485077978050	610	**105.**	3928413764606871165730	610
46.	1836311903	831663	**76.**	3416454622906707	987	**106.**	6356306993006846248183	831663
47.	2971215073	233	**77.**	5527939700884757	1597	**107.**	10284720757613717413913	233
48.	4807526976	831896	**78.**	8944394323791464	2584	**108.**	16641027750620563662096	831896
49.	7778742049	89	**79.**	14472334024676221	4181	**109.**	26925748508234281076009	89
50.	12586269025	831985	**80.**	23416728348467685	6765	**110.**	43566776258854844738105	831985
51.	20365011074	34	**81.**	37889062373143906	10946	**111.**	70492524767089125814114	34
52.	32951280099	832019	**82.**	61305790721611591	17711	**112.**	114059301025943970552219	832019
53.	53316291173	13	**83.**	99194853094755497	28657	**113.**	184551825793033096366333	13
54.	86267571272	832032	**84.**	160500643816367088	46368	**114.**	298611126818977066918552	832032
55.	139583862445	5	**85.**	259695496911122585	75025	**115.**	483162952612010163284885	5
56.	225851433717	832037	**86.**	420196140727489673	121393	**116.**	781774079430987230203437	832037
57.	365435296162	2	**87.**	679891637638612258	196418	**117.**	1264937032042997393488322	2
58.	591286729879	832039	**88.**	1100087778366101931	317811	**118.**	2046711111473984623691759	832039
59.	956722026041	1	**89.**	1779979416004714189	514229	**119.**	3311648143516982017180081	1

Some Fibonacci numbers together with their residues modulo `F(30)`=832040

```
\newcounter{index}
\tabskip 1ex
  \fdef\Fibxxx{\FibonacciN {30}}%
  \setcounter{index}{30}%
\vbox{\halign{\bfseries#.\hfil&#\hfil &\hfil #\cr
  \xintFor* #1 in {\FibonacciSeq {30}{59}}\do
  {\theindex &\xintthe#1 &
    \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}\vrule
\vbox{\halign{\bfseries#.\hfil&#\hfil &\hfil #\cr
  \xintFor* #1 in {\FibonacciSeq {60}{89}}\do
  {\theindex &\xintthe#1 &
    \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}\vrule
\vbox{\halign{\bfseries#.\hfil&#\hfil &\hfil #\cr
  \xintFor* #1 in {\FibonacciSeq {90}{119}}\do
  {\theindex &\xintthe#1 &
   \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}%
```

This produces the Fibonacci numbers from F(30) to F(119), and computes also all the congruence classes modulo F(30). The output has been put in a float, which appears on the previous page. I leave to the mathematically inclined readers the task to explain the visible patterns... ;-).

6.25 \xintForpair, \xintForthree, \xintForfour

on The syntax is illustrated in this example. The notation is the usual one for n-uples, with parentheses and commas. Spaces around commas and parentheses are ignored.

```
{\centering\begin{tabular}{cccc}
    \xintForpair #1#2 in { ( A , a ) , ( B , b ) , ( C , c ) } \do {%
      \xintForpair #3#4 in { ( X , x ) , ( Y , y ) , ( Z , z ) } \do {%
        $\Biggl($\begin{tabular}{cc}
          -#1- & -#3-\\
          -#4- & -#2-\\
        \end{tabular}$\Biggr)$&}\\\noalign{\vskip1\jot}}%
\end{tabular}\\}
```

$$\begin{pmatrix} \text{-A-} & \text{-X-} \\ \text{-x-} & \text{-a-} \end{pmatrix} \begin{pmatrix} \text{-A-} & \text{-Y-} \\ \text{-y-} & \text{-a-} \end{pmatrix} \begin{pmatrix} \text{-A-} & \text{-Z-} \\ \text{-z-} & \text{-a-} \end{pmatrix}$$

$$\begin{pmatrix} \text{-B-} & \text{-X-} \\ \text{-x-} & \text{-b-} \end{pmatrix} \begin{pmatrix} \text{-B-} & \text{-Y-} \\ \text{-y-} & \text{-b-} \end{pmatrix} \begin{pmatrix} \text{-B-} & \text{-Z-} \\ \text{-z-} & \text{-b-} \end{pmatrix}$$

$$\begin{pmatrix} \text{-C-} & \text{-X-} \\ \text{-x-} & \text{-c-} \end{pmatrix} \begin{pmatrix} \text{-C-} & \text{-Y-} \\ \text{-y-} & \text{-c-} \end{pmatrix} \begin{pmatrix} \text{-C-} & \text{-Z-} \\ \text{-z-} & \text{-c-} \end{pmatrix}$$

Only #1#2, #2#3, #3#4, ..., #8#9 are valid (no error check is done on the input syntax, #1#3 or similar all end up in errors). One can nest with \xintFor, for disjoint sets of macro parameters. There is also \xintForthree (from #1#2#3 to #7#8#9) and \xintForfour (from #1#2#3#4 to #6#7#8#9). \par tokens are accepted in both the comma separated list and the replacement text.

6.26 \xintAssign

\xintAssign⟨*braced things*⟩\to⟨*as many cs as they are things*⟩ defines (without checking if something gets overwritten) the control sequences on the right of \to to expand to the successive tokens or braced items found one after the other on the left of \to. It is not expandable.

A `full' expansion is first applied to the material in front of \xintAssign, which may thus be a macro expanding to a list of braced items.

Special case: if after this initial expansion no brace is found immediately after \xintAssign, it is assumed that there is only one control sequence following \to, and this control sequence is then defined via \def to expand to the material between \xintAssign and \to. Other types of expansions are specified through an optional parameter to \xintAssign, see *infra*.

```
\xintAssign \xintiiDivision{1000000000000}{133333333}\to\Q\R
\meaning\Q:macro:->7500, \meaning\R: macro:->2500
\xintAssign \xintiiPow {7}{13}\to\SevenToThePowerThirteen
\SevenToThePowerThirteen=96889010407
(same as \edef\SevenToThePowerThirteen{\xintiPow {7}{13}})
```

\xintAssign admits since 1.09i an optional parameter, for example \xintAssign [e]... or \xintAssign [oo] .... With [f] for example the definitions of the macros initially on the right of \to will be made with \fdef which *f*-expands the replacement text. The default is simply to make the definitions with \def, corresponding to an empty optional paramter []. Possibilities: [], [g], [e], [x], [o], [go], [oo], [goo], [f], [gf].

In all cases, recall that \xintAssign starts with an *f*-expansion of what comes next; this produces some list of tokens or braced items, and the optional parameter only intervenes to decide the expansion type to be applied then to each one of these items.

Note: prior to release 1.09j, \xintAssign did an \edef by default, but it now does \def. Use the optional parameter [e] to force use of \edef.

Remark: since xinttools 1.1c, \xintAssign is less picky and a stray space right before the \to causes no surprises, and the successive braced items may be separated by spaces, which will get discarded. In case the contents up to \to did not start with a brace a single macro is defined and it will contain the spaces. Contrarily to the earlier version, there is no problem if such contents do contain braces after the first non-brace token.

6.27 \xintAssignArray

\xintAssignArray⟨*braced things*⟩\to\myArray first expands fully what comes immediately after \xintAssignArray and expects to find a list of braced things {A}{B}... (or tokens). It then defines \myArray as a macro with one parameter, such that \myArray{x} expands to give the xth braced thing of this original list (the argument {x} itself is fed to a \numexpr by \myArray, and \myArray expands in two steps to its output). With 0 as parameter, \myArray{0} returns the number M of elements of the array so that the successive elements are \myArray{1}, . . . , \myArray{M}.

```
\xintAssignArray \xintBezout {1000}{113}\to\Bez
```

will set \Bez{0} to 5, \Bez{1} to 1000, \Bez{2} to 113, \Bez{3} to -20, \Bez{4} to -177, and \Bez{5} to 1: $(-20)\times 1000 - (-177)\times 113 = 1$. This macro is incompatible with expansion-only contexts.

\xintAssignArray admits now an optional parameter, for example \xintAssignArray [e].... This means that the definitions of the macros will be made with \edef. The default is [], which makes the definitions with \def. Other possibilities: [], [o], [oo], [f]. Contrarily to \xintAssign one can not use the g here to make the definitions global. For this, one should rather do \xintAssignArray within a group starting with \globaldefs 1.

Note that prior to release 1.09j each item (token or braced material) was submitted to an \edef, but the default is now to use \def.

6.28 \xintDigitsOf

fN This is a synonym for \xintAssignArray, to be used to define an array giving all the digits of a given (positive, else the minus sign will be treated as first item) number.

```
\xintDigitsOf\xintiPow {7}{500}\to\digits
```

7^{500} has \digits{0}=423 digits, and the 123rd among them (starting from the most significant) is \digits{123}=3.

6.29 \xintRelaxArray

\xintRelaxArray\myArray (globally) sets to \relax all macros which were defined by the previous \xintAssignArray with \myArray as array macro.

6.30 The Quick Sort algorithm illustrated

First a completely expandable macro which sorts a comma separated list of numbers.[61]

The \QSx macro expands its list argument, which may thus be a macro; its comma separated items must expand to integers or decimal numbers or fractions or scientific notation as acceptable to xintfrac, but if an item is itself some (expandable) macro, this macro will be expanded each time the item is considered in a comparison test! This is actually good if the macro expands in one step to the digits, and there are many many digits, but bad if the macro needs to do many computations.

[61] The code in earlier versions of this manual handled inputs composed of braced items. I have switched to comma separated inputs on the occasion of http://tex.stackexchange.com/a/273084. The version here is like code 3 on http://tex.stackexchange.com (which is about 3x faster than the earlier code it replaced in this manual) with a modification to make it more efficient if the data has many repeated values. A faster routine (for sorting hundreds of values) is provided as code 6 at the link mentioned in the footnote, it is based on Merge Sort, but limited to inputs which one can handle as TeX dimensions.This code 6 could be extended to handle more general numbers, as acceptable by xintfrac. I have also written a non expandable version, which is even faster, but this matters really only when handling hundreds or rather thousands of values.

Thus \QSx should be used with either explicit numbers or with items being macros expanding in one step to the numbers (particularly if these numbers are very big).

If the interest is only in TeX integers, then one should replace the \xintifCmp macro with a suitable conditional, possibly helped by tools such as \ifnumgreater, \ifnumequal and \ifnumless from etoolbox (LaTeX only; I didn't see a direct equivalent to \xintifCmp.) Or, if we are dealing with decimal numbers with at most four+four digits, then one should use suitable \ifdim tests. Naturally this will boost consequently the speed, from having skipped all the overhead in parsing fractions and scientific numbers as are acceptable by xintfrac macros, and subsequent treatment.

```
% THE QUICK SORT ALGORITHM EXPANDABLY
% \usepackage{xintfrac} in the preamble (latex)
\makeatletter
% use extra safe delimiters
\catcode`! 3 \catcode`? 3
\def\QSx {\romannumeral0\qsx }%
% first we check if empty list (else \qsx@finish will not find a comma)
\def\qsx   #1{\expandafter\qsx@a\romannumeral-`0#1,!,?}%
\def\qsx@a #1{\ifx,#1\expandafter\qsx@abort\else
                     \expandafter\qsx@start\fi #1}%
\def\qsx@abort #1?{ }%
\def\qsx@start {\expandafter\qsx@finish\romannumeral0\qsx@b,}%
\def\qsx@finish ,#1{ #1}%
%
% we check if empty of single and if not pick up the first as Pivot:
\def\qsx@b ,#1#2,#3{\ifx?#3\xintdothis\qsx@empty\fi
                    \ifx!#3\xintdothis\qsx@single\fi
                    \xintorthat\qsx@separate {#1#2}{}{}{#1#2}#3}%
\def\qsx@empty  #1#2#3#4#5{ }%
\def\qsx@single #1#2#3#4#5?{, #4}%
\def\qsx@separate #1#2#3#4#5#6,%
{%
    \ifx!#5\expandafter\qsx@separate@done\fi
    \xintifCmp {#5#6}{#4}%
          \qsx@separate@appendtosmaller
          \qsx@separate@appendtoequal
          \qsx@separate@appendtogreater {#5#6}{#1}{#2}{#3}{#4}%
}%
%
\def\qsx@separate@appendtoequal   #1#2{\qsx@separate {#2,#1}}%
\def\qsx@separate@appendtogreater #1#2#3{\qsx@separate {#2}{#3,#1}}%
\def\qsx@separate@appendtosmaller #1#2#3#4{\qsx@separate {#2}{#3}{#4,#1}}%
%
\def\qsx@separate@done\xintifCmp #1%
          \qsx@separate@appendtosmaller
          \qsx@separate@appendtoequal
          \qsx@separate@appendtogreater #2#3#4#5#6#7?%
{%
    \expandafter\qsx@f\expandafter {\romannumeral0\qsx@b #4,!,?}{\qsx@b #5,!,?}{#3}%
}%
%
\def\qsx@f #1#2#3{#2, #3#1}%
%
\catcode`! 12 \catcode`? 12
\makeatother

% EXAMPLE
\begingroup
\edef\z {\QSx {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
```

```
                1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}}
\meaning\z

\def\a {3.123456789123456789}\def\b {3.123456789123456788}
\def\c {3.123456789123456790}\def\d {3.123456789123456787}
\oodef\z {\QSx { \a, \b, \c, \d}}%
% The space before \a to let it not be expanded during the conversion from CSV
% values to List. The \oodef expands exactly twice (via a bunch of \expandafter's)
\meaning\z
\endgroup
```

macro:->0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0

macro:->\d , \b , \a , \c (the spaces after \d, etc... come from the use of the \meaning primitive.)

The choice of pivot as first element is bad if the list is already almost sorted. Let's add a variant which will pick up the pivot index randomly. The previous routine worked also internally with comma separated lists, but for a change this one will use internally lists of braced items (the initial conversion via \xintCSVtoList handles all potential spurious space problems).

```
% QuickSort expandably on comma separated values with random choice of pivots
% ====> Requires availability of \pdfuniformdeviate <====
% \usepackage{xintfrac, xinttools} in preamble
\makeatletter
\def\QSx {\romannumeral0\qsx }% This is a f-expandable macro.
% This converts from comma separated values on input and back on output.
% **** NOTE: these steps (and the other ones too, actually) are costly if input
%             has thousands of items.
\def\qsx #1{\xintlistwithsep{, }%
            {\expandafter\qsx@sort@a\expandafter{\romannumeral0\xintcsvtolist{#1}}}}%
%
% we check if empty or single or double and if not pick up the first as Pivot:
\def\qsx@sort@a #1%
    {\expandafter\qsx@sort@b\expandafter{\romannumeral0\xintlength{#1}}{#1}}%
\def\qsx@sort@b #1{\ifcase #1
                      \expandafter\qsx@sort@empty
                      \or\expandafter\qsx@sort@single
                      \or\expandafter\qsx@sort@double
                      \else\expandafter\qsx@sort@c\fi {#1}}%
\def\qsx@sort@empty  #1#2{ }%
\def\qsx@sort@single #1#2{#2}%
\catcode`_ 11
\def\qsx@sort@double #1#2{\xintifGt #2{\xint_exchangetwo_keepbraces}{}#2}%
\catcode`_ 8
\def\qsx@sort@c      #1#2{%
    \expandafter\qsx@sort@sep@a\expandafter
                {\romannumeral0\xintnthelt{\pdfuniformdeviate #1+\@ne}{#2}}#2?}%
\def\qsx@sort@sep@a #1{\qsx@sort@sep@loop {}{}{}{#1}}%
\def\qsx@sort@sep@loop #1#2#3#4#5%
{%
    \ifx?#5\expandafter\qsx@sort@sep@done\fi
    \xintifCmp {#5}{#4}%
          \qsx@sort@sep@appendtosmaller
          \qsx@sort@sep@appendtoequal
          \qsx@sort@sep@appendtogreater {#5}{#1}{#2}{#3}{#4}%
}%
%
\def\qsx@sort@sep@appendtoequal   #1#2{\qsx@sort@sep@loop {#2{#1}}}%
```

```
\def\qsx@sort@sep@appendtogreater #1#2#3{\qsx@sort@sep@loop {#2}{#3{#1}}}%
\def\qsx@sort@sep@appendtosmaller #1#2#3#4{\qsx@sort@sep@loop {#2}{#3}{#4{#1}}}%
%
\def\qsx@sort@sep@done\xintifCmp #1%
          \qsx@sort@sep@appendtosmaller
          \qsx@sort@sep@appendtoequal
          \qsx@sort@sep@appendtogreater #2#3#4#5#6%
{%
    \expandafter\qsx@sort@recurse\expandafter
               {\romannumeral0\qsx@sort@a {#4}}{\qsx@sort@a {#5}}{#3}%
}%
%
\def\qsx@sort@recurse #1#2#3{#2#3#1}%
%
\makeatother

% EXAMPLES
\begingroup
\edef\z {\QSx {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
               1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}}
\meaning\z

\def\a {3.123456789123456789}\def\b {3.123456789123456788}
\def\c {3.123456789123456790}\def\d {3.123456789123456787}
\oodef\z {\QSx { \a, \b, \c, \d}}%
% The space before \a to let it not be expanded during the conversion from CSV
% values to List. The \oodef expands exactly twice (via a bunch of \expandafter's)
\meaning\z

\def\somenumbers{%
3997.6421, 8809.9358, 1805.4976, 5673.6478, 3179.1328, 1425.4503, 4417.7691,
2166.9040, 9279.7159, 3797.6992, 8057.1926, 2971.9166, 9372.2699, 9128.4052,
1228.0931, 3859.5459, 8561.7670, 2949.6929, 3512.1873, 1698.3952, 5282.9359,
1055.2154, 8760.8428, 7543.6015, 4934.4302, 7526.2729, 6246.0052, 9512.4667,
7423.1124, 5601.8436, 4433.5361, 9970.4849, 1519.3302, 7944.4953, 4910.7662,
3679.1515, 8167.6824, 2644.4325, 8239.4799, 4595.1908, 1560.2458, 6098.9677,
3116.3850, 9130.5298, 3236.2895, 3177.6830, 5373.1193, 5118.4922, 2743.8513,
8008.5975, 4189.2614, 1883.2764, 9090.9641, 2625.5400, 2899.3257, 9157.1094,
8048.4216, 3875.6233, 5684.3375, 8399.4277, 4528.5308, 6926.7729, 6941.6278,
9745.4137, 1875.1205, 2755.0443, 9161.1524, 9491.1593, 8857.3519, 4290.0451,
2382.4218, 3678.2963, 5647.0379, 1528.7301, 2627.8957, 9007.9860, 1988.5417,
2405.1911, 5065.8063, 5856.2141, 8989.8105, 9349.7840, 9970.3013, 8105.4062,
3041.7779, 5058.0480, 8165.0721, 9637.7196, 1795.0894, 7275.3838, 5997.0429,
7562.6481, 8084.0163, 3481.6319, 8078.8512, 2983.7624, 3925.4026, 4931.5812,
1323.1517, 6253.0945}%

\oodef\z {\QSx \somenumbers}%
\hsize 87\fontcharwd\font`0 \setbox0\hbox{\kern\fontcharwd\font`0}%
\lccode`~=32 \lowercase{\def~}{\discretionary{}{}{\copy0}}\catcode32 13
\noindent\ \ \ \scantokens\expandafter{\meaning\z}\par
\endgroup
```

macro:->0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0

macro:->\d , \b , \a , \c

macro:->1055.2154, 1228.0931, 1323.1517, 1425.4503, 1519.3302, 1528.7301, 1560.2458, 1698.3952, 1795.0894, 1805.4976, 1875.1205, 1883.2764, 1988.5417, 2166.9040, 2382.4218, 2405.1911, 2625.5400, 2627.8957, 2644.4325, 2743.8513, 2755.0443, 2899.3257, 2949.6929, 2971.9166, 2983.7624, 3041.7779, 3116.3850, 3177.6830, 3179.1328, 3236.2895, 3481.6319, 3512.1873, 3678.2963, 3679.1515, 3797.6992, 3859.5459, 3875.6233, 3925.4026, 3997.6421, 4189.2614, 4290.0451, 4417.7691, 4433.5361, 4528.5308, 4595.1908, 4910.7662, 4931.5812, 4934.4302, 5058.0480, 5065.8063, 5118.4922, 5282.9359, 5373.1193, 5601.8436, 5647.0379, 5673.6478, 5684.3375, 5856.2141, 5997.0429, 6098.9677, 6246.0052, 6253.0945, 6926.7729, 6941.6278, 7275.3838, 7423.1124, 7526.2729, 7543.6015, 7562.6481, 7944.4953, 8008.5975, 8048.4216, 8057.1926, 8078.8512, 8084.0163, 8105.4062, 8165.0721, 8167.6824, 8239.4799, 8399.4277, 8561.7670, 8760.8428, 8809.9358, 8857.3519, 8989.8105, 9007.9860, 9090.9641, 9128.4052, 9130.5298, 9157.1094, 9161.1524, 9279.7159, 9349.7840, 9372.2699, 9491.1593, 9512.4667, 9637.7196, 9745.4137, 9970.3013, 9970.4849

All these examples were with numbers which may have been handled via \ifdim tests rather than \xintifCmp from xintfrac; naturally that would have been faster. For a yet faster routine (based however on the Merge Sort and using the \pdfescapestring PDFTeX primitive) see code 6 at http://tex.stackexchange.com/a/273084.

We then turn to a graphical illustration of the algorithm.[62] For simplicity the pivot is always chosen as the first list item. Then we also give a variant which picks up the last item as pivot.

```
% in LaTeX preamble:
% \usepackage{xintfrac, xinttools}
% \usepackage{color}
% or, when using Plain TeX:
% \input xintfrac.sty \input xinttools.sty
% \input color.tex
%
% Color definitions
\definecolor{LEFT}{RGB}{216,195,88}
\definecolor{RIGHT}{RGB}{208,231,153}
\definecolor{INERT}{RGB}{199,200,194}
\definecolor{INERTpiv}{RGB}{237,237,237}
\definecolor{PIVOT}{RGB}{109,8,57}
% Start of macro defintions
\makeatletter
% \catcode`? 3 % a bit too paranoid. Normal ? will do.
%
% argument will never be empty
\def\QS@cmp@a     #1{\QS@cmp@b  #1??}%
\def\QS@cmp@b     #1{\noexpand\QS@sep@A\@ne{#1}\QS@cmp@d {#1}}%
\def\QS@cmp@d     #1#2{\ifx ?#2\expandafter\QS@cmp@done\fi
                       \xintifCmp {#1}{#2}\tw@\@ne\z@{#2}\QS@cmp@d {#1}}%
\def\QS@cmp@done #1?{?}%
%
\def\QS@sep@A #1?{\QSLr\QS@sep@L #1\thr@@?#1\thr@@?#1\thr@@?}%
\def\QS@sep@L #1#2{\ifcase #1{#2}\or\or\else\expandafter\QS@sep@I@start\fi \QS@sep@L}%
\def\QS@sep@I@start\QS@sep@L {\noexpand\empty?\QSIr\QS@sep@I}%
\def\QS@sep@I #1#2{\ifcase#1\or{#2}\or\else\expandafter\QS@sep@R@start\fi\QS@sep@I}%
\def\QS@sep@R@start\QS@sep@I {\noexpand\empty?\QSRr\QS@sep@R}%
\def\QS@sep@R #1#2{\ifcase#1\or\or{#2}\else\expandafter\QS@sep@done\fi\QS@sep@R}%
\def\QS@sep@done\QS@sep@R {\noexpand\empty?}%
%
\def\QS@loop {%
    \xintloop
    % pivot phase
    \def\QS@pivotcount{0}%
```

[62] I have rewritten the routine to do only once (and not thrice) the needed calls to \xintifCmp, up to the price of one additional \edef, although due to the context execution time on our side is not an issue and moreover is anyhow overwhelmed by the TikZ's activities. Simultaneously I have updated the code http://tex.stackexchange.com/a/142634/4686. The variant with the choice of pivot on the right has more overhead: the reason is simply that we do not convert the data into an array, but maintain a list of tokens with self-reorganizing delimiters.

```
    \let\QSLr\DecoLEFTwithPivot  \let\QSIr \DecoINERT
    \let\QSRr\DecoRIGHTwithPivot \let\QSIrr\DecoINERT
    \centerline{\QS@list}%
    % sorting phase
    \ifnum\QS@pivotcount>\z@
            \def\QSLr {\QS@cmp@a}\def\QSRr {\QS@cmp@a}%
            \def\QSIr {\QSIrr}\let\QSIrr\relax
                \edef\QS@list{\QS@list}% compare
            \let\QSLr\relax\let\QSRr\relax\let\QSIr\relax
                \edef\QS@list{\QS@list}% separate
            \def\QSLr ##1##2?{\ifx\empty##1\else\noexpand \QSLr {{##1}##2}\fi}%
            \def\QSIr ##1##2?{\ifx\empty##1\else\noexpand \QSIr {{##1}##2}\fi}%
            \def\QSRr ##1##2?{\ifx\empty##1\else\noexpand \QSRr {{##1}##2}\fi}%
                \edef\QS@list{\QS@list}% gather
            \let\QSLr\DecoLEFT \let\QSRr\DecoRIGHT
            \let\QSIr\DecoINERTwithPivot \let\QSIrr\DecoINERT
            \centerline{\QS@list}%
    \repeat }%
%
% \xintFor* loops handle gracefully empty lists.
\def\DecoLEFT  #1{\xintFor* ##1 in {#1} \do {\colorbox{LEFT}{##1}}}%
\def\DecoINERT #1{\xintFor* ##1 in {#1} \do {\colorbox{INERT}{##1}}}%
\def\DecoRIGHT #1{\xintFor* ##1 in {#1} \do {\colorbox{RIGHT}{##1}}}%
\def\DecoPivot #1{\begingroup\color{PIVOT}\advance\fboxsep-\fboxrule\fbox{#1}\endgroup}%
%
\def\DecoLEFTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1} \do
        {\xintifForFirst {\DecoPivot {##1}}{\colorbox{LEFT}{##1}}}}%
\def\DecoINERTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1} \do
        {\xintifForFirst {\colorbox{INERTpiv}{##1}}{\colorbox{INERT}{##1}}}}%
\def\DecoRIGHTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1}  \do
        {\xintifForFirst {\DecoPivot {##1}}{\colorbox{RIGHT}{##1}}}}%
%
\def\QuickSort #1{% warning: not compatible with empty #1.
    % initialize, doing conversion from comma separated values to a list of braced items
    \edef\QS@list{\noexpand\QSRr{\xintCSVtoList{#1}}}% many \edef's are to follow anyhow
% earlier I did a first drawing of the list, here with the color of RIGHT elements,
% but the color should have been for example white, anyway I drop this first line
    %\let\QSRr\DecoRIGHT
    %\par\centerline{\QS@list}%
%
    % loop as many times as needed
    \QS@loop }%
%
% \catcode`? 12 % in case we had used a funny ? as delimiter.
\makeatother
%% End of macro definitions.
%% Start of Example
\begingroup\offinterlineskip
\small
% \QuickSort {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
%                 1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}
% \medskip
% with repeated values
\QuickSort {1.0, 0.5, 0.3, 0.8, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
```

```
                 1.3, 1.1, 0.7, 0.3, 1.6, 0.6, 0.3, 0.8, 0.2, 0.8, 0.7, 1.2}
\endgroup
```

1.0	0.5	0.3	0.8	1.5	1.8	2.0	1.7	0.4	1.2	1.4	1.3	1.1	0.7	0.3	1.6	0.6	0.3	0.8	0.2	0.8	0.7	1.2
0.5	0.3	0.8	0.4	0.7	0.3	0.6	0.3	0.8	0.2	0.8	0.7	1.0	1.5	1.8	2.0	1.7	1.2	1.4	1.3	1.1	1.6	1.2
0.5	0.3	0.8	0.4	0.7	0.3	0.6	0.3	0.8	0.2	0.8	0.7	1.0	1.5	1.8	2.0	1.7	1.2	1.4	1.3	1.1	1.6	1.2
0.3	0.4	0.3	0.3	0.2	0.5	0.8	0.7	0.6	0.8	0.8	0.7	1.0	1.2	1.4	1.3	1.1	1.2	1.5	1.8	2.0	1.7	1.6
0.3	0.4	0.3	0.3	0.2	0.5	0.8	0.7	0.6	0.8	0.8	0.7	1.0	1.2	1.4	1.3	1.1	1.2	1.5	1.8	2.0	1.7	1.6
0.2	0.3	0.3	0.3	0.4	0.5	0.7	0.6	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.4	1.3	1.5	1.7	1.6	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.7	0.6	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.4	1.3	1.5	1.7	1.6	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0

Here is the variant which always picks the pivot as the rightmost element.

```
\makeatletter
%
\def\QS@cmp@a #1{\noexpand\QS@sep@A\expandafter\QS@cmp@d\expandafter
                 {\romannumeral0\xintnthelt{-1}{#1}}#1??}%
%
\def\DecoLEFTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1} \do
        {\xintifForLast {\DecoPivot {##1}}{\colorbox{LEFT}{##1}}}}
\def\DecoINERTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1} \do
        {\xintifForLast {\colorbox{INERTpiv}{##1}}{\colorbox{INERT}{##1}}}}
\def\DecoRIGHTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
    \xintFor* ##1 in {#1}  \do
        {\xintifForLast {\DecoPivot {##1}}{\colorbox{RIGHT}{##1}}}}
\def\QuickSort #1{%
    % initialize, doing conversion from comma separated values to a list of braced items
    \edef\QS@list{\noexpand\QSLr {\xintCSVtoList{#1}}}% many \edef's are to follow anyhow
    %
    % loop as many times as needed
    \QS@loop }%
\makeatother
\begingroup\offinterlineskip
\small
% \QuickSort {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
%                 1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}
% \medskip
% with repeated values
\QuickSort {1.0, 0.5, 0.3, 0.8, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
                 1.3, 1.1, 0.7, 0.3, 1.6, 0.6, 0.3, 0.8, 0.2, 0.8, 0.7, 1.2}
\endgroup
```

1.0	0.5	0.3	0.8	1.5	1.8	2.0	1.7	0.4	1.2	1.4	1.3	1.1	0.7	0.3	1.6	0.6	0.3	0.8	0.2	0.8	0.7	1.2
1.0	0.5	0.3	0.8	0.4	1.1	0.7	0.3	0.6	0.3	0.8	0.2	0.8	0.7	1.2	1.2	1.5	1.8	2.0	1.7	1.4	1.3	1.6
1.0	0.5	0.3	0.8	0.4	1.1	0.7	0.3	0.6	0.3	0.8	0.2	0.8	0.7	1.2	1.2	1.5	1.8	2.0	1.7	1.4	1.3	1.6
0.5	0.3	0.4	0.3	0.6	0.3	0.2	0.7	0.7	1.0	0.8	1.1	0.8	0.8	1.2	1.2	1.5	1.4	1.3	1.6	1.8	2.0	1.7
0.5	0.3	0.4	0.3	0.6	0.3	0.2	0.7	0.7	1.0	0.8	1.1	0.8	0.8	1.2	1.2	1.5	1.4	1.3	1.6	1.8	2.0	1.7
0.2	0.5	0.3	0.4	0.3	0.6	0.3	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.5	1.4	1.6	1.7	1.8	2.0
0.2	0.5	0.3	0.4	0.3	0.6	0.3	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.5	1.4	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.5	0.4	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.5	0.4	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.5	0.4	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.5	0.4	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0

0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0
0.2	0.3	0.3	0.3	0.4	0.5	0.6	0.7	0.7	0.8	0.8	0.8	1.0	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.0

The choice of the first or last item as pivot is not a good one as nearly ordered lists will take quadratic time. But for explaining the algorithm via a graphical interpretation, it is not that bad. If one wanted to pick up the pivot randomly, the routine would have to be substantially rewritten: in particular the \Deco..withPivot macros need to know where the pivot is, and currently this is implemented by using either \xintifForFirst or \xintifForLast.

7 Commands of the xintcore package

☞ Prior to release 1.1 the macros which are now included in the separate package xintcore were part of xint. Package xintcore is automatically loaded by xint.

xintcore provides the five basic arithmetic operations on big integers: addition, subtraction, multiplication, division and powers. Division may be either rounded (\xintiiDivRound) (the rounding of 0.5 is 1 and the one of -0.5 is -1) or Euclidean (\xintiiQuo) (which for positive operands is the same as truncated division), or truncated (\xintiiDivTrunc).

In the description of the macros the {N} and {M} symbols stand for explicit (big) integers within braces or more generally any control sequence (possibly within braces) *f*-expanding to such a big integer.

The macros with a single i in their names parse their arguments automatically through \xintNum. This type of expansion applied to an argument is signaled by a $\overset{\text{Num}}{f}$ in the margin. The accepted input format is then a sequence of plus and minus signs, followed by some string of zeroes, followed by digits.

If xintfrac additionally to xintcore is loaded, \xintNum becomes a synonym to \xintTTrunc; this means that arbitrary fractions will be accepted as arguments of the macros with a single i in their names, but get truncated to integers before further processing. The format of the output will be as with only xint loaded. The only extension is in allowing a wider variety of inputs.

The macros with ii in their names have arguments which will only be *f*-expanded, but will not be parsed via \xintNum. Arguments of this type are signaled by the margin annotation *f*. For such big integers only one minus sign and no plus sign, nor leading zeros, are accepted. -0 is not valid in this strict input format. Loading xintfrac does not bring any modification to these macros whether for input or output.

The letter x (with margin annotation $\overset{\text{num}}{x}$) stands for something which will be inserted in-between a \numexpr and a \relax. It will thus be completely expanded and must give an integer obeying the TeX bounds. Thus, it may be for example a count register, or itself a \numexpr expression, or just a number written explicitely with digits or something like 4*\count 255 + 17, etc...

For the rules regarding direct use of count registers or \numexpr expression, in the arguments to the package macros, see the Use of count section.

☞ Earlier releases of xintcore also provided macros \xintAdd, \xintMul,... as synonyms to \xintiAdd, \xintiMul,..., destined to be re-defined by xintfrac. It was announced some time ago that their usage was deprecated, because the output formats depended on whether xintfrac was loaded or not. They now have been removed. (Changed →)

The macros \xintiAdd, \xintiMul, ..., or \xintiiAdd, \xintiiMul, ...which come with xintcore are guaranteed to always output an integer without a trailing /B[n]. The latter have the lesser overhead, and the former do not complain, if xintfrac is loaded, even if used with true fractions, as they will then truncate their arguments to integers. But their output format remains unmodified: integers with no fraction slash nor [N] thingy.

The ★'s in the margin are there to remind of the complete expandability, even *f*-expandability of the macros, as discussed in subsubsection 2.3.2.

7.1 \xintNum, \xintiNum

f ★ \xintNum{N} removes chains of plus or minus signs, followed by zeroes.

```
\xintNum{+---++----+--000000000367941789479}=-367941789479
```

All xint macros with a single i in their names, such as \xintiAdd, \xintiMul apply \xintNum to their arguments.

When xintfrac is loaded, \xintNum becomes a synonym to \xintTTrunc. And \xintiNum preserved the original integer only meaning.

7.2 \xintSgn, \xintiiSgn

f ★ \xintiiSgn{N} returns 1 if the number is positive, 0 if it is zero and -1 if it is negative. It skips the \xintNum overhead.

Num *f* ★ \xintSgn is the variant using \xintNum and getting extended by xintfrac to fractions.

7.3 \xintiOpp, \xintiiOpp

Num *f* ★ *f* ★ \xintiOpp{N} return the opposite -N of the number N. \xintiiOpp is the strict integer-only variant which skips the \xintNum overhead.

7.4 \xintiAbs, \xintiiAbs

Num *ff* ★ \xintiAbs{N} returns the absolute value of the number. \xintiiAbs skips the \xintNum overhead.

7.5 \xintiiFDg

f ★ Num *f* ★ \xintiiFDg{N} returns the first digit (most significant) of the decimal expansion. It skips the overhead of parsing via \xintNum. The variant \xintFDg uses \xintNum and gets extended by xintfrac.

7.6 \xintiiLDg

f ★ \xintiiLDg{N} returns the least significant digit. When the number is positive, this is the same as the remainder in the euclidean division by ten. It skips the overhead of parsing via \xintNum.

Num *f* ★ The variant \xintLDg uses \xintNum and gets extended by xintfrac.

7.7 \xintDouble, \xintHalf

f ★ \xintDouble{N} returns 2N and \xintHalf{N}is N/2 rounded towards zero. These macros remain integer-only, even with xintfrac loaded.

7.8 \xintInc, \xintDec

f ★ \xintInc{N} is N+1 and \xintDec{N} is N-1. These macros remain integer-only, even with xintfrac loaded. They skip the overhead of parsing via \xintNum.

7.9 \xintiAdd, \xintiiAdd

Num Num *f* *ff* ★ \xintiAdd{N}{M} returns the sum of the two numbers. \xintiiAdd skips the \xintNum overhead.

7.10 \xintiSub, \xintiiSub

Num Num *f* *ff* ★ \xintiSub{N}{M} returns the difference N-M. \xintiiSub skips the \xintNum overhead.

7.11 \xintiMul, \xintiiMul

Num Num
f fff ★ \xintiMul{N}{M} returns the product of the two numbers. \xintiiMul skips the \xintNum overhead.

7.12 \xintiSqr, \xintiiSqr

Num
ff ★ \xintiSqr{N} returns the square. \xintiiSqr skips the \xintNum overhead.

7.13 \xintiPow, \xintiiPow

Num num
f x ★ \xintiPow{N}{x} returns N^x. When x is zero, this is 1. If N=0 and x<0, if |N|>1 and x<0, an error is raised. There will also be an error naturally if x exceeds the maximal ε-TeX number 2147483647, but the real limit for huge exponents comes from either the computation time or the settings of some tex memory parameters.

> Indeed, the maximal power of 2 which xint is able to compute explicitely is 2^(2^17)=2^131072 which has 39457 digits. This exceeds the maximal size on input for the xintcore multiplication, hence any 2^N with a higher N will fail. On the other hand 2^(2^16) has 19729 digits, thus it can be squared once to obtain 2^(2^17) or multiplied by anything smaller, thus all exponents up to and including 2^17 are allowed (because the power operation works by squaring things and making products).

Side remark: after all it does pay to think! I almost melted my CPU trying by dichotomy to pinpoint the exact maximal allowable N for \xintiiPow 2{N} before finally making the reasoning above. Indeed, each such computation with N>130000 activates the fan of my laptop and results in so warm a keyboard that I can hardly go on working on it! And it takes about 12 minutes for each \xintiiPow2{N} with such N's of the order of 130000 (a.t.t.o.w.).

When xintfrac is loaded the type of the second argument to \xintiPow becomes Num f: fractional input is accepted but will be truncated to an integer; it still must be non-negative else the macro would produce fractions. For the version accepting negative (but still integer) exponents see \xintPow.

num
f x ★ \xintiiPow is the variant which skips the \xintNum overhead for the first argument.

7.14 \xintiFac, \xintiiFac

num
x ★ \xintiiFac{x} computes the factorial.

> The (theoretically) allowable range is $0 \leqslant x \leqslant 10000$.
> However the maximal possible computation depends on the values of some memory parameters of the tex executable: with the current default settings of TeXLive 2015, the maximal computable factorial (a.t.t.o.w. 2015/10/06) turns out to be 5971! which has 19956 digits.

\xintiFac is originally a synonym. With xintfrac loaded it applies \xintNum to its argument and thus accepts a fractional input but truncates it to an integer.

The factorial function, or equivalently ! as post-fix operator is available in \xintiiexpr, \xintexpr:

```
\printnumber{\xinttheiiexpr 200!\relax}\par
```

78865786736479050355236321393218506229513597768717326329474253324435944996340334292030428401
19846239041772121389196388302576427902426371050619266249528299311134628572707633172373969889
43922445621451664240254033291864131227428294853277524242407573903240321257405579568660226031
90417032406235170085879617892222789623703897374720000000000000000000000000000000000000000000
0000000

See also \xintFloatFac from package xintfrac for the float variant, used in \xintfloatexpr.

7.15 \xintiDivision, \xintiiDivision

f f ★ \xintiiDivision{N}{M} returns {quotient Q}{remainder R}. This is euclidean division: N = QM + R, 0 ≤ R < |M|. So the remainder is always non-negative and the formula N = QM + R always holds independently of the signs of N or M. Division by zero is an error (even if N vanishes) and returns {0}{0}. It skips the overhead of parsing via \xintNum.

Num Num *f f* ★ \xintiDivision submits its arguments to \xintNum and is extended by xintfrac to accept fractions on input, which it truncates first, and is not to be confused with the xintfrac macro \xintDiv which divides one fraction by another.

7.16 \xintiQuo, \xintiiQuo

f f ★ \xintiiQuo{N}{M} returns the quotient from the euclidean division. It skips the overhead of parsing via \xintNum.

Num Num *f f* ★ \xintiQuo submits its arguments to \xintNum and is extended by xintfrac to accept fractions on input, which it truncates first.

Note: \xintQuo is the former name of \xintiQuo. Its use is deprecated.

7.17 \xintiRem, \xintiiRem

f f ★ \xintiiRem{N}{M} returns the remainder from the euclidean division. It skips the overhead of parsing via \xintNum.

Num Num *f f* ★ \xintiRem submits its arguments to \xintNum and is extended by xintfrac to accept fractions on input, which it truncates first.

Note: \xintRem is the former name of \xintiRem. Its use is deprecated.

7.18 \xintiDivRound, \xintiiDivRound

f f ★ \xintiiDivRound{N}{M} returns the rounded value of the algebraic quotient N/M of two big integers. The rounding of half integers is towards the nearest integer of bigger absolute value. The macro skips the overhead of parsing via \xintNum. The rounding is away from zero.

```
\xintiiDivRound {100}{3}, \xintiiDivRound {101}{3}
```

33, 34

Num Num *f f* ★ \xintiDivRound submits its arguments to \xintNum. It is extended by xintfrac to accept fractions on input, which it truncates first before computing the rounded quotient.

7.19 \xintiDivTrunc, \xintiiDivTrunc

f f ★ \xintiiDivTrunc{N}{M} computes the truncation towards zero of the algebraic quotient N/M. It skips the overhead of parsing the operands with \xintNum. For M > 0 it is the same as \xintiiQuo.

```
$\xintiiQuo {1000}{-57}, \xintiiDivRound {1000}{-57}, \xintiiDivTrunc {1000}{-57}$
```

-17, -18, -17

Num Num *f f* ★ \xintiDivTrunc submits first its arguments to \xintNum.

7.20 \xintiMod, \xintiiMod

f f ★ \xintiiMod{N}{M} computes N - M * t(N/M), where t(N/M) is the algebraic quotient truncated towards zero . The macro skips the overhead of parsing the operands with \xintNum. For M > 0 it is the same as \xintiiRem.

```
$\xintiiRem {1000}{-57}, \xintiiMod {1000}{-57},
 \xintiiRem {-1000}{57}, \xintiiMod {-1000}{57}$
```

31, 31, 26, -31

Num Num *f f* ★ \xintiMod submits first its arguments to \xintNum.

8 Commands of the xint package

> This package loads automatically xintcore (and xintkernel) hence all macros described in section 7 are still available. Notice though that it does *not* load package xinttools.

Version 1.0 was released 2013/03/28. This is 1.2g of 2016/03/19. The core arithmetic macros have been moved to separate package xintcore, which is automatically loaded by xint.

See the documentation of xintcore or subsubsection 2.3.2 for the significance of the Num *f*, *f*, num *x* and ★ margin annotations and some important background information.

8.1 \xintReverseDigits

f ★ \xintReverseDigits{N} will reverse the order of the digits of the number, preserving an optional upfront minus sign. \xintRev is the former denomination and is kept as an alias to it. Leading zeroes resulting from the operation are not removed. Contrarily to \xintReverseOrder this macro can only be used with digits and it first expands its argument (but beware that -\x will give an unexpected result as the minus sign immediately stops this expansion; one can use \xintiiOpp{\x} as argument.)

This command has been rewritten for 1.2 and is faster for very long inputs. It is (almost) not used internally by the xintcore code, but the use of related routines explains to some extent the higher speed of release 1.2.

```
\fdef\x{\xintReverseDigits
```

```
    {-98765432109876543210987654321098765432109876543210}}\meaning\x\par
  \noindent\fdef\x{\xintReverseDigits {\xintReverseDigits
    {-98765432109876543210987654321098765432109876543210}}}\meaning\x\par
```
macro:->-01234567890123456789012345678901234567890123456789
macro:->-98765432109876543210987654321098765432109876543210

Notice that the output in this case with its leading zero is not in the strict integer format expected by the `ii' arithmetic macros.

8.2 \xintLen

Num
f ★ \xintLen{N} returns the length of the number, not counting the sign.

```
  \xintLen{-12345678901234567890123456789}=29
```
Extended by xintfrac to fractions: the length of A/B[n] is the length of A plus the length of B plus the absolute value of n and minus one (an integer input as N is internally represented in a form equivalent to N/1[0] so the minus one means that the extended \xintLen behaves the same as the original for integers).

```
  \xintLen{-1e3/5.425}=10
```
The length is computed on the A/B[n] which would have been returned by \xintRaw: \xintRaw {-1e3/5.425}=-1/5425[6].

Let's point out that the whole thing should sum up to less than circa 2^{31}, but this is a bit theoretical.

\xintLen is only for numbers or fractions. See also \xintNthElt from xinttools. See also \xint-Length from xintkernel for counting tokens (or rather braced groups), more generally.

8.3 \xintCmp, \xintiiCmp

Num Num
f *f* ★ \xintCmp{N}{M} returns 1 if N>M, 0 if N=M, and -1 if N<M. Extended by xintfrac to fractions (its output naturally still being either 1, 0, or -1).

ff ★ \xintiiCmp skips the \xintNum overhead.

8.4 \xintEq, \xintiiEq

Num Num
f *f* ★ \xintEq{N}{M} returns 1 if N=M, 0 otherwise. Extended by xintfrac to fractions.

ff ★ \xintiiEq skips the \xintNum overhead.

8.5 \xintNeq, \xintiiNeq

Num Num
f *f* ★ \xintNeq{N}{M} returns 0 if N=M, 1 otherwise. Extended by xintfrac to fractions.

ff ★ \xintiiNeq skips the \xintNum overhead.

8.6 \xintGt, \xintiiGt

Num Num
f *f* ★ \xintGt{N}{M} returns 1 if N>M, 0 otherwise. Extended by xintfrac to fractions.

ff ★ \xintiiGt skips the \xintNum overhead.

8.7 \xintLt, \xintiiLt

Num Num
f *f* ★ \xintLt{N}{M} returns 1 if N<M, 0 otherwise. Extended by xintfrac to fractions.

ff ★ \xintiiLt skips the \xintNum overhead.

8.8 \xintLtorEq, \xintiiLtorEq

Num Num
f *f* ★ \xintLtorEq{N}{M} returns 1 if N⩽M, 0 otherwise. Extended by xintfrac to fractions.

ff ★ \xintiiLtorEq skips the \xintNum overhead.

8.9 \xintGtorEq, \xintiiGtorEq

\xintGtorEq{N}{M} returns 1 if N⩾M, 0 otherwise. Extended by xintfrac to fractions.

\xintiiGtorEq skips the \xintNum overhead.

8.10 \xintIsZero, \xintiiIsZero

\xintIsZero{N} returns 1 if N=0, 0 otherwise. Extended by xintfrac to fractions.

\xintiiIsZero skips the \xintNum overhead.

8.11 \xintNot

\xintNot is a synonym for \xintIsZero.

8.12 \xintIsNotZero, \xintiiIsNotZero

\xintIsNotZero{N} returns 1 if N<>0, 0 otherwise. Extended by xintfrac to fractions.

\xintiiIsNotZero skips the \xintNum overhead.

8.13 \xintIsOne, \xintiiIsOne

\xintIsOne{N} returns 1 if N=1, 0 otherwise. Extended by xintfrac to fractions.

\xintiiIsOne skips the \xintNum overhead.

8.14 \xintAND

\xintAND{N}{M} returns 1 if N<>0 and M<>0 and zero otherwise. Extended by xintfrac to fractions.

8.15 \xintOR

\xintOR{N}{M} returns 1 if N<>0 or M<>0 and zero otherwise. Extended by xintfrac to fractions.

8.16 \xintXOR

\xintXOR{N}{M} returns 1 if exactly one of N or M is true (i.e. non-zero). Extended by xintfrac to fractions.

8.17 \xintANDof

\xintANDof{{a}{b}{c}...} returns 1 if all are true (i.e. non zero) and zero otherwise. The list argument may be a macro, it (or rather its first token) is *f*-expanded first (each item also is *f*-expanded). Extended by xintfrac to fractions.

8.18 \xintORof

\xintORof{{a}{b}{c}...} returns 1 if at least one is true (i.e. does not vanish). The list argument may be a macro, it is *f*-expanded first. Extended by xintfrac to fractions.

8.19 \xintXORof

\xintXORof{{a}{b}{c}...} returns 1 if an odd number of them are true (i.e. does not vanish). The list argument may be a macro, it is *f*-expanded first. Extended by xintfrac to fractions.

8.20 \xintGeq

Num Num f f ★ \xintGeq{N}{M} returns 1 if the *absolute value* of the first number is at least equal to the absolute value of the second number. If |N|<|M| it returns 0. Extended by xintfrac to fractions. Important: the macro compares *absolute values*.

8.21 \xintiMax, \xintiiMax

Num Num f f ★ \xintiMax{N}{M} returns the largest of the two in the sense of the order structure on the relative integers (*i.e.* the right-most number if they are put on a line with positive numbers on the right): \xintiMax {-5}{-6}=-5.

ff ★ The \xintiiMax macro skips the overhead of parsing the operands with \xintNum.

8.22 \xintiMin, \xintiiMin

Num Num f f ★ \xintiMin{N}{M} returns the smallest of the two in the sense of the order structure on the relative integers (*i.e.* the left-most number if they are put on a line with positive numbers on the right): \xintiMin {-5}{-6}=-6.

ff ★ The \xintiiMin macro skips the overhead of parsing the operands with \xintNum.

8.23 \xintiMaxof, \xintiiMaxof

Num f→ * f ★ \xintiMaxof{{a}{b}{c}...} returns the maximum. The list argument may be a macro, it is *f*-expanded first. Each item is submitted to \xintNum normalization.

New with 1.2a \xintiiMaxof does the same, skips \xintNum normalization of items.

8.24 \xintiMinof, \xintiiMinof

Num f→ * f ★ \xintiMinof{{a}{b}{c}...} returns the minimum. The list argument may be a macro, it is *f*-expanded first. Each item is submitted to \xintNum normalization.

New with 1.2a \xintiiMinof does the same, skips \xintNum normalization of items.

8.25 \xintiiSum

*f ★ \xintiiSum{⟨*braced things*⟩} after expanding its argument expects to find a sequence of tokens (or braced material). Each is *f*-expanded, and the sum of all these numbers is returned. Note: the summands are *not* parsed by \xintNum.

```
\xintiiSum{{123}{-98763450}{\xintiiFac{7}}{\xintiMul{3347}{591}}}=-96780210
\xintiiSum{1234567890}=45
```

An empty sum is no error and returns zero: \xintiiSum {}=0. A sum with only one term returns that number: \xintiiSum {{-1234}}=-1234. Attention that \xintiiSum {-1234} is not legal input and will make the TEX run fail. On the other hand \xintiiSum {1234}=10.

8.26 \xintiiPrd

*f ★ \xintiiPrd{⟨*braced things*⟩} after expanding its argument expects to find a sequence of (of braced items or unbraced single tokens). Each is expanded (with the usual meaning), and the product of all these numbers is returned. Note: the operands are *not* parsed by \xintNum.

```
\xintiiPrd{{-9876}{\xintiiFac{7}}{\xintiMul{3347}{591}}}=-98458861798080
\xintiiPrd{123456789123456789}=131681894400
```

An empty product is no error and returns 1: \xintiiPrd {}=1. A product reduced to a single term returns this number: \xintiiPrd {{-1234}}=-1234. Attention that \xintiiPrd {-1234} is not legal input and will make the TEX compilation fail. On the other hand \xintiiPrd {1234}=24.

```
$2^{200}3^{100}7^{100}=\printnumber
        {\xintiiPrd {{\xintiPow {2}{200}}{\xintiPow {3}{100}}{\xintiPow {7}{100}}}}$
```

$2^{200}3^{100}7^{100}$ = 2678727931661577575766279517007548402324740266374015348974459614815426412965499
490000444007240765727130000165312076406545621180143571994015903343539244028212438966822248927862988084382716133376

With xintexpr, this would be easier:

```
\xinttheiiexpr 2^200*3^100*7^100\relax
```

8.27 \xintSgnFork

x n n n ★ \xintSgnFork{-1|0|1}{⟨A⟩}{⟨B⟩}{⟨C⟩} expandably chooses to execute either the ⟨A⟩, ⟨B⟩ or ⟨C⟩ code, depending on its first argument. This first argument should be anything expanding to either -1, 0 or 1 in a non self-delimiting way (i.e. a count register must be prefixed by \the and a \numexpr...\relax also must be prefixed by \the). This utility is provided to help construct expandable macros choosing depending on a condition which one of the package macros to use, or which values to confer to their arguments.

8.28 \xintifSgn, \xintiiifSgn

Num *f n n n ★* Similar to \xintSgnFork except that the first argument may expand to a (big) integer (or a fraction if xintfrac is loaded), and it is its sign which decides which of the three branches is taken. Furthermore this first argument may be a count register, with no \the or \number prefix.

f ★ \xintiiifSgn skips the \xintNum overhead.

8.29 \xintifZero, \xintiiifZero

Num *f n n ★* \xintifZero{⟨N⟩}{⟨IsZero⟩}{⟨IsNotZero⟩} expandably checks if the first mandatory argument N (a number, possibly a fraction if xintfrac is loaded, or a macro expanding to one such) is zero or not. It then either executes the first or the second branch. Beware that both branches must be present.

f ★ \xintiiifZero skips the \xintNum overhead.

8.30 \xintifNotZero, \xintiiifNotZero

Num *f n n ★* \xintifNotZero{⟨N⟩}{⟨IsNotZero⟩}{⟨IsZero⟩} expandably checks if the first mandatory argument N (a number, possibly a fraction if xintfrac is loaded, or a macro expanding to one such) is not zero or is zero. It then either executes the first or the second branch. Beware that both branches must be present.

f ★ \xintiiifNotZero skips the \xintNum overhead.

8.31 \xintifOne, \xintiiifOne

Num *f n n ★* \xintifOne{⟨N⟩}{⟨IsOne⟩}{⟨IsNotOne⟩} expandably checks if the first mandatory argument N (a number, possibly a fraction if xintfrac is loaded, or a macro expanding to one such) is one or not. It then either executes the first or the second branch. Beware that both branches must be present.

f ★ \xintiiifOne skips the \xintNum overhead.

8.32 \xintifTrueAelseB, \xintifFalseAelseB

Num *f n n ★* \xintifTrueAelseB{⟨N⟩}{⟨true branch⟩}{⟨false branch⟩} is a synonym for \xintifNotZero.

1. with 1.09i, the synonyms \xintifTrueFalse and \xintifTrue are deprecated and will be removed in next release.
2. These macros have no lowercase versions, use \xintifzero, \xintifnotzero.

Num *f n n ★* \xintifFalseAelseB{⟨N⟩}{⟨false branch⟩}{⟨true branch⟩} is a synonym for \xintifZero.

8.33 \xintifCmp, \xintiiifCmp

Num Num f f nnn ★ \xintifCmp{⟨A⟩}{⟨B⟩}{⟨if A<B⟩}{⟨if A=B⟩}{⟨if A>B⟩} compares its arguments and chooses accordingly the correct branch.

f f ★ \xintiiifCmp skips the \xintNum overhead.

8.34 \xintifEq, \xintiiifEq

Num Num f f nn ★ \xintifEq{⟨A⟩}{⟨B⟩}{⟨YES⟩}{⟨NO⟩} checks equality of its two first arguments (numbers, or fractions if xintfrac is loaded) and does the YES or the NO branch.

f f ★ \xintiiifEq skips the \xintNum overhead.

8.35 \xintifGt, \xintiiifGt

Num Num f f nn ★ \xintifGt{⟨A⟩}{⟨B⟩}{⟨YES⟩}{⟨NO⟩} checks if A > B and in that case executes the YES branch. Extended to fractions (in particular decimal numbers) by xintfrac.

f f ★ \xintiiifGt skips the \xintNum overhead.

8.36 \xintifLt, \xintiiifLt

Num Num f f nn ★ \xintifLt{⟨A⟩}{⟨B⟩}{⟨YES⟩}{⟨NO⟩} checks if A < B and in that case executes the YES branch. Extended to fractions (in particular decimal numbers) by xintfrac.

f f ★ \xintiiifLt skips the \xintNum overhead.

8.37 \xintifOdd, \xintiiifOdd

Num f nn ★ \xintifOdd{⟨A⟩}{⟨YES⟩}{⟨NO⟩} checks if A is and odd integer and in that case executes the YES branch.

f ★ \xintiiifOdd skips the \xintNum overhead.

> The macros described next are all integer-only on input. Those with ii in their names skip the \xintNum parsing. The others, with xintfrac loaded, can have fractions as arguments, which will get truncated to integers via \xintTTrunc. On output, the macros here always produce integers (with no /B[N]).

8.38 \xintiiMON, \xintiiMMON

f ★ \xintiiMON{N} returns (-1)^N and \xintiiMMON{N} returns (-1)^{N-1}. They skip the overhead of parsing via \xintNum.

```
\xintiiMON {-280914019374101929}=-1, \xintiiMMON {-280914019374101929}=1
```

Num f ★ The variants \xintMON and \xintMMON use \xintNum and get extended to fractions by xintfrac.

8.39 \xintiiOdd

f ★ Num f ★ \xintiiOdd{N} is 1 if the number is odd and 0 otherwise. It skips the overhead of parsing via \xintNum. \xintOdd is the variant using \xintNum and extended to fractions by xintfrac.

8.40 \xintiiEven

f ★ Num f ★ \xintiiEven{N} is 1 if the number is even and 0 otherwise. It skips the overhead of parsing via \xintNum. \xintEven is the variant using \xintNum and extended to fractions by xintfrac.

8.41 \xintiSqrt, \xintiiSqrt, \xintiiSqrtR, \xintiSquareRoot, \xintiiSquareRoot

Num f ★ / f ★ / f ★

\xintiSqrt{N} returns the largest integer whose square is at most equal to N. \xintiiSqrt is the variant skipping the \xintNum overhead. \xintiiSqrtR also skips the \xintNum overhead and it returns the rounded, not truncated, square root.

```
\begin{itemize}[nosep]
\item \xintiiSqrt  {3000000000000000000000000000000000000}
\item \xintiiSqrtR {3000000000000000000000000000000000000}
\item \xintiiSqrt  {\xintiiE {3}{100}}
\end{itemize}
```

- 1732050807568877293
- 1732050807568877294
- 173205080756887729352744634150587236694280525381038

Num f ★ / f ★

\xintiSquareRoot{N} returns {M}{d} with d>0, M^2-d=N and M smallest (hence =1+\xintiSqrt{N}). \xintiiSquareRoot is the variant skipping the \xintNum overhead.

```
\xintAssign\xintiiSquareRoot {17000000000000000000000000}\to\A\B
\xintiiSub{\xintiiSqr\A}\B=\A\string^2-\B
```

17000000000000000000000000=4123105625618^2-2799177881924

A rational approximation to $\sqrt{N}$ is $M - \frac{d}{2M}$ (this is a majorant and the error is at most 1/2M; if N is a perfect square k^2 then M=k+1 and this gives k+1/(2k+2), not k).

Package xintfrac has \xintFloatSqrt for square roots of floating point numbers.

8.42 \xintiFac, \xintiiFac

Defined in xintcore, see subsection 7.14 for more info.

8.43 \xintiBinomial, \xintiiBinomial

num num x x ★ — New with 1.2f

\xintiiBinomial{x}{y} computes binomial coefficients.

\xintiBinomial is originally a synonym. With xintfrac loaded it applies \xintNum to its arguments and thus accepts fractional inputs but truncates them to an integer.

The (theoretically) allowable range is $0 \leqslant y \leqslant x \leqslant 99999999$.

This theoretical range includes binomial coefficients with more than the roughly 19950 digits that the arithmetics of xint can handle. In such cases, the computation will end up in a low-level TeX error after a long time.

It turns out that $\binom{65000}{32500}$ has 19565 digits and $\binom{64000}{32000}$ has 19264 digits. The latter can be evaluated (this takes a long long time) but presumably not the former (I didn't try). Reasonable feasible evaluations are with binomial coefficients not exceeding about one thousand digits.

The binomial function is available in the xintexpr parsers.

```
\xinttheiiexpr seq(binomial(100,i), i=47..53)\relax
```

84413487283064039501507937600, 93206558875049876949581681100, 98913082887808032681188722800, 100891344545564193334812497256, 98913082887808032681188722800, 93206558875049876949581681100, 84413487283064039501507937600

See \xintFloatBinomial from package xintfrac for the float variant, used in \xintfloatexpr.

In order to evaluate binomial coefficients $\binom{x}{y}$ with $x > 99999999$, or even $x \geqslant 2^{31}$, but y is not too large, one may use an ad hoc function definition such as:

```
\xintdeffunc mybigbinomial(x,y):=`*`(x-y+1..[1]..x)//y!;%
%                             without [1], x would have been limited to < 2^31
\printnumber{\xinttheexpr mybigbinomial(98765432109876543210,10)\relax}
```

24338098741940755592729533173058146177070669479669793038510211146784065843698581878582323710
27360575372715482389633359878460739973726786576925067784100587971261422326652270975592667517
4871960261

To get this functionality in macro form, one can do:

```
\xintNewIIExpr\MyBigBinomial [2]{`*`(#1-#2+1..[1]..#1)//#2!}
\printnumber{\MyBigBinomial {98765432109876543210}{10}}
```

243380987419407555927295331730581461770706694796697930385102111467840658436985818785823237102736057537271548238963335987846073997372678657692506778410058797126142232665227097559266751748871960261

As we used \xintNewIIExpr, this macro will only accept strict integers. Had we used \xintNewExpr the \MyBigBinomial would have accepted general fractions or decimal numbers, and computed the product at the numerator without truncating them to integers; but the factorial at the denominator would truncate its argument.

8.44 \xintiPFactorial, \xintiiPFactorial

num num
x x ★ New with 1.2f

\xintiiPFactorial{a}{b} computes the partial factorial (a+1)(a+2)...b.

\xintiPFactorial is originally a synonym. With xintfrac loaded it applies \xintNum to its arguments and thus accepts fractional inputs but truncates them to an integer.

The (theoretically) allowable range is $0 \leqslant a \leqslant b \leqslant 99999999$. If a=b the value is 1.

```
\xintiiPFactorial {100}{130}
```

69293021885203871012298422845822803287591970060789350400000000

This theoretical range includes values with more than the roughly 19950 digits that the arithmetics of xint can handle. In such cases, the computation will end up in a low-level TeX error after a long time.

The pfactorial function is available in the xintexpr parsers.

```
\xinttheiiexpr pfactorial(100,130)\relax
```

69293021885203871012298422845822803287591970060789350400000000

See \xintFloatPFactorial from package xintfrac for the float variant, used in \xintfloatexpr.

In case values are needed with $b > 99999999$, or even $b \geqslant 2^{31}$, but $b - a$ is not too large, one may use an ad hoc function definition such as:

```
\xintdeffunc mybigpfac(a,b):=`*`(a+1..[1]..b);%
%                              without [1], b would have been limited to < 2^31
\printnumber{\xinttheexpr mybigpfac(98765432100,98765432120)\relax}
```

780008550175675280672981073130237784386530020290496474672081960281164994340505876568704893229963060448223685356640391256144991258740460784410407812147267546181544273409867628345006993332294860057301699703400956657664000

The macros described next are strictly for integer-only arguments. These arguments are *not* filtered via \xintNum. The macros are not usable with fractions, even with xintfrac loaded.

8.45 \xintDSL

f ★ \xintDSL{N} is decimal shift left, *i.e.* multiplication by ten.

8.46 \xintDSR

f ★ \xintDSR{N} is decimal shift right, *i.e.* it removes the last digit (keeping the sign), equivalently it is the closest integer to N/10 when starting at zero.

8.47 \xintDSH

num x f ★ \xintDSH{x}{N} is parametrized decimal shift. When x is negative, it is like iterating \xintDSL |x| times (*i.e.* multiplication by 10^{-x}). When x positive, it is like iterating \DSR x times (and is more efficient), and for a non-negative N this is thus the same as the quotient from the euclidean division by 10^x.

8.48 \xintDSHr, \xintDSx

num x f ★ \xintDSHr{x}{N} expects x to be zero or positive and it returns then a value R which is correlated to the value Q returned by \xintDSH{x}{N} in the following manner:

- if N is positive or zero, Q and R are the quotient and remainder in the euclidean division by 10^x (obtained in a more efficient manner than using \xintiDivision),
- if N is negative let Q1 and R1 be the quotient and remainder in the euclidean division by 10^x of the absolute value of N. If Q1 does not vanish, then Q=-Q1 and R=R1. If Q1 vanishes, then Q=0 and R=-R1.
- for x=0, Q=N and R=0.

So one has N = 10^x Q + R if Q turns out to be zero or positive, and N = 10^x Q - R if Q turns out to be negative, which is exactly the case when N is at most -10^x.

num x f ★ \xintDSx{x}{N} for x negative is exactly as \xintDSH{x}{N}, *i.e.* multiplication by 10^{-x}. For x zero or positive it returns the two numbers {Q}{R} described above, each one within braces. So Q is \xintDSH{x}{N}, and R is \xintDSHr{x}{N}, but computed simultaneously.

```
\xintAssign\xintDSx {-1}{-123456789}\to\M
\meaning\M:macro:->-1234567890.
\xintAssign\xintDSx {-20}{123456789}\to\M
\meaning\M:macro:->12345678900000000000000000000.
\xintAssign\xintDSx {0}{-123004321}\to\Q\R
\meaning\Q:macro:->-123004321, \meaning\R:macro:->0.
\xintDSH {0}{-123004321}=-123004321, \xintDSHr {0}{-123004321}=0
\xintAssign\xintDSx {6}{-123004321}\to\Q\R
\meaning\Q:macro:->-123,\meaning\R:macro:->4321.
\xintDSH {6}{-123004321}=-123, \xintDSHr {6}{-123004321}=4321
\xintAssign\xintDSx {8}{-123004321}\to\Q\R
\meaning\Q:macro:->-1,\meaning\R:macro:->23004321.
\xintDSH {8}{-123004321}=-1, \xintDSHr {8}{-123004321}=23004321
\xintAssign\xintDSx {9}{-123004321}\to\Q\R
\meaning\Q:macro:->0,\meaning\R:macro:->-123004321.
\xintDSH {9}{-123004321}=0, \xintDSHr {9}{-123004321}=-123004321
```

8.49 \xintDecSplit

num x f ★ \xintDecSplit{x}{N} cuts the number into two pieces (each one within a pair of enclosing braces). First the sign if present is *removed*. Then, for x positive or null, the second piece contains the x least significant digits (*empty* if x=0) and the first piece the remaining digits (*empty* when x equals or exceeds the length of N). Leading zeroes in the second piece are not removed. When x is negative the first piece contains the |x| most significant digits and the second piece the remaining digits (*empty* if x equals or exceeds the length of N). Leading zeroes in this second piece are not removed. So the absolute value of the original number is always the concatenation of the first and second piece.

This macro's behavior for N non-negative is final and will not change. I am still hesitant about what to do with the sign of a negative N.

```
\xintAssign\xintDecSplit {0}{-123004321}\to\L\R
\meaning\L:macro:->123004321, \meaning\R:macro:->.
\xintAssign\xintDecSplit {5}{-123004321}\to\L\R
\meaning\L:macro:->1230, \meaning\R:macro:->04321.
\xintAssign\xintDecSplit {9}{-123004321}\to\L\R
\meaning\L:macro:->, \meaning\R:macro:->123004321.
\xintAssign\xintDecSplit {10}{-123004321}\to\L\R
\meaning\L:macro:->, \meaning\R:macro:->123004321.
\xintAssign\xintDecSplit {-5}{-12300004321}\to\L\R
\meaning\L:macro:->12300, \meaning\R:macro:->004321.
\xintAssign\xintDecSplit {-11}{-12300004321}\to\L\R
\meaning\L:macro:->12300004321, \meaning\R:macro:->.
\xintAssign\xintDecSplit {-15}{-12300004321}\to\L\R
\meaning\L:macro:->12300004321, \meaning\R:macro:->.
```

8.50 \xintDecSplitL

num x f ★ `\xintDecSplitL{x}{N}` returns the first piece after the action of `\xintDecSplit`.

8.51 \xintDecSplitR

num x f ★ `\xintDecSplitR{x}{N}` returns the second piece after the action of `\xintDecSplit`.

8.52 \xintiiE

f num x ★ `\xintiiE{N}{x}` serves to add zeros to the right of `N`.

```
\xintiiE {123}{89}
```

12300000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000

9 Commands of the xintfrac package

This package loads automatically xint and xintcore, hence all macros described in section 8 and section 7 are available; but some among them will only accept integers on input, not fractions or numbers in scientific notation. This is the case particularly for all those having ii in their names.

This package was first included in release 1.03 (2013/04/14) of the xint bundle. The general rule of the bundle that each macro first expands (what comes first, fully) each one of its arguments applies.

Frac *f* f stands for an integer or a fraction (see subsection 2.7 for the accepted input formats) or something which expands to an integer or fraction. It is possible to use in the numerator or the denominator of f count registers and even expressions with infix arithmetic operators, under some rules which are explained in the previous Use of count registers section.

num *x* As in the xint.sty documentation, x stands for something which will internally be embedded in a \numexpr. It may thus be a count register or something like 4*\count 255 + 17, etc..., but must expand to an integer obeying the TeX bound.

The fraction format on output is the scientific notation for the `float' macros, and the A/B[n] format for all other fraction macros, with the exception of \xintTrunc, \xintRound (which produce decimal numbers) and \xintIrr, \xintJrr, \xintRawWithZeros (which returns an A/B with no trailing [n], and prints the B even if it is 1), and \xintPRaw which does not print the [n] if n=0 or the B if B=1.

To be certain to print an integer output without trailing [n] nor fraction slash, one should use either \xintPRaw {\xintIrr {f}} or \xintNum {f} when it is already known that f evaluates to a (big) integer. For example \xintPRaw {\xintAdd {2/5}{3/5}} gives a perhaps disappointing 5/5 whereas \xintPRaw {\xintIrr {\xintAdd {2/5}{3/5}}} returns 1. As we knew the result was an integer we could have used \xintNum {\xintAdd {2/5}{3/5}}=1.

Some macros (such as \xintiTrunc, \xintiRound, and \xintiFac) always produce integers on output.

☞ The macro \xintXTrunc uses \xintiloop from package xinttools, hence there is a partial dependency of xintfrac on xinttools, and the latter must be required explicitely by the user intending to use \xintXTrunc.

Refer to subsection 2.2 for general background information on how floating point numbers and evaluations are implemented.

9.1 \xintNum

f ★ The macro from xint is made a synonym to \xintTTrunc.[63]

The original (which normalizes big integers to strict format) is still available as \xintiNum. It is imprudent to apply \xintNum to numbers with a large power of ten given either in scientific notation or with the [n] notation, as the macro will according to its definition add all the needed zeroes to produce an explicit integer in strict format.

9.2 \xintifInt

Frac f nn ★ \xintifInt{f}{YES branch}{NO branch} expandably chooses the YES branch if f reveals itself after expansion and simplification to be an integer. As with the other xint conditionals, both branches must be present although one of the two (or both, but why then?) may well be an empty brace pair {}. Spaces in-between the braced things do not matter, but a space after the closing brace of the NO branch is significant.

9.3 \xintLen

Frac f ★ The original macro is extended to accept a fraction on input.

```
\xintLen {201710/298219}=11, \xintLen {1234/1}=4, \xintLen {1234}=4
```

9.4 \xintRaw

Frac f ★ This macro `prints' the fraction f as it is received by the package after its parsing and expansion, in a form A/B[n] equivalent to the internal representation: the denominator B is always strictly positive and is printed even if it has value 1.

```
\xintRaw{\the\numexpr 571*987\relax.123e-10/\the\numexpr -201+59\relax e-7}=
-563577123/142[-6]
```

9.5 \xintPRaw

Frac f ★ PRaw stands for ``pretty raw''. It does *not* show the [n] if n=0 and does *not* show the B if B=1.

```
\xintPRaw {123e10/321e10}=123/321, \xintPRaw {123e9/321e10}=123/321[-1]
```

[63] In earlier releases than 1.1, \xintNum did \xintIrr and then complained if the denominator was not 1, else, it silently removed the denominator.

```
\xintPRaw {\xintIrr{861/123}}=7 vz. \xintIrr{861/123}=7/1
```

See also \xintFrac (or \xintFwOver) for math mode. As is examplified above the \xintIrr macro which puts the fraction into irreducible form does not remove the /1 if the fraction is an integer. One can use \xintNum{f} or \xintPRaw{\xintIrr{f}} which produces the same output only if f is an integer (after simplication).

9.6 \xintNumerator

Frac *f* ★ This returns the numerator corresponding to the internal representation of a fraction, with positive powers of ten converted into zeroes of this numerator:

```
\xintNumerator {178000/25600000[17]}=17800000000000000000000
\xintNumerator {312.289001/20198.27}=312289001
\xintNumerator {178000e-3/256e5}=178000
\xintNumerator {178.000/25600000}=178000
```

As shown by the examples, no simplification of the input is done. For a result uniquely associated to the value of the fraction first apply \xintIrr.

9.7 \xintDenominator

Frac *f* ★ This returns the denominator corresponding to the internal representation of the fraction:[64]

```
\xintDenominator {178000/25600000[17]}=25600000
\xintDenominator {312.289001/20198.27}=20198270000
\xintDenominator {178000e-3/256e5}=25600000000
\xintDenominator {178.000/25600000}=25600000000
```

As shown by the examples, no simplification of the input is done. The denominator looks wrong in the last example, but the numerator was tacitly multiplied by 1000 through the removal of the decimal point. For a result uniquely associated to the value of the fraction first apply \xintIrr.

9.8 \xintRawWithZeros

Frac *f* ★ This macro `prints' the fraction f (after its parsing and expansion) in A/B form, with A as returned by \xintNumerator{f} and B as returned by \xintDenominator{f}.

```
\xintRawWithZeros{\the\numexpr 571*987\relax.123e-10/\the\numexpr -201+59\relax e-7}=
-563577123/142000000
```

9.9 \xintREZ

Frac *f* ★ This command normalizes a fraction by removing the powers of ten from its numerator and denominator:

```
\xintREZ {178000/25600000[17]}=178/256[15]
\xintREZ {1780000000000e30/2560000000000e15}=178/256[15]
```

As shown by the example, it does not otherwise simplify the fraction.

9.10 \xintFrac

Frac *f* ★ This is a LaTeX only command, to be used in math mode only. It will print a fraction, internally represented as something equivalent to A/B[n] as \frac {A}{B}10^n. The power of ten is omitted when n=0, the denominator is omitted when it has value one, the number being separated from the power of ten by a \cdot. $\xintFrac {178.000/25600000}$ gives $\frac{178000}{25600000}10^{-3}$, $\xintFrac {178.000/1}$ gives $178000 \cdot 10^{-3}$, $\xintFrac {3.5/5.7}$ gives $\frac{35}{57}$, and $\xintFrac {\xintNum {\xintiiFac{10}/\xintiSqr{\xintiiFac {5}}}}$ gives 252. As shown by the examples, simplification of the input (apart from removing the decimal points and moving the minus sign to the numerator) is not

64 recall that the [] construct excludes presence of a decimal point.

done automatically and must be the result of macros such as \xintIrr, \xintREZ, or \xintNum (for fractions being in fact integers.)

9.11 \xintSignedFrac

Frac f ★ This is as \xintFrac except that a negative fraction has the sign put in front, not in the numerator.

```
\[\xintFrac {-355/113}=\xintSignedFrac {-355/113}\]
```

$$\frac{-355}{113} = -\frac{355}{113}$$

9.12 \xintFwOver

Frac f ★ This does the same as \xintFrac except that the \over primitive is used for the fraction (in case the denominator is not one; and a pair of braces contains the A\over B part). $\xintFwOver {178.000/25600000}$ gives $\frac{178000}{25600000}10^{-3}$, $\xintFwOver {178.000/1}$ gives $178000\cdot 10^{-3}$, $\xintFwOver {3.5/5.7}$ gives $\frac{35}{57}$, and $\xintFwOver {\xintNum {\xintiiFac{10}/\xintiSqr{\xintiiFac {5}}}}$ gives 252.

9.13 \xintSignedFwOver

Frac f ★ This is as \xintFwOver except that a negative fraction has the sign put in front, not in the numerator.

```
\[\xintFwOver {-355/113}=\xintSignedFwOver {-355/113}\]
```

$$\frac{-355}{113} = -\frac{355}{113}$$

9.14 \xintIrr

Frac f ★ This puts the fraction into its unique irreducible form:

\xintIrr {178.256/256.178}=6856/9853 = $\frac{6856}{9853}$

Note that the current implementation does not cleverly first factor powers of 2 and 5, so input such as \xintIrr {2/3[100]} will make xintfrac do the Euclidean division of 2·10^{100} by 3, which is a bit stupid.

Starting with release 1.08, \xintIrr does not remove the trailing /1 when the output is an integer. This was deemed better for various (stupid?) reasons and thus the output format is now *always* A/B with B>0. Use \xintPRaw on top of \xintIrr if it is needed to get rid of a possible trailing /1. For display in math mode, use rather \xintFrac{\xintIrr {f}} or \xintFwOver{\xintIrr {f}}.

9.15 \xintJrr

Frac f ★ This also puts the fraction into its unique irreducible form:

\xintJrr {178.256/256.178}=6856/9853

This is faster than \xintIrr for fractions having some big common factor in the numerator and the denominator.

\xintJrr {\xintiPow{\xintiiFac {15}}{3}/\xintiiPrdExpr {\xintiiFac{10}}{\xintiiFac{30}}{\xintiiFac{5}}\relax }=1001/51705840

But to notice the difference one would need computations with much bigger numbers than in this example. Starting with release 1.08, \xintJrr does not remove the trailing /1 when the output is an integer.

9.16 \xintTrunc

num x Frac f ★ \xintTrunc{x}{f} returns the integral part, a dot, and then the first x digits of the decimal expansion of the fraction f. The argument x should be non-negative.

In the special case when f evaluates to 0, the output is 0 with no decimal point nor decimal digits, else the post decimal mark digits are always printed. A non-zero negative f which is smaller in absolute value than 10^{-x} will give -0.000....

```
\xintTrunc {16}{-803.2028/20905.298}=-0.0384210165289200
\xintTrunc {20}{-803.2028/20905.298}=-0.03842101652892008523
\xintTrunc {10}{\xintPow {-11}{-11}}=-0.0000000000
\xintTrunc {12}{\xintPow {-11}{-11}}=-0.000000000003
\xintTrunc {12}{\xintAdd {-1/3}{3/9}}=0
```

The digits printed are exact up to and including the last one.

9.17 \xintiTrunc

num x Frac f ★ \xintiTrunc{x}{f} returns the integer equal to 10^x times what \xintTrunc{x}{f} would produce.

```
\xintiTrunc {16}{-803.2028/20905.298}=-384210165289200
\xintiTrunc {10}{\xintPow {-11}{-11}}=0
\xintiTrunc {12}{\xintPow {-11}{-11}}=-3
```

The difference between \xintTrunc{0}{f} and \xintiTrunc{0}{f} is that the latter never has the decimal mark always present in the former except for f=0. And \xintTrunc{0}{-0.5} returns ``-0.'' whereas \xintiTrunc{0}{-0.5} simply returns ``0''.

9.18 \xintTTrunc

Frac f ★ \xintTTrunc{f} truncates to an integer (truncation towards zero). This is the same as \xintiTrunc {0}{f} and as \xintNum.

9.19 \xintXTrunc

num x Frac f ☆ \xintXTrunc{x}{f} is completely expandable but not f-expandable, as is indicated by the hollow star in the margin. It can not be used as argument to the other package macros, but is designed to be used inside an \edef, or rather a \write. Here is an example session where the user after some warming up checks that $1/66049 = 1/257^2$ has period $257 * 256 = 65792$ (it is also checked here that this is indeed the smallest period).

> To use \xintXTrunc the user must load xinttools, additionally to xintfrac. The interactive session below does not show this because it was done at a time when xint (hence also xintfrac) automatically loaded xinttools. This is not the case anymore. \xintXTrunc is the sole dependency of xintfrac on xinttools.

```
xxx:_xint $ etex -jobname worksheet-66049
This is pdfTeX, Version 3.1415926-2.5-1.40.14 (TeX Live 2013)
 restricted \write18 enabled.
**\relax
entering extended mode

*\input xintfrac.sty
(./xintfrac.sty (./xint.sty (./xinttools.sty)))
*\message{\xintTrunc {100}{1/71}}% Warming up!

0.0140845070422535211267605633802816901408450704225352112676056338028169014084
```

```
0704225352112676056338O
*\message{\xintTrunc {350}{1/71}}% period is 35

0.01408450704225352112676056338028169014084507042253521126760563380281690140845
07042253521126760563380281690140845070422535211267605633802816901408450704225 35
21126760563380281690140845070422535211267605633802816901408450704225352112676 05
63380281690140845070422535211267605633802816901408450704225352112676056338028 16
90140845070422535211267605633802816 9
*\edef\Z {\xintXTrunc  {65792}{1/66049}}% getting serious...

*\def\trim 0.{}\oodef\Z {\expandafter\trim\Z}% removing 0.

*\edef\W {\xintXTrunc {131584}{1/66049}}% a few seconds

*\oodef\W {\expandafter\trim\W}

*\oodef\ZZ {\expandafter\Z\Z}% doubling the period

*\ifx\W\ZZ \message{YES!}\else\message{BUG!}\fi % xint never has bugs...
YES!
*\message{\xintTrunc {260}{1/66049}}% check visually that 256 is not a period

0.00001514027464458205271843631243470756559523989765174340262532362337052794137
68565761782918742145982528123060152311162924495450347469303093158109888113370 37
65386304107556511075110902511771563536162545988584232917985132250299020424230 49
554118911717058547442050 5
*\edef\X {\xintXTrunc {257*128}{1/66049}}% infix here ok, less than 8 tokens

*\oodef\X {\expandafter\trim\X}% we now have the first 257*128 digits

*\oodef\XX {\expandafter\X\X}% was 257*128 a period?

*\ifx\XX\Z \message{257*128 is a period}\else \message{257 * 128 not a period}\fi
257 * 128 not a period
*\immediate\write-1 {1/66049=0.\Z... (repeat)}

*\oodef\ZA {\xintNum {\Z}}% we remove the 0000, or we could use next \xintiMul

*\immediate\write-1 {10\string^65792-1=\xintiiMul {\ZA}{66049}}

*% This was slow :( I should write a multiplication, still completely

*% expandable, but not f-expandable, which could be much faster on such cases.

*\bye
No pages of output.
Transcript written on worksheet-66049.log.
xxx:_xint $
```

The \xintiiMul {\ZA}{66049} above can sadly *not* be executed with xint 1.2, due to the new limitation to at most about 19950 digits for multiplication. On the other hand \edef\W {\xintXTrunc {131584}{1/66049}} produces the 131584 digits four times faster. The macro \xintXTrunc has not yet been adapted to the new integer model underlying the 1.2 xintcore macros, and perhaps some future improvements are possible. So far it only benefits from a faster division routine, in that specific case for a divisor having more than four but less than nine digits.

Fraction arguments to \xintXTrunc corresponding to a A/B[N] with a negative N are treated somewhat less efficiently (additional memory impact) than for positive or zero N. This is because the algorithm tries to work with the smallest denominator hence does not extend B with zeroes, and technical reasons lead to the use of some tricks.[65]

Contrarily to \xintTrunc, in the case of the second argument revealing itself to be exactly zero, \xintXTrunc will output 0.000..., not 0. Also, the first argument must be at least 1.

9.20 \xintRound

num x Frac f ★ \xintRound{x}{f} returns the start of the decimal expansion of the fraction f, rounded to x digits precision after the decimal point. The argument x should be non-negative. Only when f evaluates exactly to zero does \xintRound return 0 without decimal point. When f is not zero, its sign is given in the output, also when the digits printed are all zero.

```
\xintRound {16}{-803.2028/20905.298}=-0.0384210165289201
\xintRound {20}{-803.2028/20905.298}=-0.03842101652892008523
\xintRound {10}{\xintPow {-11}{-11}}=-0.0000000000
\xintRound {12}{\xintPow {-11}{-11}}=-0.000000000004
\xintRound {12}{\xintAdd {-1/3}{3/9}}=0
```

The identity \xintRound {x}{-f}=-\xintRound {x}{f} holds. And regarding $(-11)^{-11}$ here is some more of its expansion:

```
-0.00000000000350493899481392497604003313162598556370...
```

9.21 \xintiRound

num x Frac f ★ \xintiRound{x}{f} returns the integer equal to 10^x times what \xintRound{x}{f} would return.

```
\xintiRound {16}{-803.2028/20905.298}=-384210165289201
\xintiRound {10}{\xintPow {-11}{-11}}=0
```

Differences between \xintRound{0}{f} and \xintiRound{0}{f}: the former cannot be used inside integer-only macros, and the latter removes the decimal point, and never returns -0 (and removes all superfluous leading zeroes.)

9.22 \xintFloor, \xintiFloor

Frac f ★ \xintFloor {f} returns the largest relative integer N with N ⩽ f.

```
\xintFloor {-2.13}=-3/1[0], \xintFloor {-2}=-2/1[0], \xintFloor {2.13}=2/1[0]
```

Frac f ★ \xintiFloor {f} does the same but without adding the /1[0].

```
\xintiFloor {-2.13}=-3, \xintiFloor {-2}=-2, \xintiFloor {2.13}=2
```

9.23 \xintCeil, \xintiCeil

Frac f ★ \xintCeil {f} returns the smallest relative integer N with N > f.

```
\xintCeil {-2.13}=-2/1[0], \xintCeil {-2}=-2/1[0], \xintCeil {2.13}=3/1[0]
```

Frac f ★ \xintiCeil {f} does the same but without adding the /1[0].

9.24 \xintTFrac

Frac f ★ \xintTFrac{f} returns the fractional part, f=trunc(f)+frac(f). Thus if f<0, then -1<frac(f)<=0 and if f>0 one has 0<= frac(f)<1. The T stands for `Trunc', and there should exist also similar macros associated respectively with `Round', `Floor', and `Ceil', each type of rounding to

[65] Technical note: I do not provide an \xintXFloat because this would almost certainly mean having to clone the entire core division routines into a "long division" variant. But this could have given another approach to the implementation of \xintXTrunc, especially for the case of a negative N. Doing these things with TEX is an effort. Besides an \xintXFloat would be interesting only if also for example the square root routine was provided in an X version (I have not given thought to that). If feasible X routines would be interesting in the \xintexpr context where things are expanded inside \csname ..\endcsname.

an integer deserving arguably to be associated with a fractional ``modulo''. By sheer laziness, the package currently implements only the ``modulo'' associated with `Truncation'. Other types of modulo may be obtained more cumbersomely via a combination of the rounding with a subsequent subtraction from f.

Notice that the result is filtered through \xintREZ, and will thus be of the form A/B[N], where

Documentation updated. → neither A nor B has trailing zeros. But the output fraction is not reduced to smallest terms.

The function call in expressions (\xintexpr, \xintfloatexpr) is frac. Inside \xintexpr..\relax, the function frac is mapped to \xintTFrac. Inside \xintfloatexpr..\relax, frac first applies \xintTFrac to its argument (which may be an exact fraction with more digits than the floating point precision) and only in a second stage makes the conversion to a floating point number with the precision as set by \xintDigits (default is 16).

```
\xintTFrac {1235/97}=71/97[0]   \xintTFrac {-1235/97}=-71/97[0]
\xintTFrac {1235.973}=973/1[-3]   \xintTFrac {-1235.973}=-973/1[-3]
\xintTFrac {1.122435727e5}=5727/1[-4]
```

9.25 \xintE

Frac num
f x ★ \xintE {f}{x} multiplies the fraction f by 10^x. The *second* argument x must obey the TeX bounds. Example:

```
\count 255 123456789 \xintE {10}{\count 255}->10/1[123456789]
```

Be careful that for obvious reasons such gigantic numbers should not be given to \xintNum, or added to something with a widely different order of magnitude, as the package always works to get the *exact* result. There is *no problem* using them for *float* operations:

```
\xintFloatAdd {1e1234567890}{1}=1.000000000000000e1234567890
```

9.26 \xintAdd

Frac Frac
f f ★ Computes the addition of two fractions. To keep for integers the integer format on output use \xintiAdd.

Checks if one denominator is a multiple of the other. Else multiplies the denominators.

9.27 \xintSub

Frac Frac
f f ★ Computes the difference of two fractions (\xintSub{F}{G} computes F-G). To keep for integers the integer format on output use \xintiSub.

Checks if one denominator is a multiple of the other. Else multiplies the denominators.

9.28 \xintMul

Frac Frac
f f ★ Computes the product of two fractions. To keep for integers the integer format on output use \xintiMul.

No reduction attempted.

9.29 \xintSqr

Frac
f ★ Computes the square of one fraction. To maintain for integer input an integer format on output use \xintiSqr.

9.30 \xintDiv

Frac Frac
f f ★ Computes the quotient of two fractions. (\xintDiv{F}{G} computes F/G). To keep for integers the integer format on output use \xintiMul.

No reduction attempted.

9.31 \xintDivTrunc, \xintDivRound

Frac Frac
f f ★ Computes the quotient of the two arguments then either truncates or rounds to an integer.

9.32 \xintiFac

Num
f ★ With xintfrac loaded \xintiFac is extended to allow a fraction f as input, it will be truncated first to an integer n before the evaluation of the factorial. The output is an integer in strict format, without a trailing /1[0]. See the \xintiiFac doc for more info.

9.33 \xintiBinomial

Num Num
f f ★ With xintfrac loaded \xintiBinomial is extended to allow fractional inputs which will be truncated to integers before the evaluation of the binomial. The output is an integer in strict format, without a trailing /1[0]. See the \xintiiBinomial doc for the current allowable range.

9.34 \xintiPFactorial

Num Num
f f ★ With xintfrac loaded \xintiPFactorial is extended to allow fractional inputs which will be truncated to integers before the evaluation of the partial factorial. The output is an integer in strict format, without a trailing /1[0]. See the \xintiiPFactorial doc for more info.

9.35 \xintPow

Frac Num
f f ★ \xintPow{f}{x}: computes f^x with f a fraction and x possibly also, but x will first get truncated to a (positive or negative) integer.

The output will now always be in the form A/B[n] (even when the exponent vanishes: \xintPow {2/3}{0}=1/1[0]).

The macro handling only integers is available as \xintiPow. Only \xintPow accepts negative exponent, as this produces fractions.

Within an \xintiiexpr..\relax the infix operator ^ is mapped to \xintiiPow; within an \xintexpr-ession it is mapped to \xintPow.

9.36 \xintSum

Frac
f→ * f ★ This computes the sum of fractions. The output will now always be in the form A/B[n]. The original, for big integers only (in strict format), is available as \xintiiSum.

```
\xintSum {{1282/2196921}{-281710/291927}{4028/28612}}
```

-15113784906302076/18350036010217404[0]

No simplification attempted.

9.37 \xintPrd

Frac
f→ * f ★ TThis computes the product of fractions. The output will now always be in the form A/B[n]. The original, for big integers only (in strict format), is available as \xintiiPrd.

```
\xintPrd {{1282/2196921}{-281710/291927}{4028/28612}}
```

-1454721142160/18350036010217404[0]

No simplification attempted.

9.38 \xintCmp

Frac Frac
f f ★ This compares two fractions F and G and produces -1, 0, or 1 according to F<G, F=G, F>G.

For choosing branches according to the result of comparing f and g, see \xintifCmp.

9.39 \xintIsOne

Frac f ★ This returns 1 if the fraction is 1 and 0 if not.

```
\xintIsOne {21921379213/21921379213} but \xintIsOne {1.000000000000000000000000000000001}
```

1 but 0

9.40 \xintGeq

Frac Frac f f ★ This compares the *absolute values* of two fractions. \xintGeq{f}{g} returns 1 if |f| ⩾ |g| and 0 if not.

May be used for expandably branching as: \xintSgnFork{\xintGeq{f}{g}}{}{code for |f|<|g|}{code for |f|⩾|g|}

9.41 \xintMax

Frac Frac f f ★ The maximum of two fractions. But now \xintMax {2}{3} returns 3/1[0]. The original, for use with (possibly big) integers only with no need of normalization, is available as \xintiiMax: \xintiiMax {2}{3}=3.

Num Num f f ★ There is also \xintiMax which works with fractions but first truncates them to integers.

```
\xintMax {2.5}{7.2} but \xintiMax {2.5}{7.2}
```

72/1[-1] but 7

9.42 \xintMin

Frac Frac f f ★ The maximum of two fractions. The original, for use with (possibly big) integers only with no need of normalization, is available as \xintiiMin: \xintiiMin {2}{3}=2.

Num Num f f ★ There is also \xintiMin which works with fractions but first truncates them to integers.

```
\xintMin {2.5}{7.2} but \xintiMin {2.5}{7.2}
```

25/1[-1] but 2

9.43 \xintMaxof

f → * Frac f ★ The maximum of any number of fractions, each within braces, and the whole thing within braces.

```
\xintMaxof {{1.23}{1.2299}{1.2301}} and \xintMaxof {{-1.23}{-1.2299}{-1.2301}}
```

12301/1[-4] and -12299/1[-4]

9.44 \xintMinof

f → * Frac f ★ The minimum of any number of fractions, each within braces, and the whole thing within braces.

```
\xintMinof {{1.23}{1.2299}{1.2301}} and \xintMinof {{-1.23}{-1.2299}{-1.2301}}
```

12299/1[-4] and -12301/1[-4]

9.45 \xintAbs

Frac f ★ The absolute value. Note that \xintAbs {-2}=2/1[0] whereas \xintiAbs {-2}=2.

9.46 \xintSgn

Frac f ★ The sign of a fraction.

9.47 \xintOpp

Frac f ★ The opposite of a fraction. Note that \xintOpp {3} now outputs -3/1[0] whereas \xintiOpp {3} returns -3.

9.48 \xintDigits, \xinttheDigits

The syntax \xintDigits := D; (where spaces do not matter) assigns the value of D to the number of digits to be used by floating point operations. The default is 16. The maximal value is 32767. The ★ macro \xinttheDigits serves to print the current value.

9.49 \xintFloat

[num x] Frac f ★ The macro \xintFloat [P]{f} has an optional argument P which replaces the current value of \xinttheDigits. The (rounded truncation of the) fraction f is then printed in scientific form, with P digits, a lowercase e and an exponent N. The first digit is from 1 to 9, it is preceded by an optional minus sign and is followed by a dot and P-1 digits, the trailing zeroes are not trimmed. In the exceptional case where the rounding went to the next power of ten, the output is 10.0...0eN (with a sign, perhaps). The sole exception is for a zero value, which then gets output as 0.e0 (in an \xintCmp test it is the only possible output of \xintFloat or one of the `Float' macros which will test positive for equality with zero).

```
\xintFloat[32]{1234567/7654321}=1.6129020457856418616360615134902e-1
\xintFloat[32]{1/\xintiiFac{100}}=1.0715102881254669231835467595192e-158
```

The argument to \xintFloat may be an \xinttheexpr-ession, like the other macros; only its final evaluation is submitted to \xintFloat: the inner evaluations of chained arguments are not at all done in `floating' mode. For this one must use \xintthefloatexpr.

9.50 \xintPFloat

[num x] Frac f ★ The macro \xintPFloat [P]{f} is like \xintFloat but ``pretty-prints'' the output. Its behaviour has changed with release 1.2f: there is only one simplification rule now which is that decimal notation (with possibly needed extra zeros) is used in place of scientific notation when the exponent would end up being between -5 and 5 inclusive.[66]

Changed (1.2f) If the input vanishes the output will be 0. with a a decimal mark (the original version printed 0 with no decimal mark).[67]

\xintthefloatexpr applies this macro to its output (or each of its outputs, if comma separated). Currently trailing zeros are not trimmed.

```
\begingroup\def\test #1{#1${}\to{}$\xintPFloat{#1}}%
\string\xintDigits\ at \xinttheDigits
\begin{itemize}[nosep]
\item \test {0}
\item \test {1.23456789e-7}
\item \test {1.23456789e-6}
\item \test {1.23456789e-5}
\item \test {1.23456789e-4}
\item \test {1.23456789e-3}
\item \test {1.23456789e-2}
\item \test {1.23456789e-1}
\item \test {1.23456789e0}
\item \test {1.23456789e1}
\item \test {1.23456789e2}
\item \test {1.23456789e3}
\item \test {1.23456789e4}
\item \test {1.23456789e5}
\item \test {1.23456789e6}
\item \test {1.23456789e7}
\end{itemize}
```

[66] In the exceptional case of an input rounded up towards next power of ten, the exponent referred-to here is the integer N in 10.0..0eN with a total number of zeroes equal to the precision. [67] Currently there are no subnormal numbers, and no underflow because the exponent is only limited by the maximal TeX number; thus underflow situations would manifest themselves via low-level arithmetic overflow errors.

```
    \endgroup
```

\xintDigits at 16

- 0 → 0.
- 1.23456789e-7 → 1.234567890000000e-7
- 1.23456789e-6 → 1.234567890000000e-6
- 1.23456789e-5 → 0.00001234567890000000
- 1.23456789e-4 → 0.0001234567890000000
- 1.23456789e-3 → 0.001234567890000000
- 1.23456789e-2 → 0.01234567890000000
- 1.23456789e-1 → 0.1234567890000000
- 1.23456789e0 → 1.234567890000000
- 1.23456789e1 → 12.34567890000000
- 1.23456789e2 → 123.4567890000000
- 1.23456789e3 → 1234.567890000000
- 1.23456789e4 → 12345.67890000000
- 1.23456789e5 → 123456.7890000000
- 1.23456789e6 → 1.234567890000000e6
- 1.23456789e7 → 1.234567890000000e7

9.51 \xintFloatE

[num x] Frac f num x ★

\xintFloatE [P]{f}{x} multiplies the input f by 10^x, and converts it to float format according to the optional first argument or current value of \xinttheDigits.

```
    \xintFloatE {1.23e37}{53}=1.230000000000000e90
```

9.52 \xintFloatAdd

[num x] Frac f Frac f ★ Changed (1.2f)

\xintFloatAdd [P]{f}{g} first replaces f and g with their float approximations f' and g' to P significant places or to the precision from \xintDigits. It then produces the sum f'+g', correctly rounded to nearest with the same number of significant places.

As for \xintFloat, in case of rounding up to next power of ten, the value may exceptionally come out as 10.0...0eN with a total of P+1 digits.

See subsection 2.2 for more.

9.53 \xintFloatSub

[num x] Frac f Frac f ★ Changed (1.2f)

\xintFloatSub [P]{f}{g} first replaces f and g with their float approximations f' and g' to P significant places or to the precision from \xintDigits. It then produces the difference f'-g' correctly rounded to nearest P-float.

As for \xintFloat, in case of rounding up to next power of ten, the value may exceptionally come out as 10.0...0eN with a total of P+1 digits.

See subsection 2.2 for more.

9.54 \xintFloatMul

[num x] Frac f Frac f ★ Changed (1.2f)

\xintFloatMul [P]{f}{g} first replaces f and g with their float approximations f' and g' to P (or \xinttheDigits) significant places. It then correctly rounds the product f'*g' to nearest P-float.

See subsection 2.2 for more.

It is obviously much needed that the author improves its algorithms to avoid going through the exact 2P or 2P-1 digits before throwing to the waste-bin half of those digits !

9.55 \xintFloatDiv

num x Frac f Frac f ★ Changed (1.2f)

\xintFloatDiv [P]{f}{g} first replaces f and g with their float approximations f' and g' to P (or \xinttheDigits) significant places. It then correctly rounds the fraction f'/g' to nearest P-float.

See subsection 2.2 for more.

9.56 \xintFloatFac

num x Num f ★ New with 1.2

\xintFloatFac[P]{f} returns the factorial with either \xinttheDigits or P digits of precision.

The exact theoretical value differs from the calculated one Y by an absolute error strictly less than 0.6 ulp(Y).

```
$1000!\approx{}$\xintFloatFac [30]{1000}
```

$1000! \approx 4.02387260077093773543702433923e2567$ The computation proceeds via doing explicitely the product, as the Stirling formula cannot be used for lack so far of exp/log.

The maximal allowed argument is 99999999, but already 100000! currently takes, for 16 digits of precision, a few seconds on my laptop (it returns 2.824229407960348e456573).

The factorial function is available in \xintfloatexpr:

```
\xintthefloatexpr factorial(1000)\relax % same as 1000!
```

4.023872600770938e2567

9.57 \xintFloatBinomial

num x Num f Num f ★ New with 1.2f

\xintFloatBinomial[P]{x}{y} computes binomial coefficients with either \xinttheDigits or P digits of precision.

The exact theoretical value differs from the calculated one Y by an absolute error strictly less than 0.6 ulp(Y).

```
${3000\choose 1500}\approx{}$\xintFloatBinomial [24]{3000}{1500}
```

${3000\choose 1500}\approx 1.79196793754756005073269e901$

The binomial function is available in \xintfloatexpr:

```
\xintthefloatexpr binomial(3000,1500)\relax
```

1.791967937547560e901

The computation is based on the formula (x-y+1)...x/y! (here one arranges y<=x-y naturally).

9.58 \xintFloatPFactorial

num x Num f Num f ★ New with 1.2f

\xintFloatPFactorial[P]{x}{y} computes the product (x+1)...y.

The inputs x and y must evaluate to non-negative integers less than 10^8.

The exact theoretical value differs from the calculated one Y by an absolute error strictly less than 0.6 ulp(Y).

The pfactorial function is available in \xintfloatexpr:

```
\xintthefloatexpr pfactorial(2500,5000)\relax
```

2.595989917947957e8914

9.59 \xintFloatPow

num x Frac f num x ★

\xintFloatPow [P]{f}{x} uses either the optional argument P or the value of \xinttheDigits. It computes a floating approximation to f^x. The precision P must be at least 1, naturally.

The exponent x will be fed to a \numexpr, hence count registers are accepted on input for this x. And the absolute value |x| must obey the TeX bound. For larger exponents use the slightly slower routine \xintFloatPower which allows the exponent to be a fraction simplifying to an integer and does not limit its size. This slightly slower routine is the one to which ^ is mapped inside \xintthefloatexpr...\relax.

Changed (1.2f) The argument f is first rounded to P significant places to give f'. The output Z is such that the exact f'^x differs from Z by an absolute error less than 0.52 ulp(Z).

```
\xintFloatPow [8]{3.1415}{1234567890}=1.6122066e613749456
```

9.60 \xintFloatPower

num x Frac f Num f ★

\xintFloatPower[P]{f}{g} computes a floating point value f^g where the exponent g is not constrained to be at most the TeX bound 2147483647. It may even be a fraction A/B but must simplify to a (possibly big) integer. The exponent of the *output* however *must* at any rate obey the TeX 2147483647 bound.

Changed (1.2f)

The argument f is first rounded to P significant places to give f'. The output Z is such that the exact f'^g differs from Z by an absolute error less than 0.6 ulp(Z) (actually 0.52 ulp(Z)).[68]

```
\xintFloatPower [8]{3.1415}{3e9}=1.4317729e1491411192
```

Notice that 3e9>2^31.

Here is another example:

```
\xintFloatPower [48] {1.1547}{\xintiiPow {2}{35}}
```

computes $(1.1547)^{2^{35}} = (1.1547)^{34359738368}$ to be approximately

$2.785,837,382,571,371,438,495,789,880,733,698,213,205,183,990,48 \times 10^{2,146,424,193}$

Again, 2^{35} exceeds TeX's bound, but \xintFloatPower allows it, what counts is the exponent of the result which, while dangerously close to 2^{31} is not quite there yet.[69]

Inside an \xintfloatexpr-ession, \xintFloatPower is the function to which ^ is mapped.[70][71] Thus the same computation as above can be done via the non-expandable assignment \xintDigits:=48; and

```
\xintthefloatexpr 1.1547^(2^35)\relax
```

There is a subtlety here that the 2^35 will be evaluated as a floating point number but fortunately it only has 11 digits, hence the final evaluation is done with a correct exponent. It is safer, and also more efficient to code the above rather as:

```
\xintthefloatexpr 1.1547^\xintiiexpr 2^35\relax\relax
```

to guarantee no loss of digits in the exponent.

There is an important difference between \xintFloatPower [Q]{X}{Y} and \xintthefloatexpr [Q] X^Y \relax: in the former case the computation is done with Q digits or precision (but if X and Y themselves stand for some floating point macros with arguments, their respective evaluations obey the precision \xinttheDigits or as set optionally in the macro calls themselves), whereas with \xintthefloatexpr[Q] the evaluation of the expression proceeds with \xinttheDigits digits of precision, and the final result is then rounded to Q digits: thus this makes real sense only if used with Q<\xinttheDigits.

9.61 \xintFloatSqrt

num x Frac f ★

\xintFloatSqrt[P]{f} computes a floating point approximation of $\sqrt{f}$, either using the optional precision P or the value of \xinttheDigits.

☞ More precisely the macro achieves so-called *correct rounding*: the produced value is the rounding to P significant places of the abstract exact value, *if the input has itself at most P digits* (and an arbitrary exponent).

New with 1.2f

```
\xintFloatSqrt [89]{10}\newline
\xintFloatSqrt [89]{100}\newline
\xintFloatSqrt [89]{123456789}\newline
And now some tests to check that correct rounding applies correctly (sic):\newline
(we observe in passing illustrations that rounding to nearest is not transitive)\newline
\xintFloatSqrt {5625000075000001}\newline
```

[68] For the 0.6 ulp(Z) bound, earlier releases had potentially a problem for negative exponents; the final reverse was done with only two guard digits (due to the implementation of \xintFloat), and as a result the final error could conceivably exceed 0.6 ulp(Z), although remaining smaller than 0.7 ulp(Z). [69] The printing of the result was done via the \numprint command from the http://ctan.org/pkg/numprint package. [70] I am constantly hesitating about this. In my testing, switching to \xintFloatPow would only bring from 5% to perhaps 20% gain for computations with 16 digits of precision and moderately sized exponent. [71] Since 1.2f, the ^ operator handles also half-integer exponents.

New with 1.2f

```
\xintFloatSqrt [24]{5625000075000001}\newline
\xintFloatSqrt [32]{5625000075000001}\newline
\xintFloatSqrt [48]{562500000000000000000000007500000000000000000000001}\newline
\xintFloatSqrt [64]{562500000000000000000000007500000000000000000000001}\newline
\xintFloatSqrt [80]{562500000000000000000000007500000000000000000000001}\par
```

```
3.1622776601683793319988935444327185337195551393252168268575048527925944386392382213442481e0
1.0000000000000000000000000000000000000000000000000000000000000000000000000000000000000000e1
1.1111111060555555440541666143353469245878409860134351071458570675251471479496366736579136e4
And now some tests to check that correct rounding applies correctly (sic):
(we observe in passing illustrations that rounding to nearest is not transitive)
7.500000050000000e7
7.5000000500000000500000000e7
7.500000050000000049999999966666667e7
7.50000000000000000000000005000000000000000000000000e23
7.500000000000000000000000050000000000000000000000005000000000000000e23
7.5000000000000000000000000500000000000000000000000004999999999999999999999999966666667e23
```

9.62 \xintiDivision, \xintiQuo, \xintiRem, \xintFDg, \xintLDg, \xintMON, \xintMMON, \xintOdd

Frac Frac
f *f* ★ These macros accept a fraction (or two) on input but will truncate it (them) to an integer using \xintNum (which is the same as \xintTTrunc). On output they produce integers without / nor [N].

All have variants from package xint whose names start with xintii rather than xint; these variants accept on input only integers in the strict format (they do not use \xintNum). They thus have less overhead, and may be used when one is dealing exclusively with (big) integers.

```
\xintNum {1e80}
100000000000000000000000000000000000000000000000000000000000000000000000000000000
```

10 Commands of the xintexpr package

The xintexpr package was first released with version 1.07 (2013/05/25) of the xint bundle. It was substantially enhanced with release 1.1 from 2014/10/28.

Release 1.2 removed a limitation to numbers of at most 5000 digits, and there is now a float variant of the factorial. Also the ``pseudo-functions'' qint, qfrac, qfloat ('q' for quick), were added to handle very big inputs and avoid scanning it digit per digit.

The package loads automatically xintfrac and xinttools (it is now the only arithmetic package from the xint bundle which loads xinttools).

- for using the gcd and lcm functions, it is necessary to load package xintgcd.

```
\xinttheexpr lcm (2^5*7*13^10*17^5,2^3*13^15*19^3,7^3*13*23^2)\relax
```

28943794413380000367610460876088864

- for allowing hexadecimal numbers (uppercase letters) on input, it is necessary to load package xintbinhex.

```
\xinttheexpr "A*"B*"C*"D*"D*"F, "FF.FF, reduce("FF.FFF + 16^-3)\relax
```

3346200, 25599609375[-8], 256

Please refer to section 4 for a more detailed description of the syntax elements for expressions.

10.1 The \xintexpr expressions

x ★ An xintexpression is a construct \xintexpr⟨*expandable_expression*⟩\relax where the expandable expression is read and completely expanded from left to right.

An \xintexpr...\relax *must* end in a \relax (which will be absorbed). Like a \numexpr expression, it is not printable as is, nor can it be directly employed as argument to the other package macros. For this one must use one of the two equivalent forms:

x ★ • \xinttheexpr⟨*expandable_expression*⟩\relax, or

x ★ • \xintthe\xintexpr⟨*expandable_expression*⟩\relax.

The computations are done *exactly*, and with no simplification of the result. See \xintfloatexpr for a similar parser which rounds each operation inside the expression to \xinttheDigits digits of precision.

It is possible to wrap the whole expression (or only some portions) inside the \xintexpr...\relax in one of the functions round, trunc, or float: in round and trunc the second optional parameter is the number of digits of the fractional part; in float it is the total number of digits of the mantissa.

The reduce will reduce the fraction to smallest terms, or more precisely it reduces to smallest terms the A/B part from its argument evaluating to A/B[N].

As an alternative and equivalent syntax to

```
\xintexpr round(<expression>, D)\relax
```

there is[72]

```
\xintiexpr [D] <expression> \relax
```

The parameter D must be zero or positive.[73] Perhaps some future version will give a meaning to using a negative D.[74]

- the expression may contain arbitrarily many levels of nested parenthesized sub-expressions.
- to let sub-contents evaluate as a sub-unit it should be either
  1. parenthesized,
  2. or a sub-expression \xintexpr...\relax.
- to either give an expression as argument to the other package macros, or more generally to macros which expand their arguments, one must use the \xinttheexpr...\relax or \xintthe\xintexpr...\relax forms. Similarly, printing the result itself must be done with these forms.
- one should not use \xinttheexpr...\relax as a sub-constituent of an \xintexpr...\relax but only the \xintexpr...\relax form which is more efficient in this context.
- each xintexpression, whether prefixed or not with \xintthe, is completely expandable and obtains its result in two expansion steps.

10.2 Expression syntax overview

An expression is enclosed between either \xintexpr, or \xintiexpr, or \xintiiexpr, or \xintfloatexpr, or \xintboolexpr and a *mandatory* ending \relax. An expression may be a sub-unit of another one.

Apart from \xintfloatexpr the evaluations of algebraic operations are *exact*. The variant \xintiiexpr does not know fractions and is provided for integer-only calculations. The variant \xintiexpr is exactly like \xintexpr except that it either rounds the final result to an integer, or in case of an optional parameter [d], rounds to a fixed point number with d digits after decimal mark. The variant \xintfloatexpr does float calculations according to the current value of the precision set by \xintDigits.

The whole expression should be prefixed by \xintthe when it is destined to be printed on the typeset page, or given as argument to a macro (assuming this macro systematically expands its argument). As a shortcut to \xintthe\xintexpr there is \xinttheexpr. One gets used to not forget the two t's.

\xintexpr-essions and \xinttheexpr-essions are completely expandable, in two steps.

- An expression is built the standard way with opening and closing parentheses, infix operators, and (big) numbers, with possibly a fractional part, and/or scientific notation (except for \xintiiexpr which only admits big integers). All variants work with comma separated expressions. On output each comma will be followed by a space. A decimal number must have digits either before or after the decimal mark.

[72] For truncation rather than rounding, one uses \xintexpr trunc(<expression>, D)\relax. [73] Notice that D=0 corresponds to using round(<expression>) not round(<expression>,0) which would leave a trailing dot. Same for trunc. There is also function float for floating point rounding to \xinttheDigits or the given number of significant digits as second argument. [74] Thanks to KT for this suggestion. Sorry for the delay in implementing it... matter of formatting the output and corresponding choice of user interface are still in need of some additional thinking.

- as everything gets expanded, the characters ., +, -, *, /, ^, !, &, |, ?, :, <, >, =, (,), ",], [, @ and the comma , should not (if used in the expression) be active. For example, the French language in Babel system, for pdfLaTeX, activates !, ?, ; and :. Turn off the activity before the expressions.

  Alternatively the command \xintexprSafeCatcodes resets all characters potentially needed by \xintexpr to their standard catcodes and \xintexprRestoreCatcodes restores the status prevailing at the time of the previous \xintexprSafeCatcodes.
- The infix operators are +, -, *, /, ^ (or **) for exact or floating point algebra (currently, only integer and half-integer exponents are allowed for float power operations), && and || [75] for combining ``true'' (non zero) and ``false'' (zero) conditions, as can be formed for example with the = (or ==), <, >, <=, >=, != comparison operators.
- The ! is either a function (the logical not) requiring an argument within parentheses, or a post-fix operator which does the factorial. In \xintfloatexpr it is mapped to \xintFloatFac, else it computes the exact factorial.
- The ? may serve either as a function (the truth value) requiring an argument within parentheses, or as two-way post-fix branching operator (cond)?{YES}{NO}. The false branch will *not* be evaluated.
- There is also ?? which branches according to the scheme (x)??{<0}{=0}{>0}.
- Comma separated lists may be generated with a..b and a..[d]..b and they may be manipulated to some extent once put into bracket. See subsection 4.3. There is no real concept of a list object, nor list operations, although itemwise manipulation are possible. There is no notion of an ``nuple'' object. The variable nil is reserved, it represents an empty list.
- Count registers and \numexpr-essions are accepted (LaTeX's counters can be inserted using \value) natively without \the or \number as prefix. Also dimen registers and control sequences, skip registers and control sequences (LaTeX's lengths), \dimexpr-essions, \glueexpr-essions are automatically unpacked using \number, discarding the stretch and shrink components and giving the dimension value in sp units (1/65536th of a TeX point). Furthermore, tacit multiplication is implied, when the (count or dimen or glue) register or variable, or the (\numexpr or \dimexpr or \glueexpr) expression is immediately prefixed by a (decimal) number.
- See subsection 4.2 for the complete rules of tacit multiplication.
- With a macro \x defined like this:

  ```
  \def\x {\xintexpr \a + \b \relax} or \edef\x {\xintexpr \a+\b\relax}
  ```

  one may then do \xintthe\x, either for printing the result on the page or to use it in some other macros expanding their arguments. The \edef does the computation immediately but keeps it in an internal private format. Naturally, the \edef is only possible if \a and \b are already defined. With both approaches the \x can be inserted in other expressions, as for example (assuming naturally as we use an \edef that in the `yet-to-be computed' case the \a and \b now have some suitable meaning):

  ```
  \edef\y {\xintexpr \x^3\relax}
  ```
- There is also \xintboolexpr ... \relax and \xinttheboolexpr ... \relax. Same as \xintexpr with the final result converted to 1 if it is not zero. See also \xintifboolexpr (subsection 10.16) and the discussion of the bool and togl functions in section 10. Here is an example:

  ```
  \xintNewBoolExpr \AssertionA[3]{ #1 && (#2|#3) }
  \xintNewBoolExpr \AssertionB[3]{ #1 || (#2&#3) }
  \xintNewBoolExpr \AssertionC[3]{ xor(#1,#2,#3) }
  {\centering\normalcolor\xintFor #1 in {0,1} \do {%
    \xintFor #2 in {0,1} \do {%
      \xintFor #3 in {0,1} \do {%
  ```

[75] with releases earlier than 1.1, only single character operators & and | were available, because the parser did not handle multi-character operators. Their usage in this rôle is now deprecated, as they may be assigned some new meaning in the future.

```
#1 AND (#2 OR #3) is \textcolor[named]{OrangeRed}{\AssertionA {#1}{#2}{#3}}\hfil
#1 OR (#2 AND #3) is \textcolor[named]{OrangeRed}{\AssertionB {#1}{#2}{#3}}\hfil
#1 XOR #2 XOR #3  is \textcolor[named]{OrangeRed}{\AssertionC {#1}{#2}{#3}}\\}}}}
```

0 AND (0 OR 0) is 0	0 OR (0 AND 0) is 0	0 XOR 0 XOR 0 is 0
0 AND (0 OR 1) is 0	0 OR (0 AND 1) is 0	0 XOR 0 XOR 1 is 1
0 AND (1 OR 0) is 0	0 OR (1 AND 0) is 0	0 XOR 1 XOR 0 is 1
0 AND (1 OR 1) is 0	0 OR (1 AND 1) is 1	0 XOR 1 XOR 1 is 0
1 AND (0 OR 0) is 0	1 OR (0 AND 0) is 1	1 XOR 0 XOR 0 is 1
1 AND (0 OR 1) is 1	1 OR (0 AND 1) is 1	1 XOR 0 XOR 1 is 0
1 AND (1 OR 0) is 1	1 OR (1 AND 0) is 1	1 XOR 1 XOR 0 is 0
1 AND (1 OR 1) is 1	1 OR (1 AND 1) is 1	1 XOR 1 XOR 1 is 1

This example used for efficiency \xintNewBoolExpr. See also the subsection 10.10.

- There is \xintfloatexpr ... \relax where the algebra is done in floating point approximation (also for each intermediate result). Use the syntax \xintDigits:=N; to set the precision. Default: 16 digits.

  \xintthefloatexpr 2^100000\relax: 9.990020930143845e30102

  The square-root operation can be used in \xintexpr, it is computed as a float with the precision set by \xintDigits or by the optional second argument:

```
\xinttheexpr sqrt(2,60)\relax

Here the [60] is to avoid truncation to |\xinttheDigits| of precision on output.
\printnumber{\xintthefloatexpr [60] sqrt(2,60)\relax}
```

  141421356237309504880168872420969807856967187537694807317668[-59]

  Here the [60] is to avoid truncation to \xinttheDigits of precision on output. 1.414213562373095048801688724209698078569671875376948073176668

  Floats are quickly indispensable when using the power function , as exact results will easily have hundreds, if not thousands, of digits.

```
\xintDigits:=48;\xintthefloatexpr 2^100000\relax
```

  9.99002093014384507944032764330033590980429139054e30102

  Only integer and half-integer exponents are allowed.

- Hexadecimal TeX number denotations (*i.e.*, with a " prefix) are recognized by the \xintexpr parser and its variants. This requires xintbinhex. Except in \xintiiexpr, a (possibly empty) fractional part with the dot . as ``hexadecimal'' mark is allowed.

  \xinttheexpr "FEDCBA9876543210\relax→18364758544493064720

  \xinttheiexpr 16^5-("F75DE.0A8B9+"8A21.F5746+16^-5)\relax→0

  Letters must be uppercased, as with standard TeX hexadecimal denotations.

- 2^-10 is perfectly accepted input, no need for parentheses; and operators of power ^, division /, and subtraction - are all left-associative: 2^4^8 is evaluated as (2^4)^8. The minus sign as prefix has various precedence levels: \xintexpr -3-4*-5^-7\relax evaluates as (-3)-(4*(-(5^(-7)))) and -3^-4*-5-7 as (-((3^(-4))*(-5)))-7.

  An exception to left associativity is applied in case of tacit multiplication x/2y is interpreted as x/(2y) and (1+2)/(3+4)(5+6) is computed as (1+2)/((3+4)*(5+6)). See subsection 4.2.

- if one uses *macros* within \xintexpr..\relax one should obviously take into account that the parser will *not* see the macro arguments. This applies also to macros defined via \xintNewExpr. Functions declared with \xintdeffunc on the other hand behave exactly like the native functions of the package.

10.3 More examples with dummy variables

These examples were first added to this manual at the time of the 1.1 release (2014/10/29).

```
Prime numbers are always cool
```

```
\xinttheiiexpr seq((seq((subs((x/:m)?{(m*m>x)?{1}{0}}{-1},m=2n+1))
                        ??{break(0)}{omit}{break(1)},n=1++))?{x}{omit},
               x=10001..[2]..10200)\relax
```

Prime numbers are always cool 10007, 10009, 10037, 10039, 10061, 10067, 10069, 10079, 10091, 10093, 10099, 10103, 10111, 10133, 10139, 10141, 10151, 10159, 10163, 10169, 10177, 10181, 10193

The syntax in this last example may look a bit involved. First `x/:m` computes `x modulo m` (this is the modulo with respect to truncated division, which here for positive arguments is like Euclidean division; in `\xintexpr...\relax`, `a/:b` is such that `a = b*(a//b)+a/:b`, with `a//b` the algebraic quotient `a/b` truncated to an integer.). The `(x)?{yes}{no}` construct checks if `x` (which *must* be within parentheses) is true or false, i.e. non zero or zero. It then executes either the `yes` or the `no` branch, the non chosen branch is *not* evaluated. Thus if `m` divides `x` we are in the second (``false'') branch. This gives a `-1`. This `-1` is the argument to a `??` branch which is of the type `(y)??{y<0}{y=0}{y>0}`, thus here the `y<0`, i.e., `break(0)` is chosen. This `0` is thus given to another `?` which consequently chooses `omit`, hence the number is not kept in the list. The numbers which survive are the prime numbers.

```
The first Fibonacci number beyond |2^64| bound is
\xinttheiiexpr subs(iterr(0,1;(@1>N)?{break(i)}{@1+@2},i=1++),N=2^64)\relax{}
and the previous number was its index.
```

The first Fibonacci number beyond 2^64 bound is 19740274219868223167, 94 and the previous number was its index.

One more recursion:

```
\def\syr #1{\xinttheiiexpr rseq(#1; (@<=1)?{break(i)}{odd(@)?{3@+1}{@//2}},i=0++)\relax}
The 3x+1 problem: \syr{231}\par
```

The 3x+1 problem: 231, 694, 347, 1042, 521, 1564, 782, 391, 1174, 587, 1762, 881, 2644, 1322, 661, 1984, 992, 496, 248, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1, 127

OK, a final one:

```
\def\syrMax #1{\xinttheiiexpr iterr(#1,#1;even(i)?
                                {(@2<=1)?{break(i/2)}{odd(@2)?{3@2+1}{@2//2}}}
                                {(@1>@2)?{@1}{@2}},i=0++)\relax }
With initial value 1161, the maximal number attained is \syrMax{1161} and that latter
number is the number of steps which was needed to reach 1.\par
```

With initial value 1161, the maximal number attained is 190996, 181 and that latter number is the number of steps which was needed to reach 1.

Well, one more:

```
\newcommand\GCD [2]{\xinttheiiexpr rrseq(#1,#2; (@1=0)?{abort}{@2/:@1}, i=1++)\relax }
\GCD {13^10*17^5*29^5}{2^5*3^6*17^2}
```

4014838863509162883616357, 6741792, 3367717, 6358, 4335, 2023, 289, 0

and the ultimate:

```
\newcommand\Factors [1]{\xinttheiiexpr
    subs(seq((i/:3=1)?{omit}{[L][i]},i=0..len(L)-1),
    L=rseq(#1;(p^2>[@][0])?{([@][0]>1)?{break(1,[@][0],1)}{abort}}
                      {(([@][0])/:p)?{omit}
    {iter(([@][0])//p; (@/:p)?{break(@,p,e)}{@//p},e=1++)}},p=2++))\relax }
\Factors {41^4*59^2*29^3*13^5*17^8*29^2*59^4*37^6}
```

16246355912554185673266068721806243461403654781833, 13, 5, 17, 8, 29, 5, 37, 6, 41, 4, 59, 6

This might look a bit scary, I admit.[76] xintexpr has minimal tools and is obstinate about doing everything expandably! We are hampered by absence of a notion of ``nuple''. The algorithm divides

[76] Look at the Brent-Salamin algorithm implementation for a much saner example.

N by 2 until no more possible, then by 3, then by 4 (which is silly), then by 5, then by 6 (silly again), ...

The variable L=rseq(#1;...) expands, if one follows the steps, to a comma separated list starting with the initial (evaluated) N=#1 and then pseudo-triplets where the first item is N trimmed of small primes, the second item is the last prime divisor found, and the third item is its exponent in original N.

The algorithm needs to keep handy the last computed quotient by prime powers, hence all of them, but at the very end it will be cleaner to get rid of them (this corresponds to the first line in the code above). This is achieved in a cumbersome inefficient way; indeed each item extraction [L][i] is costly: it is not like accessing an array stored in memory, due to expandability, nothing can be stored in memory! Nevertheless, this step could be done here in a far less inefficient manner if there was a variant of seq which, in the spirit of \xintiloopindex, would know how many steps it had been through so far. This is a feature to be added to \xintexpr! (as well as a ++ construct allowing a non unit step).

Notice that in iter(([@][0])//p; the @ refers to the previous triplet (or in the first step to N), but the latter @ showing up in (@/:p)? refers to the previous value computed by iter.

Parentheses are essential in ..([y][0]) else the parser will see ..[and end up in ultimate confusion, and also in ([@][0])/:p else the parser will see the itemwise operator]/ on lists and again be very confused (I could implement a]/: on lists, but in this situation this would also be very confusing to the parser.)

For comparison, here is an *f*-expandable macro expanding to the same result, but coded directly with the xint macros. Here #1 can not be itself an expression, naturally. But at least we let \Factorize *f*-expand its argument.

```
\makeatletter
\newcommand\Factorize [1]
      {\romannumeral0\expandafter\factorize\expandafter{\romannumeral-`0#1}}%
\newcommand\factorize [1]{\xintiiifOne{#1}{ 1}{\factors@a #1.{#1};}}%
\def\factors@a #1.{\xintiiifOdd{#1}
   {\factors@c 3.#1.}%
   {\expandafter\factors@b \expandafter1\expandafter.\romannumeral0\xinthalf{#1}.}}%
\def\factors@b #1.#2.{\xintiiifOne{#2}
   {\factors@end {2, #1}}%
   {\xintiiifOdd{#2}{\factors@c 3.#2.{2, #1}}%
                    {\expandafter\factors@b \the\numexpr #1+\@ne\expandafter.%
                        \romannumeral0\xinthalf{#2}.}}%
}%
\def\factors@c #1.#2.{%
    \expandafter\factors@d\romannumeral0\xintiidivision {#2}{#1}{#1}{#2}%
}%
\def\factors@d #1#2#3#4{\xintiiifNotZero{#2}
   {\xintiiifGt{#3}{#1}
        {\factors@end {#4, 1}}% ultimate quotient is a prime with power 1
        {\expandafter\factors@c\the\numexpr #3+\tw@.#4.}}%
   {\factors@e 1.#3.#1.}%
}%
\def\factors@e #1.#2.#3.{\xintiiifOne{#3}
   {\factors@end {#2, #1}}%
   {\expandafter\factors@f\romannumeral0\xintiidivision {#3}{#2}{#1}{#2}{#3}}%
}%
\def\factors@f #1#2#3#4#5{\xintiiifNotZero{#2}
   {\expandafter\factors@c\the\numexpr #4+\tw@.#5.{#4, #3}}%
   {\expandafter\factors@e\the\numexpr #3+\@ne.#4.#1.}%
```

```
}%
\def\factors@end #1;{\xintlistwithsep{, }{\xintRevWithBraces {#1}}}%
\makeatother
```

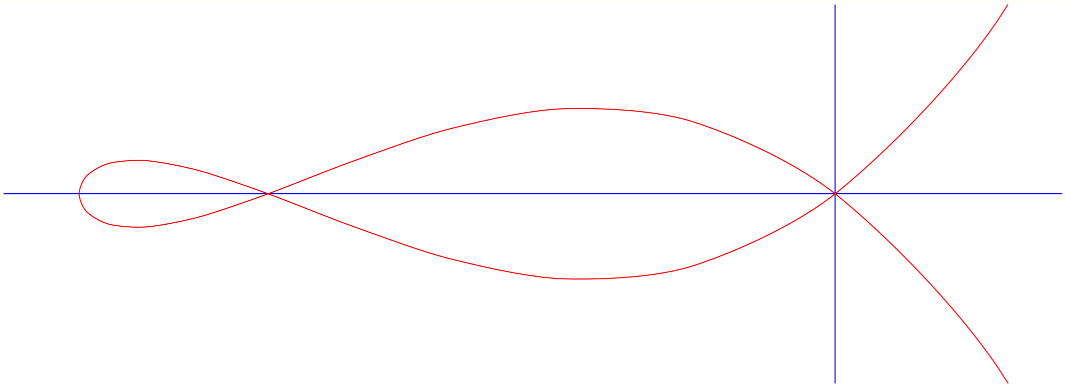
The macro \Factorize puts a little stress on the input save stack in order not be bothered with previously gathered things. I timed it to be about seven times faster than \Factors in test cases such as 16246355912554185673266068721806243461403654781833 and others. Among the various things explaining the speed-up, there is fact that we step by increments of two, not one, and also that we divide only once, obtaining quotient and remainder in one go. These two things already make for a speed-up factor of about four. Thus, our earlier \Factors was not completely inefficient, and was quite easier to come up with than \Factorize.[77]

If we only considered small integers, we could write pure \numexpr methods which would be very much faster (especially if we had a table of small primes prepared first) but still ridiculously slow compared to any non expandable implementation, not to mention use of programming languages directly accessing the CPU registers. . .

10.4 The \xintthecoords command

It converts a comma separated list into the format for list of coordinates as expected by the TikZ coordinates syntax. The code had to work around the problem that TikZ seemingly allows only a maximal number of about one hundred expansion steps for the list to be entirely produced. Presumably to catch an infinite loop, but one hundred is ridiculously low a number of expansion steps for any serious work in TEX for dealing with tokens.

```
{\centering\begin{tikzpicture}[scale=10]\xintDigits:=8;
  \clip (-1.1,-.25) rectangle (.3,.25);
  \draw [blue] (-1.1,0)--(1,0);
  \draw [blue] (0,-1)--(0,+1);
  \draw [red] plot[smooth] coordinates {\xintthecoords
    % converts into (x1, y1) (x2, y2)... format
    \xintfloatexpr seq((x^2-1,mul(x-t,t=-1+[0..4]/2)),x=-1.2..[0.1]..+1.2) \relax };
\end{tikzpicture}\par }
```



\xintthecoords should be followed immediately by \xintfloatexpr or \xintiexpr or \xintiiexpr, but not \xintthefloatexpr, etc. . .

Besides, as TikZ will not understand the A/B[N] format which is used on output by \xintexpr, \xintexpr is not really usable with \xintthecoords for a TikZ picture, but one may use it on its own, and the reason for the spaces in and between coordinate pairs is to allow if necessary to print on the page for examination with about correct line-breaks.

```
\edef\x{\xintthecoords \xintexpr rrseq(1/2,1/3; @1+@2, x=1..20)\relax }
\meaning\x +++
```

macro:->(1/2, 1/3) (5/6, 7/6) (12/6, 19/6) (31/6, 50/6) (81/6, 131/6) (212/6, 343/6) (555/6, 898/6) (1453/6, 2351/6) (3804/6, 6155/6) (9959/6, 16114/6) (26073/6, 42187/6)+++

77 2015/11/18 I have not revisited this code for a long time, and perhaps I could improve it now with some new techniques.

10.5 Breaking changes which came with the 1.1 release of xintexpr

Release 1.1 of 2014/10/29 brought many changes to xintexpr whose description has been incorporated to previous sections. Here we keep only a short list of the then breaking changes, for the record.

- in \xintiiexpr, / does *rounded* division, rather than as in earlier releases the Euclidean division (for positive arguments, this is truncated division). The new // operator does truncated division,
- the : operator for three-way branching is gone, replaced with ??,
- 1e(3+5) is now illegal. The number parser identifies e and E in the same way it does for the decimal mark, earlier versions treated e as E rather as postfix operators,
- the add and mul have a new syntax, old syntax is with \`+\` and \`*\` (quotes mandatory), sum and prd are gone,
- no more special treatment for encountered brace pairs {..} by the number scanner, a/b[N] notation can be used without use of braces (the N will end up space-stripped in a \numexpr, it is not parsed by the \xintexpr-ession scanner).
- although & and | are still available as Boolean operators the use of && and || is strongly recommended. The single letter operators might be assigned some other meaning in later releases (bitwise operations, perhaps). Do not use them.
- place holders for \xintNewExpr could be denoted #1, #2, ... or also, for special purposes $1, $2, ... Only the first form is now accepted and the special cases previously treated via the second form are now managed via a protect(...) function.

10.6 \numexpr or \dimexpr expressions, count and dimension registers and variables

Count registers, count control sequences, dimen registers, dimen control sequences (like \parindent), skips and skip control sequences, \numexpr, \dimexpr, \glueexpr, \fontdimen can be inserted directly, they will be unpacked using \number which gives the internal value in terms of scaled points for the dimensional variables: 1 pt = 65536 sp (stretch and shrink components are thus discarded).

Tacit multiplication (see subsection 4.2) is implied, when a number or decimal number prefixes such a register or control sequence. LaTeX lengths are skip control sequences and LaTeX counters should be inserted using \value.

Release 1.2 of the \xintexpr parser also recognizes and prefixes with \number the \ht, \dp, and \wd TeX primitives as well as the \fontcharht, \fontcharwd, \fontchardp and \fontcharic ε-TeX primitives.

In the case of numbered registers like \count255 or \dimen0 (or \ht0), the resulting digits will be re-parsed, so for example \count255 0 is like 100 if \the\count255 would give 10. The same happens with inputs such as \fontdimen6\font. And \numexpr 35+52\relax will be exactly as if 87 as been encountered by the parser, thus more digits may follow: \numexpr 35+52\relax 000 is like 87000. If a new \numexpr follows, it is treated as what would happen when \xintexpr scans a number and finds a non-digit: it does a tacit multiplication.

```
\xinttheexpr \numexpr 351+877\relax\numexpr 1000-125\relax\relax{} is the same
as \xinttheexpr 1228*875\relax.
```

1074500 is the same as 1074500.

Control sequences however (such as \parindent) are picked up as a whole by \xintexpr, and the numbers they define cannot be extended extra digits, a syntax error is raised if the parser finds digits rather than a legal operation after such a control sequence.

A token list variable must be prefixed by \the, it will not be unpacked automatically (the parser will actually try \number, and thus fail). Do not use \the but only \number with a dimen or skip, as the \xintexpr parser doesn't understand pt and its presence is a syntax error. To use a dimension expressed in terms of points or other TEX recognized units, incorporate it in \dimexpr...\relax.

Regarding how dimensional expressions are converted by TEX into scaled points see also subsection 2.10.

10.7 Catcodes and spaces

Active characters may (and will) break the functioning of \xintexpr. Inside an expression one may prefix, for example a : with \string. Or, for a more radical way, there is \xintexprSafeCatcodes. This is a non-expandable step as it changes catcodes.

10.7.1 \xintexprSafeCatcodes

This command sets the catcodes of the relevant characters to safe values. This is used internally by \xintNewExpr (restoring the catcodes on exit), hence \xintNewExpr does not have to be protected against active characters.

Attention however that if the whole

```
\xintNewExpr \foo [N] {<expression with #1,...>}
```

has been fetched as a macro argument, it will be too late then for \xintNewExpr to sanitize the catcodes of the (active) characters within the expression.

10.7.2 \xintexprRestoreCatcodes

Restores the catcodes to the earlier state.

Spaces inside an \xinttheexpr...\relax should mostly be innocuous (except inside macro arguments).

\xintexpr and \xinttheexpr are for the most part agnostic regarding catcodes: (unbraced) digits, binary operators, minus and plus signs as prefixes, dot as decimal mark, parentheses, may be indifferently of catcode letter or other or subscript or superscript, ..., it doesn't matter.[78]

The characters +, -, *, /, ^, !, &, |, ?, :, <, >, =, (,), ", [,], ;, the dot and the comma should not be active if in the expression, as everything is expanded along the way. If one of them is active, it should be prefixed with \string.

The exclamation mark ! should have its standard catcode: with catcode letter it is used internally and hence will confuse the parsers if it comes from the expression.

Digits, slash, square brackets, minus sign, in the output from an \xinttheexpr are all of catcode 12. For \xintthefloatexpr the `e' in the output has its standard catcode ``letter''.

A macro with arguments will expand and grab its arguments before the parser may get a chance to see them, so the situation with catcodes and spaces is not the same within such macro arguments.

10.8 Expandability, \xinteval

As is the case with all other package macros \xintexpr *f*-expands (in two steps) to its final (non-printable) result; and \xinttheexpr *f*-expands (in two steps) to the chain of digits (and possibly minus sign -, decimal mark ., fraction slash /, scientific e, square brackets [,]) representing the result.

Starting with 1.09j, an \xintexpr..\relax can be inserted without \xintthe prefix inside an \edef, or a \write. It expands to a private more compact representation (five tokens) than \xinttheexpr or \xintthe\xintexpr.

The material between \xintexpr and \relax should contain only expandable material.

The once expanded \xintexpr is \romannumeral0\xinteval. And there is similarly \xintieval, \xintiieval, and \xintfloateval. For the other cases one can use \romannumeral-`0 as prefix. For an

[78] Furthermore, although \xintexpr uses \string, it is escape-char agnostic. It should work with any \escapechar setting including -1.

example of expandable algorithms making use of chains of \xinteval-uations connected via \expandafter see subsection 6.24.

An expression can only be legally finished by a \relax token, which will be absorbed.

It is quite possible to nest expressions among themselves; for example, if one needs inside an \xintiiexpr...\relax to do some computations with fractions, rounding the final result to an integer, one just has to insert \xintiexpr...\relax. The functioning of the infix operators will not be in the least affected from the fact that the surrounding ``environment'' is the \xintiiexpr one.

10.9 Memory considerations

The parser creates an undefined control sequence for each intermediate computation evaluation: addition, subtraction, etc... Thus, a moderately sized expression might create 10, or 20 such control sequences. On my TEX installation, the memory available for such things is of circa 200, 000 multi-letter control words. So this means that a document containing hundreds, perhaps even thousands of expressions will compile with no problem.

Besides the hash table, also TEX main memory is impacted. Thus, if xintexpr is used for computing plots[79], this may cause a problem. In my testing and with current TL2015 memory settings, I ran into problems after doing about *ten thousand* evaluations (for example (#1+#2)*#3-#1*#3-#2*#3)) each with number having *hundreds* of digits. Typical error message can be:

```
./testaleatoires.tex:243: TeX capacity exceeded, sorry [pool size=6134970].
<argument> ...19140037877484848545931233090884903
```

There is a (partial) solution.[80]

A document can possibly do tens of thousands of evaluations only if some identical formulae are being used repeatedly, with varying arguments (from previous computations possibly) or coming from data being fetched from a file. Most certainly, there will be a a few dozens formulae at most, but they will be used again and again with varying inputs.

With the \xintNewExpr command, it is possible to convert once and for all an expression containing parameters into an expandable macro with parameters. Only this initial definition of this macro actually activates the \xintexpr parser and will (very moderately) impact the hash-table: once this unique parsing is done, a macro with parameters is produced which is built-up recursively from the \xintAdd, \xintMul, etc... macros, exactly as it would be necessary to do without the facilities of the xintexpr package.

Notice that since 1.2c the \xintdeffunc construct allows an alternative to \xintNewExpr whose syntax uses arbitrary letters rather than macro parameters #1, #2, ... The declared function must still be used inside an expression, thus the \csname...\endcsname problem is not completely eliminated, but the computation it realizes will need only as many \csname's as there are arguments to the function and will create only one more to store the result of the evaluation.

10.10 The \xintNewExpr command

The command is used as:

\xintNewExpr{\myformula}[n]{⟨*stuff*⟩}, where

- ⟨*stuff*⟩ will be inserted inside \xinttheexpr . . . \relax,
- n is an integer between zero and nine, inclusive, which is the number of parameters of \myformula,
- the placeholders #1, #2, ..., #n are used inside ⟨*stuff*⟩ in their usual rôle,[81] [82]

[79] this is not very probable as so far xint does not include a mathematical library with floating point calculations, but provides only the basic operations of algebra. [80] which convinced me that I could stick with the parser implementation despite its potential impact on the hash-table and other parts of TEX's memory. [81] if \xintNewExpr is used inside a macro, the #'s must be doubled as usual. [82] the #'s will in pratice have their usual catcode, but category code other #'s are accepted too.

- the [n] is *mandatory*, even for n=0.[83]
- the macro \myformula is defined without checking if it already exists, LaTeX users might prefer to do first \newcommand*\myformula {} to get a reasonable error message in case \myformula already exists,
- the protection against active characters is done automatically (as long as the whole thing has not already been fetched as a macro argument and the catcodes correspondingly already frozen).

It will be a completely expandable macro entirely built-up using \xintAdd, \xintSub, \xintMul, \xintDiv, \xintPow, etc. . . as corresponds to the expression written with the infix operators. Macros created by \xintNewExpr can thus be nested.

```
        \xintNewFloatExpr \FA [2]{(#1+#2)^10}
        \xintNewFloatExpr \FB [2]{sqrt(#1*#2)}
    \begin{enumerate}[nosep]
        \item \FA {5}{5}
        \item \FB {30}{10}
        \item \FA {\FB {30}{10}}{\FB {40}{20}}
    \end{enumerate}
```

1. 1.000000000000000e10
2. 17.32050807568877
3. 3.891379490446502e16

The use of \xintNewExpr circumvents the impact of the \xintexpr parsers on TeX's memory: it is useful if one has a formula which has to be re-evaluated thousands of times with distinct inputs each with dozens, or hundreds of digits.

A ``formula'' created by \xintNewExpr is thus a macro whose parameters are given to a possibly very complicated combination of the various macros of xint and xintfrac. Consequently, one can not use at all any infix notation in the inputs, but only the formats which are recognized by the xintfrac macros.

This is thus quite different from a macro with parameters which one would have defined via a simple \def or \newcommand as for example:

```
    \newcommand\myformula [1]{\xinttheexpr (#1)^3\relax}
```

Such a macro \myformula, if it was used tens of thousands of times with various big inputs would end up populating large parts of TeX's memory. It would thus be better for such use cases to go for:

```
    \xintNewExpr\myformula [1]{#1^3\relax}
```

Here naturally the situation is over-simplified and it would be even simpler to go directly for the use of the macro \xintPow or \xintPower.

\xintNewExpr tries to do as many evaluations as are possible at the time the macro parameters are still parameters. Let's see a few examples. For this I will use \meaning which reveals the contents of a macro.

1. the examples use a mysterious \fixmeaning macro, which is there to get in the display \romannumeral`^^@ rather than the frankly cabalistic \romannumeral`` which made the admiration of the readers of the documentation dated 2015/10/19 (the second ` stood for an ascii code zero token as per T1 encoded newtxtt font). Thus the true meaning is ``fixed'' to display something different which is how the macro could be defined in a standard tex source file (modulo, as one can see in example, the use of characters such as : as letters in control sequence names). Prior to 1.2a, the meaning would have started with a more mundane \romannumeral-`0, but I decided at the time of releasing 1.2a to imitate the serious guys and switch for the more hacky yet \romannumeral`^^@ everywhere in the source code (not only in the macros produced by \xintNewExpr), or to be more precise for an equivalent as the caret has catcode letter in xint's source code, and I had to use another character.

83 there is some use for \xintNewExpr[0] compared to an \edef as \xintNewExpr has some built-in catcode protection.

2. the meaning reveals the use of some private macros from the xint bundle, which should not be directly used. If the things look a bit complicated, it is because they have to cater for many possibilities.

3. the point of showing the meaning is also to see what has already been evaluated in the construction of the macros.

```
\xintNewIIExpr\FA [1]{13*25*78*#1+2826*292}\fixmeaning\FA
```
```
macro:#1->\romannumeral`^^@\xintCSV::csv {\xintiiAdd {\xintiiMul {25350}{#1}}{825192}}
```
```
\xintNewIExpr\FA [2]{(3/5*9/7*13/11*#1-#2)*3^7}
\printnumber{\fixmeaning\FA}
```
```
macro:#1#2->\romannumeral`^^@\xintSPRaw::csv {\xintRound::csv {0}{\xintMul {\xintSub {\xintMu
l {351/385[0]}{#1}}{#2}}{2187/1[0]}}}
```
```
% an example with optional parameter
\xintNewIExpr\FA [3]{[24] (#1+#2)/(#1-#2)^#3}
\printnumber{\fixmeaning\FA}
```
```
macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintRound::csv {24}{\xintDiv {\xintAdd {#1}{
#2}}{\xintPow {\xintSub {#1}{#2}}{#3}}}}
```
```
\xintNewFloatExpr\FA [2]{[12] 3.1415^3*#1-#2^5}
\printnumber{\fixmeaning\FA}
```
```
macro:#1#2->\romannumeral`^^@\xintPFloat::csv {12}{\XINTinFloatSub {\XINTinFloatMul {31003533
39837500[-14]}{#1}}{\XINTinFloatPowerH {#2}{5}}}
```
```
\xintNewExpr\DET[9]{ #1*#5*#9+#2*#6*#7+#3*#4*#8-#1*#6*#8-#2*#4*#9-#3*#5*#7 }
\printnumber{\fixmeaning\DET}
```
```
macro:#1#2#3#4#5#6#7#8#9->\romannumeral`^^@\xintSPRaw::csv {\xintSub {\xintSub {\xintSub {\xi
ntAdd {\xintAdd {\xintMul {\xintMul {#1}{#5}}{#9}}{\xintMul {\xintMul {#2}{#6}}{#7}}}{\xintMul
{\xintMul {#3}{#4}}{#8}}}{\xintMul {\xintMul {#1}{#6}}{#8}}}{\xintMul {\xintMul {#2}{#4}}{#9}}
}{\xintMul {\xintMul {#3}{#5}}{#7}}}
```

Notice that since 1.2c it is perhaps more natural to do:

```
% attention that «ad» would try to use non-existent variable "ad"
\xintdeffunc det2(a, b, c, d) := a*d - b*c ;
% This is impossible because we must use single letters :
% \xintdeffunc det3(x_11, x_12, x_13, x_21, x_22, x_23, x_31, x_32, x_33) :=
% x_11 * det2 (x_22, x_23, x_32, x_33) + x_21 * det2 (x_32, x_33, x_12, x_13)
%                                       + x_31 * det2 (x_12, x_13, x_22, x_23);
\xintdeffunc det3 (a, b, c, u, v, w, x, y, z) := a*v*z + b*w*x + c*u*y - b*u*z - c*v*x - a*w*y ;
\xinttheexpr det3 (1,1,1,1,2,4,1,3,9),  det3 (1,10,100,1,100,10000,1,1000,1000000),
   90*900*990, reduce(det3 (1,1/2,1/3,1/2,1/3,1/4,1/3,1/4,1/5))\relax\newline
\xintdeffunc det3bis (a, b, c, u, v, w, x, y, z) :=
                     a*det2(v,w,y,z)-b*det2(u,w,x,z)+c*det2(u,v,x,y);
\pdfsetrandomseed 123456789 % xint.pdf should be predictable from xint.dtx !
\xinttheexpr subs(subs(subs(subs(subs(subs(subs(subs(subs(
% we use one extra pair of parentheses to hide the commas from the subs
            (a, b, c, u, v, w, x, y, z, det3    (a, b, c, u, v, w, x, y, z),
                                        det3bis (a, b, c, u, v, w, x, y, z)),
    z=\pdfuniformdeviate 1000), y=\pdfuniformdeviate 1000), x=\pdfuniformdeviate 1000),
    w=\pdfuniformdeviate 1000), v=\pdfuniformdeviate 1000), u=\pdfuniformdeviate 1000),
    c=\pdfuniformdeviate 1000), b=\pdfuniformdeviate 1000), a=\pdfuniformdeviate 1000)\relax
```

2, 80190000, 80190000, 1/2160
339, 694, 33, 664, 31, 921, 891, 763, 353, 188129929, 188129929

The last computation with its nine nested subs can be coded more economically (and efficiently), exploiting the fact that a single dummy variable can expand to a whole list:

```
\pdfsetrandomseed 123456789 % xint.pdf should be predictable from xint.dtx !
\xinttheexpr subs((L, det3(L), det3bis(L)), % parentheses used to hide the inner commas
    L=\pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000,
```

```
        \pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000,
        \pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000)\relax
```

339, 694, 33, 664, 31, 921, 891, 763, 353, 188129929, 188129929

With \xintverbosetrue we will find in the log:

```
    Function det3 for \xintexpr parser associated to \XINT_expr_userfunc_det3 w
ith meaning macro:#1,#2,#3,#4,#5,#6,#7,#8,#9,->\xintSub {\xintSub {\xintSub {\x
intAdd {\xintAdd {\xintMul {\xintMul {#1}{#5}}{#9}}{\xintMul {\xintMul {#2}{#6}
}{#7}}}{\xintMul {\xintMul {#3}{#4}}{#8}}}{\xintMul {\xintMul {#2}{#4}}{#9}}}{\
xintMul {\xintMul {#3}{#5}}{#7}}}{\xintMul {\xintMul {#1}{#6}}{#8}}
Package xintexpr Info: (on line 11)
    Function det3bis for \xintexpr parser associated to \XINT_expr_userfunc_det
3bis with meaning macro:#1,#2,#3,#4,#5,#6,#7,#8,#9,->\xintAdd {\xintSub {\xintM
ul {#1}{\xintSub {\xintMul {#5}{#9}}{\xintMul {#6}{#8}}}}{\xintMul {#2}{\xintSu
b {\xintMul {#4}{#9}}{\xintMul {#6}{#7}}}}}{\xintMul {#3}{\xintSub {\xintMul {#
4}{#8}}{\xintMul {#5}{#7}}}}
```

Lists, including Python-like selectors, are compatible with \xintNewExpr:[84]

```
\xintNewExpr\Foo[5]{\empty[#1..[#2]..#3][#4:#5]}
\begin{itemize}[nosep]
\item |\Foo{1}{3}{90}{20}{30}|->\Foo{1}{3}{90}{20}{30}
\item |\Foo{1}{3}{90}{-40}{-15}|->\Foo{1}{3}{90}{-40}{-15}
\item |\Foo{1.234}{-0.123}{-10}{3}{7}|->\Foo{1.234}{-0.123}{-10}{3}{7}
\end{itemize}
\fdef\test {\Foo {0}{10}{100}{3}{6}}\meaning\test +++
```

- \Foo{1}{3}{90}{20}{30}->61, 64, 67, 70, 73, 76, 79, 82, 85, 88
- \Foo{1}{3}{90}{-40}{-15}->1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43
- \Foo{1.234}{-0.123}{-10}{3}{7}->865[-3], 742[-3], 619[-3], 496[-3]

macro:->30, 40, 50+++

In this last example the macro \Foo will not be able to handle an empty #4 or #5: this is only possible in an expression, because the parser identifies][: or :] and handles them appropriately. During the construction of \Foo the parser will find][#4: and not][:.

> The \xintdeffunc, \xintdefiifunc, \xintdeffloatfunc declarators added to xintexpr since release 1.2c are based on the same underlying mechanism as \xintNewExpr, \xintNewIIExpr, ... The discussion that follows applies to them too.

10.10.1 Conditional operators and \NewExpr

The ? and ?? conditional operators cannot be parsed by \xintNewExpr when they contain macro parameters #1,..., #9 within their scope. However replacing them with the functions if and, respectively ifsgn, the parsing should succeed. And the created macro will *not evaluate the branches to be skipped*, thus behaving exactly like ? and ?? would have in the \xintexpr.

```
\xintNewExpr\Formula [3]{ if((#1>#2) && (#2>#3), sqrt(#1-#2)*sqrt(#2-#3),  #1^2+#3/#2) }%
\printnumber{\fixmeaning\Formula }
```

macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintiiifNotZero {\xintAND {\xintGt {#1}{#2}}{\xintGt {#2}{#3}}}{\xintMul {\XINTinFloatSqrtdigits {\xintSub {#1}{#2}}}{\XINTinFloatSqrtdigits {\xintSub {#2}{#3}}}}{\xintAdd {\xintPow {#1}{2}}{\xintDiv {#3}{#2}}}}

This formula (with its \xintiiifNotZero) will gobble the false branch without evaluating it when used with given arguments.

Remark: the meaning above reveals some of the private macros used by the package. They are not for direct use.

Another example

84 The \empty token is optional here, but it would be needed in case of \xintNewFloatExpr or \xintNewIExpr.

```
\xintNewExpr\myformula[3]{ ifsgn(#1,#2/#3,#2-#3,#2*#3) }%
\fixmeaning\myformula
```

macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintiiifSgn {#1}{\xintDiv {#2}{#3}}{\xintSub {#2}{#3}}{\xintMul {#2}{#3}}}

Again, this macro gobbles the false branches, as would have the operator ?? inside an \xintexpr-ession.

10.10.2 External macros and \NewExpr; the protect function

For macros within such a created xint-formula command, there are two cases:

- the macro does not involve the numbered parameters in its arguments: it may then be left as is, and will be evaluated once during the construction of the formula,
- it does involve at least one of the macro parameters as argument. Then:

> the whole thing (macro + argument) should be protect-ed, not in the LaTeX sense (!), but in the following way: protect(\macro {#1}).

Here is a silly example illustrating the general principle: the macros here have equivalent functional forms which are more convenient; but some of the more obscure package macros of xint dealing with integers do not have functions pre-defined to be in correspondance with them, use this mechanism could be applied to them.

```
\xintNewExpr\formulaA[2]{protect(\xintRound{#1}{#2}) - protect(\xintTrunc{#1}{#2})}%
\printnumber{\fixmeaning\formulaA}

\xintNewIIExpr\formulaB [3]{rem(#1,quo(protect(\the\numexpr #2\relax),#3))}%
\noindent\printnumber{\fixmeaning\formulaB }
```

macro:#1#2->\romannumeral`^^@\xintSPRaw::csv {\xintSub {\xintRound {#1}{#2}}{\xintTrunc {#1}{#2}}}

macro:#1#2#3->\romannumeral`^^@\xintCSV::csv {\xintiiRem {#1}{\xintiiQuo {\the \numexpr #2\relax }{#3}}}

Only macros involving the #1, #2, etc. . . should be protected in this way; the +, *, etc. . . symbols, the functions from the \xintexpr syntax, none should ever be included in a protected string.

10.10.3 Limitations of \xintNewExpr and \xintdeffunc

\xintNewExpr will pre-evaluate everything as long as it does not contain the macro parameters #1, #2, ... and the special measures to take when these are inside branches to ? and ?? (replace these operators by if and ifsgn) or as arguments to macros external to xintexpr (use protect) were discussed in subsubsection 10.10.1 and subsubsection 10.10.2.

The main remaining limitation is that expressions with dummy variables are compatible with \xintNewExpr only to the extent that the iterated-over list of values does not depend on the macro parameters #1, #2, ... For example, this works:

```
\xintNewExpr \FA [2] {reduce(add((t+#1)/(t+#2), t=0..5))}
\FA {1}{1}, \FA {1}{2}, \FA {2}{3}
```

6, 617/140, 1339/280 but the 5 can not be abstracted into a third argument #3.

There are no restriction on using macro parameters #1, #2, ... with list constructs. For example, this works:

```
\xintNewIExpr \FB [3] {[4] `+`([1/3..[#1/3]..#2]*#3)}
\begin{itemize}[nosep]
\item \FB {1}{10/3}{100} % (1/3+2/3+...+10/3)*100
\item \FB {5}{5}{20}     % (1/3+6/3+11/3)*20
\item \FB {3}{4}{1}      % (1/3+4/3+7/3+10/3)*1
```

```
\end{itemize}
```

- 1833.3333
- 120.0000
- 7.3333

Some simple expressions with add or mul can be also expressed with `+` and `*` and list operations. But there is no hope for seq, iter, etc... if the #1, #2, ... are used inside the list argument: seq(x(x+#1)(x+#2),x=1..#3) is currently not compatible with \xintNewExpr. But seq(x(x+#1)(x+#2), x=1..10) has no problem.

All the preceeding applies identically for \xintdeffunc, \xintdefiifunc, \xintdeffloatfunc which share the same routines as \xintNewExpr, \xintNewIIExpr, ..., replacing the #1, #2, ... in the discussion by the letters used as function arguments.

There is a final syntax restriction which however applies only to \xintNewExpr et. al., and not to \xintdeffunc, \xintdefiifunc, \xintdeffloatfunc : it is possible to use sub-expressions only if they use \xintexpr, those with \xinttheexpr are illegal.

```
\xintNewExpr \FC [4] {#1+\xintexpr #2*#3\relax + #4}
\printnumber{\fixmeaning\FC}
```

macro:#1#2#3#4->\romannumeral`^^@\xintSPRaw::csv {\xintAdd {\xintAdd {#1}{\xintMul {#2}{#3}}}{#4}}

works, but already

```
\xintNewExpr \FD [1] {#1+\xinttheexpr 1\relax}
```

doesn't. On the other hand

```
\xintdeffunc FD(t) := t + \xinttheexpr 1\relax ;
```

and even

```
\xintdeffunc FE(t,u) := t + \xinttheexpr u\relax ;
```

have no issue. Anyway, one should never use \xinttheexpr for sub-expressions but only \xintexpr, so this restriction on the \xintNewExpr syntax isn't really one.

10.11 \xintiexpr, \xinttheiexpr

x ★ Equivalent to doing \xintexpr round(...)\relax (more precisely, round is applied to each one of the evaluated values, if the expression was comma separated). Thus, only the *final result value* is rounded to an integer. Half integers are rounded towards $+\infty$ for positive numbers and towards $-\infty$ for negative ones.

An optional parameter d>0 within brackets, immediately after \xintiexpr is allowed: it instructs the expression to do its final rounding to the nearest value with that many digits after the decimal mark, *i.e.*, \xintiexpr [d] <expression>\relax is equivalent (in case of a single expression) to \xintexpr round(<expression>, d)\relax.

\xintiexpr [0] ... is the same as \xintiexpr ....[85]

If truncation rather than rounding is needed use (in case of a single expression, naturally) \xintexpr trunc(...)\relax for truncation to an integer or \xintexpr trunc(...,d)\relax for truncation to a decimal number with d>0 digits after the decimal mark.

Perhaps in the future some meaning will be given to using negative value for the optional parameter d.[86]

10.12 \xintiiexpr, \xinttheiiexpr

x ★ This variant does not know fractions. It deals almost only with long integers. Comma separated lists of expressions are allowed.

[85] Incidentally using round(...,0) in place of round(...) in \xintexpr would leave a trailing dot in the produced value. [86] Thanks to KT for this suggestion.

It maps / to the *rounded* quotient. The operator // is, like in \xintexpr...\relax, mapped to *truncated* division. The euclidean quotient (which for positive operands is like the truncated quotient) was, prior to release 1.1, associated to /. The function quo(a,b) can still be employed.

The \xintiiexpr-essions use the `ii' macros for addition, subtraction, multiplication, power, square, sums, products, euclidean quotient and remainder.

The round, trunc, floor, ceil functions are still available, and are about the only places where fractions can be used, but / within, if not somehow hidden will be executed as integer rounded division. To avoid this one can wrap the input in qfrac: this means however that none of the normal expression parsing will be executed on the argument.

To understand the illustrative examples, recall that round and trunc have a second (non negative) optional argument. In a normal \xintexpr-essions, round and trunc are mapped to \xintRound and \xintTrunc, in \xintiiexpr-essions, they are mapped to \xintiRound and \xintiTrunc.

```
\xinttheiiexpr 5/3, round(5/3,3), trunc(5/3,3), trunc(\xintDiv {5}{3},3),
trunc(\xintRaw {5/3},3)\relax{} are problematic, but
%
\xinttheiiexpr 5/3,  round(qfrac(5/3),3), trunc(qfrac(5/3),3), floor(qfrac(5/3)),
ceil(qfrac(5/3))\relax{} work!
```

2, 2000, 2000, 2000, 2000 are problematic, but 2, 1667, 1666, 1, 2 work!

On the other hand decimal numbers and scientific numbers can be used directly as arguments to the num, round, or any function producing an integer.

Scientific numbers will be represented with as many zeroes as necessary, thus one does not want to insert num(1e100000) for example in an \xintiiexpression !

```
\xinttheiiexpr num(13.4567e3)+num(10000123e-3)\relax % should (num truncates) compute 13456+10000
```

23456

The reduce function is not available and will raise un error. The frac function also. The sqrt function is mapped to \xintiiSqrt which gives a truncated square root. The sqrtr function is mapped to \xintiiSqrtR which gives a rounded square root.

One can use the Float macros if one is careful to use num, or round etc. . . on their output.

```
\xinttheiiexpr \xintFloatSqrt [20]{2}, \xintFloatSqrt [20]{3}\relax % no operations

\noindent The next example requires the |round|, and one could not put the |+| inside it:

\xinttheiiexpr round(\xintFloatSqrt [20]{2},19)+round(\xintFloatSqrt [20]{3},19)\relax

(the second argument of |round| and |trunc| tells how many digits from after the
decimal mark one should keep.)
```

14142135623730950488[-19], 17320508075688772935[-19]

The next example requires the round, and one could not put the + inside it:

31462643699419723423

(the second argument of round and trunc tells how many digits from after the decimal mark one should keep.)

The whole point of \xintiiexpr is to gain some speed in *integer-only* algorithms, and the above explanations related to how to nevertheless use fractions therein are a bit peripheral. We observed (2013/12/18) of the order of 30% speed gain when dealing with numbers with circa one hundred digits (1.2: this info may be obsolete).

10.13 \xintboolexpr, \xinttheboolexpr

x ★ Equivalent to doing \xintexpr ...\relax and returning 1 if the result does not vanish, and 0 is the result is zero. As \xintexpr, this can be used on comma separated lists of expressions, and will return a comma separated list of 0's and 1's.

10.14 \xintfloatexpr, \xintthefloatexpr

x ★ \xintfloatexpr...\relax is exactly like \xintexpr...\relax but with the four binary operations and the power function are mapped to \xintFloatAdd, \xintFloatSub, \xintFloatMul, \xintFloatDiv and \xintFloatPower, respectively.[87]

The target precision for the computation is from the current setting of \xintDigits. Comma separated lists of expressions are allowed.

An optional (positive) parameter within brackets is allowed: the final float will have that many digits of precision. This is provided to get rid of possibly irrelevant last digits, thus makes sense only if this parameter is less than the \xinttheDigits precision.

Since 1.2f all float operations first round their arguments; a parsed number is not rounded prior to its use as operand to such a float operation.

```
\xintDigits:=36;
  \xintthefloatexpr (1/13+1/121)*(1/179-1/173)/(1/19-1/18)\relax
  0.00564487459334466559166166079096852897
  \xintthefloatexpr\xintexpr (1/13+1/121)*(1/179-1/173)/(1/19-1/18)\relax\relax
  0.00564487459334466559166166079096852912
```

The latter is the rounding of the exact result. The former one has its last three digits wrong due to the cumulative effect of rounding errors in the intermediate computations, as compared to exact evaluations.

I recall here from subsection 2.2 that with release 1.2f the float macros for addition, subtraction, multiplication and division round their arguments first to P significant places with P the asked-for precision of the output; and similarly the power macros and the square root macro. This does not modify anything for computations with arguments having at most P significant places already.

Changed (1.2f)

10.15 Using an expression parser within another one

This was already illustrated before. In the following:

```
\xintthefloatexpr \xintexpr add(1/i, i=1234..1243)\relax ^100\relax
```

5.136088460396579e-210, the inner sum is computed exactly. Then it will be rounded to \xinttheDigits significant digits, and then its power will be evaluated as a float operation. One should avoid the "\xintthe" parsers in inner positions as this induces digit by digit parsing of the inner computation result by the outer parser. Here is the same computation done with floats all the way:

```
\xintthefloatexpr add(1/i, i=1234..1243)^100\relax
```

5.136088460396643e-210

Not surprisingly this differs from the previous one which was exact until raising to the 100th power.

The fact that the inner expression occurs inside a bigger one has nil influence on its behaviour. There is the limitation though that the outputs from \xintexpr and \xintfloatexpr can not be used directly in \xinttheiiexpr integer-only parser. But one can do:

```
\xinttheiiexpr round(\xintfloatexpr 3.14^10\relax)\relax % or trunc
```

93174

10.16 \xintifboolexpr

xnn ★ \xintifboolexpr{<expr>}{YES}{NO} does \xinttheexpr <expr>\relax and then executes the YES or the

[87] Since 1.2f the ^ handles half-integer exponents, contrarily to \xintFloatPower.

NO branch depending on whether the outcome was non-zero or zero. <expr> can involve various & and |, parentheses, all, any, xor, the bool or togl operators, but is not limited to them: the most general computation can be done, the test is on whether the outcome of the computation vanishes or not.

Will not work on an expression composed of comma separated sub-expressions.

10.17 \xintifboolfloatexpr

xnn ★ \xintifboolfloatexpr{<expr>}{YES}{NO} does \xintthefloatexpr <expr>\relax and then executes the YES or the NO branch depending on whether the outcome was non zero or zero.

10.18 \xintifbooliiexpr

xnn ★ \xintifbooliiexpr{<expr>}{YES}{NO} does \xinttheiiexpr <expr>\relax and then executes the YES or the NO branch depending on whether the outcome was non zero or zero.

10.19 \xintNewFloatExpr

This is exactly like \xintNewExpr except that the created formulas are set-up to use \xintthefloatexpr. The precision used for the computation will be the one given by \xinttheDigits at the time of use of the created formulas. However, the numbers hard-wired in the original expression will have been evaluated with the then current setting for \xintDigits.

```
\xintNewFloatExpr \f [1] {sqrt(#1)}
\f {2} (with \xinttheDigits{} of precision).

{\xintDigits := 32;\f {2} (with \xinttheDigits{} of precision).}

\xintNewFloatExpr \f [1] {sqrt(#1)*sqrt(2)}
\f {2} (with \xinttheDigits {} of precision).

{\xintDigits := 32;\f {2} (?? we thought we had a higher precision. Explanation next)}

The sqrt(2) in the second formula was computed with only \xinttheDigits{} of
precision. Setting |\xinttheDigits| to a higher value at the time of definition will
confirm that the result above is from a mismatch of the precision for |sqrt(2)| at
the time of its evaluation and the precision for the new |sqrt(2)| with |#1=2| at
the time of use.

{\xintDigits := 32;\xintNewFloatExpr \f [1] {sqrt(#1)*sqrt(2)}
\f {2} (with \xinttheDigits {} of precision)}
```

1.414213562373095 (with 16 of precision).

1.4142135623730950488016887242097 (with 32 of precision).

2.000000000000000 (with 16 of precision).

1.9999999999999999309839899395125 (?? we thought we had a higher precision. Explanation next)

The sqrt(2) in the second formula was computed with only 16 of precision. Setting \xinttheDigits to a higher value at the time of definition will confirm that the result above is from a mismatch of the precision for sqrt(2) at the time of its evaluation and the precision for the new sqrt(2) with #1=2 at the time of use.

2.0000000000000000000000000000000 (with 32 of precision)

10.20 \xintNewIExpr

Like \xintNewExpr but using \xinttheiexpr.

10.21 \xintNewIIExpr

Like \xintNewExpr but using \xinttheiiexpr.

10.22 \xintNewBoolExpr

Like \xintNewExpr but using \xinttheboolexpr.

10.23 Technicalities

As already mentioned \xintNewExpr\myformula[n] does not check the prior existence of a macro \myformula. And the number of parameters n given as mandatory argument within square brackets should be (at least) equal to the number of parameters in the expression.

Obviously I should mention that \xintNewExpr itself can not be used in an expansion-only context, as it creates a macro.

The \escapechar setting may be arbitrary when using \xintexpr.

The format of the output of \xintexpr⟨*stuff*⟩\relax is a ! (with catcode 11) followed by various things:

```
\edef\f {\xintexpr 1.23^10\relax }\meaning\f
```

macro:->!\XINT_expr_usethe \XINT_protectii \XINT_expr_print \.=792594609605189126649/1[-20]

> Note that \xintexpr expands in an \edef, contrarily to \numexpr which is non-expandable, if not prefixed by \the, \number, or \romannumeral or in some other context where TeX is building a number. See subsection 6.24 for some illustration.

I decided to put all intermediate results (from each evaluation of an infix operators, or of a parenthesized subpart of the expression, or from application of the minus as prefix, or of the exclamation sign as postfix, or any encountered braced material) inside \csname...\endcsname, as this can be done expandably and encapsulates an arbitrarily long fraction in a single token (left with undefined meaning), thus providing tremendous relief to the programmer in his/her expansion control.

> As the \xintexpr computations corresponding to functions and infix or postfix operators are done inside \csname...\endcsname, the *f*-expandability could possibly be dropped and one could imagine implementing the basic operations with expandable but not *f*-expandable macros (as \xintXTrunc.) I have not investigated that possibility.

Syntax errors in the input such as using a one-argument function with two arguments will generate low-level TeX processing unrecoverable errors, with cryptic accompanying message.

Some other problems will give rise to `error messages' macros giving some indication on the location and nature of the problem. Mainly, an attempt has been made to handle gracefully missing or extraneous parentheses.

However, this mechanism is completely inoperant for parentheses involved in the syntax of the seq, add, mul, subs, rseq and rrseq functions, and missing parentheses may cause the parser to fetch tokens beyond the ending \relax necessarily ending up in cryptic low-level TeX-errors.

Note that the ,<letter>= part must be visible, it can not arise from expansion (the equal sign does not have to be an equal sign, it can be any token and will be gobbled). However for iter, iterr, rseq, rrseq, the initial values delimited by a ; are parsed in the normal way, and in particular may be braced or arise from expansion. This is useful as the ; may be hidden from \xintdeffunc as {;} for example. Again, this remark does *not* apply to the comma , which precedes the <letter>= part. The comma will be fetched by delimited macros and must be there. Nesting is handled by checking (again using suitable delimited macros) that parentheses are suitably balanced.

Note that \relax is *mandatory* (contrarily to the situation for \numexpr).

10.24 Acknowledgements (2013/05/25)

I was greatly helped in my preparatory thinking, prior to producing such an expandable parser, by the commented source of the l3fp package, specifically the l3fp-parse.dtx file (in the version of April-May 2013; I think there was in particular a text called ``roadmap'' which was helpful). Also the source of the calc package was instructive, despite the fact that here for \xintexpr the principles are necessarily different due to the aim of achieving expandability.

11 Commands of the xintbinhex package

This package was first included in the 1.08 (2013/06/07) release of xint. It provides expandable conversions of arbitrarily big integers to and from binary and hexadecimal.

The argument is first *f*-expanded. It then may start with an optional minus sign (unique, of category code other), followed with optional leading zeroes (arbitrarily many, category code other) and then ``digits'' (hexadecimal letters may be of category code letter or other, and must be uppercased). The optional (unique) minus sign (plus sign is not allowed) is kept in the output. Leading zeroes are allowed, and stripped. The hexadecimal letters on output are of category code letter, and uppercased.

11.1 \xintDecToHex

f ★ Converts from decimal to hexadecimal.

```
\xintDecToHex{27182818284590452353602874713526624977572470936999595749669676277240766303535
47594571382178525166427427466391932003}
->11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918
814C63
```

11.2 \xintDecToBin

f ★ Converts from decimal to binary.

```
\xintDecToBin{27182818284590452353602874713526624977572470936999595749669676277240766303535
47594571382178525166427427466391932003}
->1000110101001001110010111110001100110100101001001101010010111000001010001111101111101000001
0101000000101111001000101001110001111100000101100010111110001000001101100010001110001001000
1011101011101111001010110101011101100000101110110011100011010010011100101111010001101101110011
1100100011011000110000000110010100100110110101111110011011111011010110010010001100010000001
0100110001100011
```

11.3 \xintHexToDec

f ★ Converts from hexadecimal to decimal.

```
\xintHexToDec{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C603
2936BF37DAC918814C63}
->271828182845904523536028747135266249775724709369995957496696762772407663035354759457138217
852516642742746639193200 3
```

11.4 \xintBinToDec

f ★ Converts from binary to decimal.

```
\xintBinToDec{100011010100100111001011111000110011010010100100110101001011100000101000111
1011111101000001010100000001011110010001010011100011111000001011000101111100010000011011000100
0111000100100010111010111011110010101101010111011000001011101100111000110100100111001011110
1000110110111001110010001101100011000000011001010010011011010111111001101111101101011001001000
1100010000000101001100011000111}
->271828182845904523536028747135266249775724709369995957496696762772407663035354759457138217
852516642742746639193200 3
```

11.5 \xintBinToHex

f ★ Converts from binary to hexadecimal.

```
\xintBinToHex{1000110101001001110010111110001100110100101001001101010010111000001010001111
10111110100001010100000010111100100010100111000111110000010110001011111000100000110110001000
11100010010001011101011101111001010110101011101100000101110110011100011010010011100101111010
00110110111001110010001101100011000000011001010010011011010111111001101111101101011001001000
11000100000010100110001100011}
->11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918
814C63
```

11.6 \xintHexToBin

f ★ Converts from hexadecimal to binary.

```
\xintHexToBin{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C603
2936BF37DAC918814C63}
->100011010100100111001011111000110011010010100100110101001011100000101000111110111110100001
01010000001011110010001010011100011111000001011000101111100010000011011000100011100010010001
01110101110111100101011010101110110000010111011001110001101001001110010111101000110110111001
11001000110110001100000001100101001001101101011111100110111110110101100100100011000100000010
100110001100011
```

11.7 \xintCHexToBin

f ★ Also converts from hexadecimal to binary. Faster on inputs with at least one hundred hexadecimal digits.

```
\xintCHexToBin{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C60
32936BF37DAC918814C63}
->100011010100100111001011111000110011010010100100110101001011100000101000111110111110100001
01010000001011110010001010011100011111000001011000101111100010000011011000100011100010010001
01110101110111100101011010101110110000010111011001110001101001001110010111101000110110111001
11001000110110001100000001100101001001101101011111100110111110110101100100100011000100000010
100110001100011
```

12 Commands of the xintgcd package

This package was included in the original release 1.0 (2013/03/28) of the xint bundle.

Since release 1.09a the macros filter their inputs through the \xintNum macro, so one can use count registers, or fractions as long as they reduce to integers.

Since release 1.1, the two ``typeset'' macros require the explicit loading by the user of package xinttools.

12.1 \xintGCD, \xintiiGCD

Num Num f f ★ \xintGCD{N}{M} computes the greatest common divisor. It is positive, except when both N and M vanish, in which case the macro returns zero.

```
\xintGCD{10000}{1113}=1
\xintiiGCD{123456789012345}{9876543210321}=3
```

ff★ \xintiiGCD skips the \xintNum overhead.

12.2 \xintGCDof

Num f→ * f ★ \xintGCDof{{a}{b}{c}...} computes the greatest common divisor of all integers a, b, ... The list argument may be a macro, it is *f*-expanded first and must contain at least one item.

12.3 \xintLCM, \xintiiLCM

Num Num f f ★ \xintGCD{N}{M} computes the least common multiple. It is 0 if one of the two integers vanishes.

ff★ \xintiiLCM skips the \xintNum overhead.

12.4 \xintLCMof

Num f→ * f ★ \xintLCMof{{a}{b}{c}...} computes the least common multiple of all integers a, b, ... The list argument may be a macro, it is *f*-expanded first and must contain at least one item.

12.5 \xintBezout

Num Num f f ★ \xintBezout{N}{M} returns five numbers A, B, U, V, D within braces. A is the first (expanded, as usual) input number, B the second, D is the GCD, and UA - VB = D.

```
\xintAssign {{\xintBezout {10000}{1113}}}\to\X
\meaning\X:macro:->\xintBezout {10000}{1113}.
```

\xintAssign {\xintBezout {10000}{1113}}\to\A\B\U\V\D

\A:10000, \B:1113, \U:-131, \V:-1177, \D:1.

\xintAssign {\xintBezout {123456789012345}{9876543210321}}\to\A\B\U\V\D

\A:123456789012345, \B:9876543210321, \U:256654313730, \V:3208178892607, \D:3.

12.6 \xintEuclideAlgorithm

Num Num
f f ★ \xintEuclideAlgorithm{N}{M} applies the Euclide algorithm and keeps a copy of all quotients and remainders.

```
\xintAssign {{\xintEuclideAlgorithm {10000}{1113}}}\to\X
```

\meaning\X:macro:->\xintEuclideAlgorithm {10000}{1113}.

The first token is the number of steps, the second is N, the third is the GCD, the fourth is M then the first quotient and remainder, the second quotient and remainder, . . . until the final quotient and last (zero) remainder.

12.7 \xintBezoutAlgorithm

Num Num
f f ★ \xintBezoutAlgorithm{N}{M} applies the Euclide algorithm and keeps a copy of all quotients and remainders. Furthermore it computes the entries of the successive products of the 2 by 2 matrices $\begin{pmatrix} q & 1 \\ 1 & 0 \end{pmatrix}$ formed from the quotients arising in the algorithm.

```
\xintAssign {{\xintBezoutAlgorithm {10000}{1113}}}\to\X
```

\meaning\X:macro:->\xintBezoutAlgorithm {10000}{1113}.

The first token is the number of steps, the second is N, then 0, 1, the GCD, M, 1, 0, the first quotient, the first remainder, the top left entry of the first matrix, the bottom left entry, and then these four things at each step until the end.

12.8 \xintTypesetEuclideAlgorithm

Requires explicit loading by the user of package xinttools.

Num Num
f f This macro is just an example of how to organize the data returned by \xintEuclideAlgorithm. Copy the source code to a new macro and modify it to what is needed.

```
\xintTypesetEuclideAlgorithm {123456789012345}{9876543210321}
```

$$
\begin{aligned}
123456789012345 &= 12 \times 9876543210321 + 4938270488493\\
9876543210321 &= 2 \times 4938270488493 + 2233335\\
4938270488493 &= 2211164 \times 2233335 + 536553\\
2233335 &= 4 \times 536553 + 87123\\
536553 &= 6 \times 87123 + 13815\\
87123 &= 6 \times 13815 + 4233\\
13815 &= 3 \times 4233 + 1116\\
4233 &= 3 \times 1116 + 885\\
1116 &= 1 \times 885 + 231\\
885 &= 3 \times 231 + 192\\
231 &= 1 \times 192 + 39\\
192 &= 4 \times 39 + 36\\
39 &= 1 \times 36 + 3\\
36 &= 12 \times 3 + 0
\end{aligned}
$$

12.9 \xintTypesetBezoutAlgorithm

Requires explicit loading by the user of package xinttools.

Num Num
f f This macro is just an example of how to organize the data returned by \xintBezoutAlgorithm. Copy the source code to a new macro and modify it to what is needed.

```
\xintTypesetBezoutAlgorithm {10000}{1113}
```

$$
\begin{aligned}
10000 &= 8 \times 1113 + 1096\\
8 &= 8 \times 1 + 0\\
1 &= 8 \times 0 + 1
\end{aligned}
$$

$$1113 = 1 \times 1096 + 17$$
$$9 = 1 \times 8 + 1$$
$$1 = 1 \times 1 + 0$$
$$1096 = 64 \times 17 + 8$$
$$584 = 64 \times 9 + 8$$
$$65 = 64 \times 1 + 1$$
$$17 = 2 \times 8 + 1$$
$$1177 = 2 \times 584 + 9$$
$$131 = 2 \times 65 + 1$$
$$8 = 8 \times 1 + 0$$
$$10000 = 8 \times 1177 + 584$$
$$1113 = 8 \times 131 + 65$$
$$131 \times 10000 - 1177 \times 1113 = -1$$

13 Commands of the xintseries package

This package was first released with version 1.03 (2013/04/14) of the xint bundle.

The $\overset{\text{Frac}}{f}$ expansion type of various macro arguments is only a $\overset{\text{Num}}{f}$ if only xint but not xintfrac is loaded. The macro \xintiSeries is special and expects summing big integers obeying the strict format, even if xintfrac is loaded.

The arguments serving as indices are of the $\overset{\text{num}}{x}$ expansion type.

In some cases one or two of the macro arguments are only expanded at a later stage not immediately.

13.1 \xintSeries

$\overset{\text{num}}{x}\ \overset{\text{num}}{x}\ \overset{\text{Frac}}{f}$ ★ \xintSeries{A}{B}{\coeff} computes $\sum_{n=A}^{n=B}$ "coeff-n". The initial and final indices must obey the \numexpr constraint of expanding to numbers at most 2^31-1. The \coeff macro must be a one-parameter f-expandable command, taking on input an explicit number n and producing some number or fraction \coeff{n}; it is expanded at the time it is needed.[88]

```
\def\coeff #1{\xintiiMON{#1}/#1.5} % (-1)^n/(n+1/2)
\fdef\w {\xintSeries {0}{50}{\coeff}} % we want to re-use it
\fdef\z {\xintJrr {\w}[0]} % the [0] for a microsecond gain.
% \xintJrr preferred to \xintIrr: a big common factor is suspected.
% But numbers much bigger would be needed to show the greater efficiency.
\[ \sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} = \xintFrac\z \]
```

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} = \frac{173909338287370940432112792101626602278714}{110027467159390003025279917226039729050575}$$

The definition of \coeff as \xintiiMON{#1}/#1.5 is quite suboptimal. It allows #1 to be a big integer, but anyhow only small integers are accepted as initial and final indices (they are of the $\overset{\text{num}}{x}$ type). Second, when the xintfrac parser sees the #1.5 it will remove the dot hence create a denominator with one digit more. For example 1/3.5 turns internally into 10/35 whereas it would be more efficient to have 2/7. For info here is the non-reduced \w:

$$\frac{2448921273374043981855311818957882212897907644510269165039 0625}{154936248757874299375548246172975814272155426442623138427734375} 10^1$$

It would have been bigger still in releases earlier than 1.1: now, the xintfrac \xintAdd routine does not multiply blindly denominators anymore, it checks if one is a multiple of the other. However it does not practice systematic reduction to lowest terms.

A more efficient way to code \coeff is illustrated next.

```
\def\coeff #1{\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}%
% The [0] in \coeff is a tiny optimization: in its presence the \xintfracname parser
% sees something which is already in internal format.
\fdef\w {\xintSeries {0}{50}{\coeff}}
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12}=\xintFrac\w\]
```

88 \xintiiMON is like \xintMON but does not parse its argument through \xintNum, for efficiency; other macros of this type are \xintiiAdd, \xintiiMul, \xintiiSum, \xintiiPrd, \xintiiMMON, \xintiiLDg, \xintiiFDg, \xintiiOdd, ...

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} = \frac{16434432468156553870834658853603713915338473 0}{10397595646562355285888952177860754395279337 5}$$

The reduced form \z as displayed above only differs from this one by a factor of 945.

```
\def\coeffleibnitz #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}
\cnta 1
\loop  % in this loop we recompute from scratch each partial sum!
% we can afford that, as \xintSeries is fast enough.
\noindent\hbox to 2em{\hfil\texttt{\the\cnta.} }%
         \xintTrunc {12}{\xintSeries {1}{\cnta}{\coeffleibnitz}}\dots
\endgraf
\ifnum\cnta < 30 \advance\cnta 1 \repeat
```

1. 1.000000000000...
2. 0.500000000000...
3. 0.833333333333...
4. 0.583333333333...
5. 0.783333333333...
6. 0.616666666666...
7. 0.759523809523...
8. 0.634523809523...
9. 0.745634920634...
10. 0.645634920634...
11. 0.736544011544...
12. 0.653210678210...
13. 0.730133755133...
14. 0.658705183705...
15. 0.725371850371...
16. 0.662871850371...
17. 0.721695379783...
18. 0.666139824228...
19. 0.718771403175...
20. 0.668771403175...
21. 0.716390450794...
22. 0.670935905339...
23. 0.714414166209...
24. 0.672747499542...
25. 0.712747499542...
26. 0.674285961081...
27. 0.711322998118...
28. 0.675608712404...
29. 0.710091471024...
30. 0.676758137691...

13.2 \xintiSeries

num num
x x f ★ \xintiSeries{A}{B}{\coeff} computes $\sum_{n=A}^{n=B}$ "coeff–n" where \coeff{n} must f-expand to a (possibly long) integer in the strict format.

```
\def\coeff #1{\xintiTrunc {40}{\xintMON{#1}/#1.5}}%
% better:
\def\coeff #1{\xintiTrunc {40}
   {\the\numexpr 2*\xintiiMON{#1}\relax/\the\numexpr 2*#1+1\relax [0]}}%
% better still:
\def\coeff #1{\xintiTrunc {40}
 {\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}}%
% (-1)^n/(n+1/2) times 10^40, truncated to an integer.
\[ \sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} \approx
        \xintTrunc {40}{\xintiSeries {0}{50}{\coeff}[-40]}\dots\]
```

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} \approx 1.5805993064935250412367895069567264144810$$

We should have cut out at least the last two digits: truncating errors originating with the first coefficients of the sum will never go away, and each truncation introduces an uncertainty in the last digit, so as we have 40 terms, we should trash the last two digits, or at least round at 38 digits. It is interesting to compare with the computation where rounding rather than truncation is used, and with the decimal expansion of the exactly computed partial sum of the series:

```
\def\coeff #1{\xintiRound {40} % rounding at 40
  {\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}}%
% (-1)^n/(n+1/2) times 10^40, rounded to an integer.
\[ \sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12} \approx
        \xintTrunc {40}{\xintiSeries {0}{50}{\coeff}[-40]}\]
\def\exactcoeff #1%
  {\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}%
\[ \sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac12}
   = \xintTrunc {50}{\xintSeries {0}{50}{\exactcoeff}}\dots\]
```

$$\sum_{n=0}^{n=50}\frac{(-1)^n}{n+\frac12}\approx 1.5805993064935250412367895069567264144804$$

$$\sum_{n=0}^{n=50}\frac{(-1)^n}{n+\frac12}=1.58059930649352504123678950695672641448068680288367\ldots$$

This shows indeed that our sum of truncated terms estimated wrongly the 39th and 40th digits of the exact result[89] and that the sum of rounded terms fared a bit better.

13.3 \xintRationalSeries

num x Frac f Frac f ★ `\xintRationalSeries{A}{B}{f}{\ratio}` evaluates $\sum_{n=A}^{n=B} F(n)$, where F(n) is specified indirectly via the data of f=F(A) and the one-parameter macro \ratio which must be such that \macro{n} expands to F(n)/F(n-1). The name indicates that \xintRationalSeries was designed to be useful in the cases where F(n)/F(n-1) is a rational function of n but it may be anything expanding to a fraction. The macro \ratio must be an expandable-only compatible command and expand to its value after iterated full expansion of its first token. A and B are fed to a \numexpr hence may be count registers or arithmetic expressions built with such; they must obey the TeX bound. The initial term f may be a macro \f, it will be expanded to its value representing F(A).

```
\def\ratio #1{2/#1[0]}% 2/n, to compute exp(2)
\cnta 0 % previously declared count
\begin{quote}
\loop \fdef\z {\xintRationalSeries {0}{\cnta}{1}{\ratio }}%
\noindent$\sum_{n=0}^{\the\cnta} \frac{2^n}{n!}=
          \xintTrunc{12}\z\dots=
          \xintFrac\z=\xintFrac{\xintIrr\z}$\vtop to 5pt{}\par
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{quote}
```

$\sum_{n=0}^{0}\frac{2^n}{n!}=1.000000000000\cdots=1=1$

$\sum_{n=0}^{1}\frac{2^n}{n!}=3.000000000000\cdots=3=3$

$\sum_{n=0}^{2}\frac{2^n}{n!}=5.000000000000\cdots=\frac{10}{2}=5$

$\sum_{n=0}^{3}\frac{2^n}{n!}=6.333333333333\cdots=\frac{38}{6}=\frac{19}{3}$

$\sum_{n=0}^{4}\frac{2^n}{n!}=7.000000000000\cdots=\frac{168}{24}=7$

$\sum_{n=0}^{5}\frac{2^n}{n!}=7.266666666666\cdots=\frac{872}{120}=\frac{109}{15}$

$\sum_{n=0}^{6}\frac{2^n}{n!}=7.355555555555\cdots=\frac{5296}{720}=\frac{331}{45}$

$\sum_{n=0}^{7}\frac{2^n}{n!}=7.380952380952\cdots=\frac{37200}{5040}=\frac{155}{21}$

$\sum_{n=0}^{8}\frac{2^n}{n!}=7.387301587301\cdots=\frac{297856}{40320}=\frac{2327}{315}$

$\sum_{n=0}^{9}\frac{2^n}{n!}=7.388712522045\cdots=\frac{2681216}{362880}=\frac{20947}{2835}$

$\sum_{n=0}^{10}\frac{2^n}{n!}=7.388994708994\cdots=\frac{26813184}{3628800}=\frac{34913}{4725}$

$\sum_{n=0}^{11}\frac{2^n}{n!}=7.389046015712\cdots=\frac{294947072}{39916800}=\frac{164591}{22275}$

$\sum_{n=0}^{12}\frac{2^n}{n!}=7.389054566832\cdots=\frac{3539368960}{479001600}=\frac{691283}{93555}$

$\sum_{n=0}^{13}\frac{2^n}{n!}=7.389055882389\cdots=\frac{46011804672}{6227020800}=\frac{14977801}{2027025}$

[89] as the series is alternating, we can roughly expect an error of $\sqrt{40}$ and the last two digits are off by 4 units, which is not contradictory to our expectations.

$\sum_{n=0}^{14} \frac{2^n}{n!} = 7.389056070325\cdots = \frac{644165281792}{87178291200} = \frac{314533829}{42567525}$

$\sum_{n=0}^{15} \frac{2^n}{n!} = 7.389056095384\cdots = \frac{9662479259648}{1307674368000} = \frac{4718007451}{638512875}$

$\sum_{n=0}^{16} \frac{2^n}{n!} = 7.389056098516\cdots = \frac{154599668219904}{20922789888000} = \frac{1572669151}{212837625}$

$\sum_{n=0}^{17} \frac{2^n}{n!} = 7.389056098884\cdots = \frac{2628194359869440}{355687428096000} = \frac{16041225341}{2170943775}$

$\sum_{n=0}^{18} \frac{2^n}{n!} = 7.389056098925\cdots = \frac{47307498477912064}{6402373705728000} = \frac{103122162907}{13956067125}$

$\sum_{n=0}^{19} \frac{2^n}{n!} = 7.389056098930\cdots = \frac{898842471080853504}{121645100408832000} = \frac{4571749222213}{618718975875}$

$\sum_{n=0}^{20} \frac{2^n}{n!} = 7.389056098930\cdots = \frac{17976849421618118656}{2432902008176640000} = \frac{68576238333199}{9280784638125}$

```
\def\ratio #1{-1/#1[0]}% -1/n, comes from the series of exp(-1)
\cnta 0 % previously declared count
\begin{quote}
\loop
\fdef\z {\xintRationalSeries {0}{\cnta}{1}{\ratio }}%
\noindent$\sum_{n=0}^{\the\cnta} \frac{(-1)^n}{n!}=
  \xintTrunc{20}\z\dots=\xintFrac{\z}=\xintFrac{\xintIrr\z}$%
         \vtop to 5pt{}\par
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{quote}
```

$\sum_{n=0}^{0} \frac{(-1)^n}{n!} = 1.00000000000000000000\cdots = 1 = 1$

$\sum_{n=0}^{1} \frac{(-1)^n}{n!} = 0\cdots = 0 = 0$

$\sum_{n=0}^{2} \frac{(-1)^n}{n!} = 0.50000000000000000000\cdots = \frac{1}{2} = \frac{1}{2}$

$\sum_{n=0}^{3} \frac{(-1)^n}{n!} = 0.33333333333333333333\cdots = \frac{2}{6} = \frac{1}{3}$

$\sum_{n=0}^{4} \frac{(-1)^n}{n!} = 0.37500000000000000000\cdots = \frac{9}{24} = \frac{3}{8}$

$\sum_{n=0}^{5} \frac{(-1)^n}{n!} = 0.36666666666666666666\cdots = \frac{44}{120} = \frac{11}{30}$

$\sum_{n=0}^{6} \frac{(-1)^n}{n!} = 0.36805555555555555555\cdots = \frac{265}{720} = \frac{53}{144}$

$\sum_{n=0}^{7} \frac{(-1)^n}{n!} = 0.36785714285714285714\cdots = \frac{1854}{5040} = \frac{103}{280}$

$\sum_{n=0}^{8} \frac{(-1)^n}{n!} = 0.36788194444444444444\cdots = \frac{14833}{40320} = \frac{2119}{5760}$

$\sum_{n=0}^{9} \frac{(-1)^n}{n!} = 0.36787918871252204585\cdots = \frac{133496}{362880} = \frac{16687}{45360}$

$\sum_{n=0}^{10} \frac{(-1)^n}{n!} = 0.36787946428571428571\cdots = \frac{1334961}{3628800} = \frac{16481}{44800}$

$\sum_{n=0}^{11} \frac{(-1)^n}{n!} = 0.36787943923360590027\cdots = \frac{14684570}{39916800} = \frac{1468457}{3991680}$

$\sum_{n=0}^{12} \frac{(-1)^n}{n!} = 0.36787944132128159905\cdots = \frac{176214841}{479001600} = \frac{16019531}{43545600}$

$\sum_{n=0}^{13} \frac{(-1)^n}{n!} = 0.36787944116069116069\cdots = \frac{2290792932}{6227020800} = \frac{63633137}{172972800}$

$\sum_{n=0}^{14} \frac{(-1)^n}{n!} = 0.36787944117216190628\cdots = \frac{32071101049}{87178291200} = \frac{2467007773}{6706022400}$

$\sum_{n=0}^{15} \frac{(-1)^n}{n!} = 0.36787944117139718991\cdots = \frac{481066515734}{1307674368000} = \frac{34361893981}{93405312000}$

$\sum_{n=0}^{16} \frac{(-1)^n}{n!} = 0.36787944117144498468\cdots = \frac{7697064251745}{20922789888000} = \frac{15549624751}{42268262400}$

$\sum_{n=0}^{17} \frac{(-1)^n}{n!} = 0.36787944117144217323\cdots = \frac{130850092279664}{355687428096000} = \frac{8178130767479}{22230464256000}$

$\sum_{n=0}^{18} \frac{(-1)^n}{n!} = 0.36787944117144232942\cdots = \frac{2355301661033953}{6402373705728000} = \frac{138547156531409}{376610217984000}$

$$\sum_{n=0}^{19} \frac{(-1)^n}{n!} = 0.367879441171442320\cdots = \frac{44750731559645106}{121645100408832000} = \frac{92079694567171}{250298560512000}$$

$$\sum_{n=0}^{20} \frac{(-1)^n}{n!} = 0.367879441171442321\cdots = \frac{895014631192902121}{2432902008176640000} = \frac{4282366656425369}{11640679464960000}$$

We can incorporate an indeterminate if we define \ratio to be a macro with two parameters: \def\ratioexp #1#2{\xintDiv{#1}{#2}}% x/n: x=#1, n=#2. Then, if \x expands to some fraction x, the command

```
\xintRationalSeries {0}{b}{1}{\ratioexp{\x}}
```

will compute $\sum_{n=0}^{n=b} x^n/n!$:

```
\cnta 0
\def\ratioexp #1#2{\xintDiv{#1}{#2}}% #1/#2
\loop
\noindent
$\sum_{n=0}^{\the\cnta} (.57)^n/n! = \xintTrunc {50}
     {\xintRationalSeries {0}{\cnta}{1}{\ratioexp{.57}}}\dots$
     \vtop to 5pt {}\endgraf
\ifnum\cnta<50 \advance\cnta 10 \repeat
```

$\sum_{n=0}^{0} (.57)^n/n! = 1.00000000000000000000000000000000000000000000000000\ldots$

$\sum_{n=0}^{10} (.57)^n/n! = 1.76826705137947002480668058035714285714285714285714\ldots$

$\sum_{n=0}^{20} (.57)^n/n! = 1.76826705143373515162089324271187082272833005529082\ldots$

$\sum_{n=0}^{30} (.57)^n/n! = 1.76826705143373515162089339282382144915484884979430\ldots$

$\sum_{n=0}^{40} (.57)^n/n! = 1.76826705143373515162089339282382144915485219867776\ldots$

$\sum_{n=0}^{50} (.57)^n/n! = 1.76826705143373515162089339282382144915485219867776\ldots$

Observe that in this last example the x was directly inserted; if it had been a more complicated explicit fraction it would have been worthwile to use \ratioexp\x with \x defined to expand to its value. In the further situation where this fraction x is not explicit but itself defined via a complicated, and time-costly, formula, it should be noted that \xintRationalSeries will do again the evaluation of \x for each term of the partial sum. The easiest is thus when x can be defined as an \edef. If however, you are in an expandable-only context and cannot store in a macro like \x the value to be used, a variant of \xintRationalSeries is needed which will first evaluate this \x and then use this result without recomputing it. This is \xintRationalSeriesX, documented next.

Here is a slightly more complicated evaluation:

```
\cnta 1
\begin{multicols}{2}
\loop \fdef\z {\xintRationalSeries
                  {\cnta}
                  {2*\cnta-1}
                  {\xintiPow {\the\cnta}{\cnta}/\xintiiFac{\cnta}}
                  {\ratioexp{\the\cnta}}}%
\fdef\w {\xintRationalSeries {0}{2*\cnta-1}{1}{\ratioexp{\the\cnta}}}%
\noindent
$\sum_{n=\the\cnta}^{\the\numexpr 2*\cnta-1\relax} \frac{\the\cnta^n}{n!}/%
          \sum_{n=0}^{\the\numexpr 2*\cnta-1\relax} \frac{\the\cnta^n}{n!} =
          \xintTrunc{8}{\xintDiv\z\w}\dots$ \vtop to 5pt{}\endgraf
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{multicols}
```

$\sum_{n=1}^{1} \frac{1^n}{n!} / \sum_{n=0}^{1} \frac{1^n}{n!} = 0.50000000\ldots$

$\sum_{n=2}^{3} \frac{2^n}{n!} / \sum_{n=0}^{3} \frac{2^n}{n!} = 0.52631578\ldots$

$\sum_{n=3}^{5} \frac{3^n}{n!} / \sum_{n=0}^{5} \frac{3^n}{n!} = 0.53804347\ldots$

$\sum_{n=4}^{7} \frac{4^n}{n!} / \sum_{n=0}^{7} \frac{4^n}{n!} = 0.54317053\ldots$

$\sum_{n=5}^{9} \frac{5^n}{n!} / \sum_{n=0}^{9} \frac{5^n}{n!} = 0.54502576\ldots$

$\sum_{n=6}^{11} \frac{6^n}{n!} / \sum_{n=0}^{11} \frac{6^n}{n!} = 0.54518217\ldots$

$\sum_{n=7}^{13} \frac{7^n}{n!} / \sum_{n=0}^{13} \frac{7^n}{n!} = 0.54445274\ldots$

$\sum_{n=8}^{15} \frac{8^n}{n!} / \sum_{n=0}^{15} \frac{8^n}{n!} = 0.54327992\ldots$

$\sum_{n=9}^{17} \frac{9^n}{n!} / \sum_{n=0}^{17} \frac{9^n}{n!} = 0.54191055\ldots$

$\sum_{n=10}^{19} \frac{10^n}{n!} / \sum_{n=0}^{19} \frac{10^n}{n!} = 0.54048295\ldots$

$\sum_{n=11}^{21} \frac{11^n}{n!} / \sum_{n=0}^{21} \frac{11^n}{n!} = 0.53907332\ldots$
$\sum_{n=12}^{23} \frac{12^n}{n!} / \sum_{n=0}^{23} \frac{12^n}{n!} = 0.53772178\ldots$
$\sum_{n=13}^{25} \frac{13^n}{n!} / \sum_{n=0}^{25} \frac{13^n}{n!} = 0.53644744\ldots$
$\sum_{n=14}^{27} \frac{14^n}{n!} / \sum_{n=0}^{27} \frac{14^n}{n!} = 0.53525726\ldots$
$\sum_{n=15}^{29} \frac{15^n}{n!} / \sum_{n=0}^{29} \frac{15^n}{n!} = 0.53415135\ldots$
$\sum_{n=16}^{31} \frac{16^n}{n!} / \sum_{n=0}^{31} \frac{16^n}{n!} = 0.53312615\ldots$
$\sum_{n=17}^{33} \frac{17^n}{n!} / \sum_{n=0}^{33} \frac{17^n}{n!} = 0.53217628\ldots$
$\sum_{n=18}^{35} \frac{18^n}{n!} / \sum_{n=0}^{35} \frac{18^n}{n!} = 0.53129566\ldots$
$\sum_{n=19}^{37} \frac{19^n}{n!} / \sum_{n=0}^{37} \frac{19^n}{n!} = 0.53047810\ldots$
$\sum_{n=20}^{39} \frac{20^n}{n!} / \sum_{n=0}^{39} \frac{20^n}{n!} = 0.52971771\ldots$

13.4 \xintRationalSeriesX

um Frac Frac
x f f f ★

\xintRationalSeriesX{A}{B}{\first}{\ratio}{\g} is a parametrized version of \xintRationalSeries where \first is now a one-parameter macro such that \first{\g} gives the initial term and \ratio is a two-parameter macro such that \ratio{n}{\g} represents the ratio of one term to the previous one. The parameter \g is evaluated only once at the beginning of the computation, and can thus itself be the yet unevaluated result of a previous computation.

Let \ratio be such a two-parameter macro; note the subtle differences between

```
\xintRationalSeries {A}{B}{\first}{\ratio{\g}}
and \xintRationalSeriesX {A}{B}{\first}{\ratio}{\g}.
```

First the location of braces differ... then, in the former case \first is a *no-parameter* macro expanding to a fractional number, and in the latter, it is a *one-parameter* macro which will use \g. Furthermore the X variant will expand \g at the very beginning whereas the former non-X former variant will evaluate it each time it needs it (which is bad if this evaluation is time-costly, but good if \g is a big explicit fraction encapsulated in a macro).

The example will use the macro \xintPowerSeries which computes efficiently exact partial sums of power series, and is discussed in the next section.

```
\def\firstterm #1{1[0]}% first term of the exponential series
% although it is the constant 1, here it must be defined as a
% one-parameter macro. Next comes the ratio function for exp:
\def\ratioexp  #1#2{\xintDiv {#1}{#2}}% x/n
% These are the (-1)^{n-1}/n of the log(1+h) series:
\def\coefflog #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}%
% Let L(h) be the first 10 terms of the log(1+h) series and
% let E(t) be the first 10 terms of the exp(t) series.
% The following computes E(L(a/10)) for a=1,...,12.
\begin{multicols}{3}\raggedcolumns
\cnta 0
\loop
\noindent\xintTrunc {18}{%
     \xintRationalSeriesX {0}{9}{\firstterm}{\ratioexp}
         {\xintPowerSeries{1}{10}{\coefflog}{\the\cnta[-1]}}}\dots
\endgraf
\ifnum\cnta < 12 \advance \cnta 1 \repeat
\end{multicols}
```

1.000000000000000000...
1.099999999999083906...
1.199999998111624029...
1.299999835744121464...
1.399996091955359088...
1.499954310225476533...
1.599659266069210466...
1.698137473697423757...
1.791898112718884531...
1.870485649686617459...
1.907197560339468199...
1.845117565491393752...
1.593831932293536053...

These completely exact operations rapidly create numbers with many digits. Let us print in full the raw fractions created by the operation illustrated above:

E(L(1[-1]))=837588191708763593233567174687168686839043767250040308005177927752494629866184007635859046126459421356378683685505198743151968256/7614438106449646789860733747200000000000[-90] (length of numerator: 129)

E(L(12[-2]))=852817067917170404743367455694175958595958834849647782275488039683039028140146275041077524507522163939774209451426122141868179564185521094269986199061074371702384540245466764149586292947053094306437899694978421745516544/7614438106449646789860733747200000000000[-180] (length of numerator: 219)

E(L(123[-3]))=855101399347484173823315578403329166472302575087958788170338112966624632249719142331115922733076372551695958545295492787962370297931258555533483817301699338027676688217119374626466595481228434620385193134459556100336250033905803253996685925486649200488748729357193753147231854444914269381901852177272347295744/7614438106449646789860733747200000000000[-270] (length of numerator: 309)

We see that the denominators here remain the same, as our input only had various powers of ten as denominators, and xintfrac efficiently assemble (some only, as we can see) powers of ten. Notice that 1 more digit in an input denominator seems to mean 90 more in the raw output. We can check that with some other test cases:

E(L(1/7))=246928403777284535542005925363994435551841707513610888309477702059412077893312778456047080539072743831005696/216062353313908320166111873421327988666012866884203817376345172792064291516013203517727545795543040000000000[0] (length of numerator: 108; length of denominator: 108)

E(L(1/71))=31693003114420993773601541095216519886197494696368120129960391285745860214771194161763328779174351843108906845841368954422645119964995819820710127126041746139734849644353924405185605375986452422252984559104/312528225156095910823136583628983100903663468032171880551790955992081064883107035602301495479192714019555648341802720770137585408881952071159334971943720550393746040624221901227384182534325875507200000000000[0] (length of numerator: 206; length of denominator: 206)

E(L(1/712))=300356435377840602055967040841188592538909311419930838799656013626071029784174496819290884958041362038132421744055614153154268292413172870530372734533290558141538915173257569411232002636456949536653491803143905110461048752979619205820572599964165780661590492904898946463533146662233869249/299935178105220909766968489591763105361775507557039697364359215352246041089232853253973804191120212141242471588173404925471664008247098734098515193250428149424064596788874441470533147848207863549778847000617103264666638782677019019130113930837421531810478062025966102914017525760000000000[0] (length of numerator: 288; length of denominator: 288)

Thus decimal numbers such as 0.123 (equivalently 123[-3]) give less computing intensive tasks than fractions such as 1/712: in the case of decimal numbers the (raw) denominators originate in the coefficients of the series themselves, powers of ten of the input within brackets being treated separately. And even then the numerators will grow with the size of the input in a sort of linear way, the coefficient being given by the order of series: here 10 from the log and 9 from the exp, so 90. One more digit in the input means 90 more digits in the numerator of the output: obviously we can not go on composing such partial sums of series and hope that xint will joyfully do all at the speed of light!

Hence, truncating the output (or better, rounding) is the only way to go if one needs a general calculus of special functions. This is why the package xintseries provides, besides \xintSeries, \xintRationalSeries, or \xintPowerSeries which compute *exact* sums, \xintFxPtPowerSeries for fixed-point computations and a (tentative naive) \xintFloatPowerSeries.

13.5 \xintPowerSeries

num x Frac f Frac f ★

\xintPowerSeries{A}{B}{\coeff}{f} evaluates the sum $\sum_{n=A}^{n=B}$ "coeff–n" · f^n. The initial and final indices are given to a \numexpr expression. The \coeff macro (which, as argument to \xintPowerSeries is expanded only at the time \coeff{n} is needed) should be defined as a one-parameter expandable command, its input will be an explicit number.

The f can be either a fraction directly input or a macro \f expanding to such a fraction. It is actually more efficient to encapsulate an explicit fraction f in such a macro, if it has big numerators and denominators (`big' means hundreds of digits) as it will then take less space in the processing until being (repeatedly) used.

This macro computes the *exact* result (one can use it also for polynomial evaluation), using a Horner scheme which helps avoiding a denominator build-up (this problem however, even if using a naive additive approach, is much less acute since release 1.1 and its new policy regarding \xintAdd).

```
\def\geom #1{1[0]} % the geometric series
\def\f {5/17[0]}
\[ \sum_{n=0}^{n=20} \Bigl(\frac 5{17}\Bigr)^n
 =\xintFrac{\xintIrr{\xintPowerSeries {0}{20}{\geom}{\f}}}
 =\xintFrac{\xinttheexpr (17^21-5^21)/12/17^20\relax}\]
```

$$\sum_{n=0}^{n=20}\Bigl(\frac{5}{17}\Bigr)^n=\frac{5757661159377657976885341}{4064231406647572522401601}=\frac{69091933912531895722624092}{48770776879770870268819212}$$

```
\def\coefflog #1{1/#1[0]}% 1/n
\def\f {1/2[0]}%
\[ \log 2 \approx \sum_{n=1}^{20} \frac1{n\cdot 2^n}
    = \xintFrac {\xintIrr {\xintPowerSeries {1}{20}{\coefflog}{\f}}}\]
\[ \log 2 \approx \sum_{n=1}^{50} \frac1{n\cdot 2^n}
    = \xintFrac {\xintIrr {\xintPowerSeries {1}{50}{\coefflog}{\f}}}\]
```

$$\log 2\approx\sum_{n=1}^{20}\frac{1}{n\cdot 2^n}=\frac{42299423848079}{61025172848640}$$

$$\log 2\approx\sum_{n=1}^{50}\frac{1}{n\cdot 2^n}=\frac{60463469751752265663579884559739219}{87230347965792839223946208178339840}$$

```
\setlength{\columnsep}{0pt}
\begin{multicols}{3}
\cnta 1 % previously declared count
\loop   % in this loop we recompute from scratch each partial sum!
% we can afford that, as \xintPowerSeries is fast enough.
\noindent\hbox to 2em{\hfil\texttt{\the\cnta.} }%
         \xintTrunc {12}
             {\xintPowerSeries {1}{\cnta}{\coefflog}{\f}}\dots
\endgraf
\ifnum \cnta < 30 \advance\cnta 1 \repeat
\end{multicols}
```

1. 0.500000000000...
2. 0.625000000000...
3. 0.666666666666...
4. 0.682291666666...
5. 0.688541666666...
6. 0.691145833333...
7. 0.692261904761...
8. 0.692750186011...
9. 0.692967199900...
10. 0.693064856150...
11. 0.693109245355...
12. 0.693129590407...
13. 0.693138980431...
14. 0.693143340085...
15. 0.693145374590...
16. 0.693146328265...
17. 0.693146777052...
18. 0.693146988980...
19. 0.693147089367...
20. 0.693147137051...
21. 0.693147159757...
22. 0.693147170594...
23. 0.693147175777...
24. 0.693147178261...
25. 0.693147179453...
26. 0.693147180026...
27. 0.693147180302...
28. 0.693147180435...
29. 0.693147180499...
30. 0.693147180530...

```
\def\coeffarctg  #1{1/\the\numexpr\ifodd #1 -2*#1-1\else2*#1+1\fi\relax }%
% the above gives (-1)^n/(2n+1). The sign being in the denominator,
%             **** no [0] should be added ****,
% else nothing is guaranteed to work (even if it could by sheer luck)
% Notice in passing this aspect of \numexpr:
```

```
%         ****  \numexpr -(1)\relax is ilegal !!! ****
\def\f {1/25[0]}% 1/5^2
\[\mathrm{Arctg}(\frac15)\approx \frac15\sum_{n=0}^{15} \frac{(-1)^n}{(2n+1)25^n}
= \xintFrac{\xintIrr {\xintDiv {\xintPowerSeries {0}{15}{\coeffarctg}{\f}}{5}}}\]
```

$$\mathrm{Arctg}(\frac{1}{5}) \approx \frac{1}{5}\sum_{n=0}^{15} \frac{(-1)^n}{(2n+1)25^n} = \frac{165918726519122955895391793269168}{840539304153062403202056884765625}$$

13.6 \xintPowerSeriesX

num num Frac Frac
x x f f

This is the same as \xintPowerSeries apart from the fact that the last parameter f is expanded once and for all before being then used repeatedly. If the f parameter is to be an explicit big fraction with many (dozens) digits, rather than using it directly it is slightly better to have some macro \g defined to expand to the explicit fraction and then use \xintPowerSeries with \g; but if f has not yet been evaluated and will be the output of a complicated expansion of some \f, and if, due to an expanding only context, doing \edef\g{\f} is no option, then \xintPowerSeriesX should be used with \f as last parameter.

```
\def\ratioexp #1#2{\xintDiv {#1}{#2}}% x/n
% These are the (-1)^{n-1}/n of the log(1+h) series:
\def\coefflog #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}%
% Let L(h) be the first 10 terms of the log(1+h) series and
% let E(t) be the first 10 terms of the exp(t) series.
% The following computes L(E(a/10)-1) for a=1,..., 12.
\begin{multicols}{3}\raggedcolumns
\cnta 1
\loop
\noindent\xintTrunc {18}{%
   \xintPowerSeriesX {1}{10}{\coefflog}
  {\xintSub
      {\xintRationalSeries {0}{9}{1[0]}{\ratioexp{\the\cnta[-1]}}}
      {1}}}\dots
\endgraf
\ifnum\cnta < 12 \advance \cnta 1 \repeat
\end{multicols}
```

0.099999999998556159...
0.199999995263443554...
0.299999338075041781...
0.399974460740121112...
0.499511320760604148...
0.593980619762352217...
0.645144282733914916...
0.398118280111436442...
-1.597091692317639401...
-12.648937932093322763...
-66.259639046914679687...
-304.768437445462801227...

13.7 \xintFxPtPowerSeries

num num
x x ★
Frac Frac num
f f x ★

\xintFxPtPowerSeries{A}{B}{\coeff}{f}{D} computes $\sum_{n=A}^{n=B}$ "coeff–n"$\cdot f^n$ with each term of the series truncated to D digits after the decimal point. As usual, A and B are completely expanded through their inclusion in a \numexpr expression. Regarding D it will be similarly be expanded each time it is used inside an \xintTrunc. The one-parameter macro \coeff is similarly expanded at the time it is used inside the computations. Idem for f. If f itself is some complicated macro it is thus better to use the variant \xintFxPtPowerSeriesX which expands it first and then uses the result of that expansion.

The current (1.04) implementation is: the first power f^A is computed exactly, then *truncated*. Then each successive power is obtained from the previous one by multiplication by the exact value of f, and truncated. And \coeff{n}·f^n is obtained from that by multiplying by \coeff{n} (untruncated) and then truncating. Finally the sum is computed exactly. Apart from that \xintFxPtPowerSeries (where FxPt means `fixed-point') is like \xintPowerSeries.

There should be a variant for things of the type $\sum c_n \frac{f^n}{n!}$ to avoid having to compute the factorial from scratch at each coefficient, the same way \xintFxPtPowerSeries does not compute f^n from scratch at each n. Perhaps in the next package release.

$e^{-\frac{1}{2}} \approx$

1.00000000000000000000	0.60653056795634920635	0.60653065971263344622
0.50000000000000000000	0.60653066483754960317	0.60653065971263342289
0.62500000000000000000	0.60653065945526069224	0.60653065971263342361
0.60416666666666666667	0.60653065972437513778	0.60653065971263342359
0.60677083333333333333	0.60653065971214266299	0.60653065971263342359
0.60651041666666666667	0.60653065971265234943	0.60653065971263342359
0.60653211805555555555	0.60653065971263274611	

```
\def\coeffexp #1{1/\xintiiFac {#1}[0]}% 1/n!
\def\f {-1/2[0]}% [0] for faster input parsing
\cnta 0 % previously declared \count register
\noindent\loop
$\xintFxPtPowerSeries {0}{\cnta}{\coeffexp}{\f}{20}$\\
\ifnum\cnta<19 \advance\cnta 1 \repeat\par
\xintFxPtPowerSeries {0}{19}{\coeffexp}{\f}{25}= 0.6065306597126334236037992
```

It is no difficulty for xintfrac to compute exactly, with the help of \xintPowerSeries, the nineteenth partial sum, and to then give (the start of) its exact decimal expansion:

$$\backslash\text{xintPowerSeries }\{0\}\{19\}\{\backslash\text{coeffexp}\}\{\backslash\text{f}\} = \frac{38682746160036397317757}{63777066403145711616000}$$

$$= 0.6065306597126334236037992152126\ldots$$

Thus, one should always estimate a priori how many ending digits are not reliable: if there are N terms and N has k digits, then digits up to but excluding the last k may usually be trusted. If we are optimistic and the series is alternating we may even replace N with $\sqrt{N}$ to get the number k of digits possibly of dubious significance.

13.8 \xintFxPtPowerSeriesX

Frac Frac num num num
f f x x x ★

\xintFxPtPowerSeriesX{A}{B}{\coeff}{\f}{D} computes, exactly as \xintFxPtPowerSeries, the sum of \coeff{n}·\f^n from n=A to n=B with each term of the series being *truncated* to D digits after the decimal point. The sole difference is that \f is first expanded and it is the result of this which is used in the computations.

Let us illustrate this on the numerical exploration of the identity

```
log(1+x) = -log(1/(1+x))
```

Let L(h)=log(1+h), and D(h)=L(h)+L(-h/(1+h)). Theoretically thus, D(h)=0 but we shall evaluate L(h) and -h/(1+h) keeping only 10 terms of their respective series. We will assume h < 0.5. With only ten terms kept in the power series we do not have quite 3 digits precision as $2^{10} = 1024$. So it wouldn't make sense to evaluate things more precisely than, say circa 5 digits after the decimal points.

```
\cnta 0
\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}% (-1)^{n-1}/n
\def\coeffalt #1{\the\numexpr\ifodd#1 -1\else1\fi\relax [0]}%   (-1)^n
\begin{multicols}2
\loop
\noindent \hbox to 2.5cm {\hss\texttt{D(\the\cnta/100): }}%
\xintAdd {\xintFxPtPowerSeriesX {1}{10}{\coefflog}{\the\cnta [-2]}{5}}
         {\xintFxPtPowerSeriesX {1}{10}{\coefflog}
             {\xintFxPtPowerSeriesX {1}{10}{\coeffalt}{\the\cnta [-2]}{5}}
          {5}}\endgraf
\ifnum\cnta < 49 \advance\cnta 7 \repeat
\end{multicols}
```

D(0/100): 0/1[0]
D(7/100): 2/1[-5]
D(14/100): 2/1[-5]
D(21/100): 3/1[-5]
D(28/100): 4/1[-5]
D(35/100): 4/1[-5]
D(42/100): 9/1[-5]
D(49/100): 42/1[-5]

Let's say we evaluate functions on [-1/2,+1/2] with values more or less also in [-1/2,+1/2] and we want to keep 4 digits of precision. So, roughly we need at least 14 terms in series like the geometric or log series. Let's make this 15. Then it doesn't make sense to compute intermediate summands with more than 6 digits precision. So we compute with 6 digits precision but return only 4 digits (rounded) after the decimal point. This result with 4 post-decimal points precision is then used as input to the next evaluation.

```
\begin{multicols}2
\loop
\noindent \hbox to 2.5cm {\hss\texttt{D(\the\cnta/100): }}%
\dtt{\xintRound{4}
 {\xintAdd {\xintFxPtPowerSeriesX {1}{15}{\coefflog}{\the\cnta [-2]}{6}}
          {\xintFxPtPowerSeriesX {1}{15}{\coefflog}
                 {\xintRound {4}{\xintFxPtPowerSeriesX {1}{15}{\coeffalt}
                              {\the\cnta [-2]}{6}}}
           {6}}%
 }}\endgraf
\ifnum\cnta < 49 \advance\cnta 7 \repeat
\end{multicols}
```

D(0/100): 0
D(7/100): 0.0000
D(14/100): 0.0000
D(21/100): -0.0001
D(28/100): -0.0001
D(35/100): -0.0001
D(42/100): -0.0000
D(49/100): -0.0001

Not bad... I have cheated a bit: the `four-digits precise' numeric evaluations were left unrounded in the final addition. However the inner rounding to four digits worked fine and made the next step faster than it would have been with longer inputs. The morale is that one should not use the raw results of \xintFxPtPowerSeriesX with the D digits with which it was computed, as the last are to be considered garbage. Rather, one should keep from the output only some smaller number of digits. This will make further computations faster and not less precise. I guess there should be some command to do this final truncating, or better, rounding, at a given number D'<D of digits. Maybe for the next release.

13.9 \xintFloatPowerSeries

$[^{\text{num}}_{x}]$ $^{\text{num}}_{x}$ $^{\text{num}}_{x}$ Frac Frac f f ★

\xintFloatPowerSeries[P]{A}{B}{\coeff}{f} computes $\sum_{n=A}^{n=B}$ "coeff−n" · f^n with a floating point precision given by the optional parameter P or by the current setting of \xintDigits.

In the current, preliminary, version, no attempt has been made to try to guarantee to the final result the precision P. Rather, P is used for all intermediate floating point evaluations. So rounding errors will make some of the last printed digits invalid. The operations done are first the evaluation of f^A using \xintFloatPow, then each successive power is obtained from this first one by multiplication by f using \xintFloatMul, then again with \xintFloatMul this is multiplied with \coeff{n}, and the sum is done adding one term at a time with \xintFloatAdd. To sum up, this is just the naive transformation of \xintFxPtPowerSeries from fixed point to floating point.

```
\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}%
\xintFloatPowerSeries [8]{1}{30}{\coefflog}{-1/2[0]}
-6.9314718e-1
```

13.10 \xintFloatPowerSeriesX

$[^{\text{num}}_{x}]$ $^{\text{num}}_{x}$ $^{\text{num}}_{x}$ Frac Frac f f ★

\xintFloatPowerSeriesX[P]{A}{B}{\coeff}{f} is like \xintFloatPowerSeries with the difference that f is expanded once and for all at the start of the computation, thus allowing efficient chaining of such series evaluations.

```
\def\coeffexp #1{1/\xintiiFac {#1}[0]}% 1/n! (exact, not float)
\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}%
\xintFloatPowerSeriesX [8]{0}{30}{\coeffexp}
    {\xintFloatPowerSeries [8]{1}{30}{\coefflog}{-1/2[0]}}
5.0000001e-1
```

13.11 Computing $\log 2$ and π

In this final section, the use of \xintFxPtPowerSeries (and \xintPowerSeries) will be illustrated on the (expandable... why make things simple when it is so easy to make them difficult!) computations of the first digits of the decimal expansion of the familiar constants $\log 2$ and π.

Let us start with $\log 2$. We will get it from this formula (which is left as an exercise):

$\log(2)=-2\log(1-13/256)-5\log(1-1/9)$

The number of terms to be kept in the log series, for a desired precision of 10^{-D} was roughly estimated without much theoretical analysis. Computing exactly the partial sums with \xintPowerSeries and then printing the truncated values, from D=0 up to D=100 showed that it worked in terms of quality of the approximation. Because of possible strings of zeroes or nines in the exact decimal expansion (in the present case of $\log 2$, strings of zeroes around the fourtieth and the sixtieth decimals), this does not mean though that all digits printed were always exact. In the end one always end up having to compute at some higher level of desired precision to validate the earlier result.

Then we tried with \xintFxPtPowerSeries: this is worthwile only for D's at least 50, as the exact evaluations are faster (with these short-length f's) for a lower number of digits. And as expected the degradation in the quality of approximation was in this range of the order of two or three digits. This meant roughly that the 3+1=4 ending digits were wrong. Again, we ended up having to compute with five more digits and compare with the earlier value to validate it. We use truncation rather than rounding because our goal is not to obtain the correct rounded decimal expansion but the correct exact truncated one.

```
\def\coefflog #1{1/#1[0]}% 1/n
\def\xa {13/256[0]}%  we will compute log(1-13/256)
\def\xb {1/9[0]}%     we will compute log(1-1/9)
\def\LogTwo #1%
%  get log(2)=-2log(1-13/256)- 5log(1-1/9)
{% we want to use \printnumber, hence need something expanding in two steps
 % only, so we use here the \romannumeral0 method
  \romannumeral0\expandafter\LogTwoDoIt \expandafter
    % Nb Terms for 1/9:
  {\the\numexpr #1*150/143\expandafter}\expandafter
    % Nb Terms for 13/256:
  {\the\numexpr #1*100/129\expandafter}\expandafter
    % We print #1 digits, but we know the ending ones are garbage
  {\the\numexpr #1\relax}% allows #1 to be a count register
}%
\def\LogTwoDoIt #1#2#3%
%  #1=nb of terms for 1/9, #2=nb of terms for 13/256,
{% #3=nb of digits for computations, also used for printing
 \xinttrunc {#3} % lowercase form to stop the \romannumeral0 expansion!
 {\xintAdd
  {\xintMul {2}{\xintFxPtPowerSeries {1}{#2}{\coefflog}{\xa}{#3}}}
  {\xintMul {5}{\xintFxPtPowerSeries {1}{#1}{\coefflog}{\xb}{#3}}}%
 }%
}%
\noindent $\log 2 \approx \LogTwo {60}\dots$\endgraf
\noindent\phantom{$\log 2$}${}\approx{}$\printnumber{\LogTwo {65}}\dots\endgraf
```

```
\noindent\phantom{$\log 2$}${}\approx{}$\printnumber{\LogTwo {70}}\dots\endgraf
```

log 2 ≈ 0.6931471805599453094172321214581765680755001343602552541206800094933723...

Here is the code doing an exact evaluation of the partial sums. We have added a +1 to the number of digits for estimating the number of terms to keep from the log series: we experimented that this gets exactly the first D digits, for all values from D=0 to D=100, except in one case (D=40) where the last digit is wrong. For values of D higher than 100 it is more efficient to use the code using \xintFxPtPowerSeries.

```
\def\LogTwo #1% get log(2)=-2log(1-13/256)- 5log(1-1/9)
{%
    \romannumeral0\expandafter\LogTwoDoIt \expandafter
    {\the\numexpr (#1+1)*150/143\expandafter}\expandafter
    {\the\numexpr (#1+1)*100/129\expandafter}\expandafter
    {\the\numexpr #1\relax}%
}%
\def\LogTwoDoIt #1#2#3%
{%  #3=nb of digits for truncating an EXACT partial sum
  \xinttrunc {#3}
    {\xintAdd
      {\xintMul {2}{\xintPowerSeries {1}{#2}{\coefflog}{\xa}}}
      {\xintMul {5}{\xintPowerSeries {1}{#1}{\coefflog}{\xb}}}%
    }%
}%
```

Let us turn now to Pi, computed with the Machin formula (but see also the approach via the Brent-Salamin algorithm with \xintfloatexpr) Again the numbers of terms to keep in the two arctg series were roughly estimated, and some experimentations showed that removing the last three digits was enough (at least for D=0-100 range). And the algorithm does print the correct digits when used with D=1000 (to be convinced of that one needs to run it for D=1000 and again, say for D=1010.) A theoretical analysis could help confirm that this algorithm always gets better than 10^{-D} precision, but again, strings of zeroes or nines encountered in the decimal expansion may falsify the ending digits, nines may be zeroes (and the last non-nine one should be increased) and zeroes may be nine (and the last non-zero one should be decreased).

```
\def\coeffarctg #1{\the\numexpr\ifodd#1 -1\else1\fi\relax/%
                                       \the\numexpr 2*#1+1\relax [0]}%
%\def\coeffarctg #1{\romannumeral0\xintmon{#1}/\the\numexpr 2*#1+1\relax }%
\def\xa {1/25[0]}%      1/5^2, the [0] for faster parsing
\def\xb {1/57121[0]}% 1/239^2, the [0] for faster parsing
\def\Machin #1{% #1 may be a count register, \Machin {\mycount} is allowed
    \romannumeral0\expandafter\MachinA \expandafter
     % number of terms for arctg(1/5):
    {\the\numexpr (#1+3)*5/7\expandafter}\expandafter
     % number of terms for arctg(1/239):
    {\the\numexpr (#1+3)*10/45\expandafter}\expandafter
     % do the computations with 3 additional digits:
    {\the\numexpr #1+3\expandafter}\expandafter
     % allow #1 to be a count register:
    {\the\numexpr #1\relax }}%
\def\MachinA #1#2#3#4%
{\xinttrunc {#4}
 {\xintSub
  {\xintMul {16/5}{\xintFxPtPowerSeries {0}{#1}{\coeffarctg}{\xa}{#3}}}
  {\xintMul{4/239}{\xintFxPtPowerSeries {0}{#2}{\coeffarctg}{\xb}{#3}}}%
 }}%
\begin{framed}
  \[ \pi = \Machin {60}\dots \]
```

```
\end{framed}
```

π = 3.141592653589793238462643383279502884197169399375105820974944...

Here is a variant \MachinBis, which evaluates the partial sums *exactly* using \xintPowerSeries, before their final truncation. No need for a ``+3'' then.

```
\def\MachinBis #1{% #1 may be a count register,
% the final result will be truncated to #1 digits post decimal point
    \romannumeral0\expandafter\MachinBisA \expandafter
     % number of terms for arctg(1/5):
    {\the\numexpr #1*5/7\expandafter}\expandafter
     % number of terms for arctg(1/239):
    {\the\numexpr #1*10/45\expandafter}\expandafter
      % allow #1 to be a count register:
    {\the\numexpr #1\relax }}%
\def\MachinBisA #1#2#3%
{\xinttrunc {#3} %
 {\xintSub
   {\xintMul {16/5}{\xintPowerSeries {0}{#1}{\coeffarctg}{\xa}}}
   {\xintMul{4/239}{\xintPowerSeries {0}{#2}{\coeffarctg}{\xb}}}%
}}%
```

Let us use this variant for a loop showing the build-up of digits:

```
\begin{multicols}{2}
  \cnta 0 % previously declared \count register
  \loop \noindent
        \centeredline{\dtt{\MachinBis{\cnta}}}%
  \ifnum\cnta < 30
  \advance\cnta 1 \repeat
\end{multicols}
```

3.
3.1
3.14
3.141
3.1415
3.14159
3.141592
3.1415926
3.14159265
3.141592653
3.1415926535
3.14159265358
3.141592653589
3.1415926535897
3.14159265358979
3.141592653589793
3.1415926535897932
3.14159265358979323
3.141592653589793238
3.1415926535897932384
3.14159265358979323846
3.141592653589793238462
3.1415926535897932384626
3.14159265358979323846264
3.141592653589793238462643
3.1415926535897932384626433
3.14159265358979323846264338
3.141592653589793238462643383
3.1415926535897932384626433832
3.14159265358979323846264338327
3.141592653589793238462643383279

You want more digits and have some time? compile this copy of the \Machin with etex (or pdftex):

```
% Compile with e-TeX extensions enabled (etex, pdftex, ...)
\input xintfrac.sty
\input xintseries.sty
% pi = 16 Arctg(1/5) - 4 Arctg(1/239) (John Machin's formula)
\def\coeffarctg #1{\the\numexpr\ifodd#1 -1\else1\fi\relax/%
                                       \the\numexpr 2*#1+1\relax [0]}%
\def\xa {1/25[0]}%
\def\xb {1/57121[0]}%
```

```
\def\Machin #1{%
    \romannumeral0\expandafter\MachinA \expandafter
    {\the\numexpr (#1+3)*5/7\expandafter}\expandafter
    {\the\numexpr (#1+3)*10/45\expandafter}\expandafter
    {\the\numexpr #1+3\expandafter}\expandafter
    {\the\numexpr #1\relax }}%
\def\MachinA #1#2#3#4%
{\xinttrunc {#4}
 {\xintSub
  {\xintMul {16/5}{\xintFxPtPowerSeries {0}{#1}{\coeffarctg}{\xa}{#3}}}
  {\xintMul {4/239}{\xintFxPtPowerSeries {0}{#2}{\coeffarctg}{\xb}{#3}}}%
}}%
\pdfresettimer
\fdef\Z {\Machin {1000}}
\odef\W {\the\pdfelapsedtime}
\message{\Z}
\message{computed in \xintRound {2}{\W/65536} seconds.}
\bye
```

This will log the first 1000 digits of π after the decimal point. On my laptop (a 2012 model) this took about 5.6 seconds last time I tried.[90] [91]

As mentioned in the introduction, the file pi.tex by D. Roegel shows that orders of magnitude faster computations are possible within TEX, but recall our constraints of complete expandability and be merciful, please.

Why truncating rather than rounding? One of our main competitors on the market of scientific computing, a canadian product (not encumbered with expandability constraints, and having barely ever heard of TEX ;-), prints numbers rounded in the last digit. Why didn't we follow suit in the macros \xintFxPtPowerSeries and \xintFxPtPowerSeriesX? To round at D digits, and excluding a rewrite or cloning of the division algorithm which anyhow would add to it some overhead in its final steps, xintfrac needs to truncate at D+1, then round. And rounding loses information! So, with more time spent, we obtain a worst result than the one truncated at D+1 (one could imagine that additions and so on, done with only D digits, cost less; true, but this is a negligeable effect per summand compared to the additional cost for this term of having been truncated at D+1 then rounded). Rounding is the way to go when setting up algorithms to evaluate functions destined to be composed one after the other: exact algebraic operations with many summands and an f variable which is a fraction are costly and create an even bigger fraction; replacing f with a reasonable rounding, and rounding the result, is necessary to allow arbitrary chaining.

But, for the computation of a single constant, we are really interested in the exact decimal expansion, so we truncate and compute more terms until the earlier result gets validated. Finally if we do want the rounding we can always do it on a value computed with D+1 truncation.

[90] With 1.09i and earlier xint, this used to be 42 seconds; starting with 1.09j, and prior to 1.2, it was 16 seconds (this was probably due to a more efficient division with denominators at most 9999). The 1.2 xintcore achieves a further gain at 5.6 seconds. [91] With \xintDigits :=1001;, the non-optimized implementation with the iter of xintexpr fame using the Brent-Salamin algorithm, took, last time I tried (1.2g), about 7.4 seconds on my laptop (the last two digits were wrong, which is ok as they serve as guard digits), and for obtaining about 500 digits, it was of 1.6s. This is not bad, taking into account that the syntax is almost free rolling speech, contrarily to the code above for the Machin formula computation; we would like to use the quadratically convergent Brent-Salamin algorithm for more digits, but with such computations with numbers of one thousand digits we are beyond the border of the reasonable range for xint. Innocent people not knowing what it means to compute with TEX, and with the extra constraint of expandability will wonder why this is at least thousands of times slower than with any other language (with a little Python program using the Decimal library, I timed the Brent-Salamin algorithm to 4.4ms for about 1000 digits and 1.14ms for 500 digits.) I will just say that for example digits are represented and manipulated via their ascii-code ! all computations must convert from ascii-code to cpu words; furthermore nothing can be stored away. And there is no memory storage with O(1) time access... if expandability is to be verified.

14 Commands of the xintcfrac package

This package was first included in release 1.04 (2013/04/25) of the xint bundle. It was kept almost unchanged until 1.09m of 2014/02/26 which brings some new macros: \xintFtoC, \xintCtoF, \xintCtoCv, dealing with sequences of braced partial quotients rather than comma separated ones, \xintFGtoC which is to produce ``guaranteed'' coefficients of some real number known approximately, and \xintGGCFrac for displaying arbitrary material as a continued fraction; also, some changes to existing macros: \xintFtoCs and \xintCntoCs insert spaces after the commas, \xintCstoF and \xintCstoCv authorize spaces in the input also before the commas.

This section contains:

1. an overview of the package functionalities,
2. a description of each one of the package macros,
3. further illustration of their use via the study of the convergents of e.

14.1 Package overview

The package computes partial quotients and convergents of a fraction, or conversely start from coefficients and obtain the corresponding fraction; three macros \xintCFrac, \xintGCFrac and \xintGGCFrac are for typesetting (the first two assume that the coefficients are numeric quantities acceptable by the xintfrac \xintFrac macro, the last one will display arbitrary material), the others can be nested (if applicable) or see their outputs further processed by other macros from the xint bundle, particularly the macros of xinttools dealing with sequences of braced items or comma separated lists.

A *simple* continued fraction has coefficients [c0,c1,...,cN] (usually called partial quotients, but I dislike this entrenched terminology), where c0 is a positive or negative integer and the others are positive integers.

Typesetting is usually done via the amsmath macro \cfrac:

```
\[ c_0 + \cfrac{1}{c_1+\cfrac1{c_2+\cfrac1{c_3+\cfrac1{\ddots}}}}\]
```

$$c_0 + \cfrac{1}{c_1+\cfrac1{c_2+\cfrac1{c_3+\cfrac1{\ddots}}}}$$

Here is a concrete example:

```
\[ \xintFrac {208341/66317}=\xintCFrac {208341/66317}\]%
```

$$\frac{208341}{66317} = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{2}}}}}$$

But it is the command \xintCFrac which did all the work of *computing* the continued fraction *and* using \cfrac from amsmath to typeset it.

A *generalized* continued fraction has the same structure but the numerators are not restricted to be 1, and numbers used in the continued fraction may be arbitrary, also fractions, irrationals, complex, indeterminates.[92] The *centered* continued fraction is an example:

```
\[ \xintFrac {915286/188421}=\xintGCFrac {5+-1/7+1/39+-1/53+-1/13}
  =\xintCFrac {915286/188421}\]
```

$$\frac{915286}{188421} = 5 - \cfrac{1}{7 + \cfrac{1}{39 - \cfrac{1}{53 - \cfrac{1}{13}}}} = 4 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{38 + \cfrac{1}{1 + \cfrac{1}{51 + \cfrac{1}{1 + \cfrac{1}{12}}}}}}}$$

The command \xintGCFrac, contrarily to \xintCFrac, does not compute anything, it just typesets starting from a generalized continued fraction in inline format, which in this example was input literally. We also used \xintCFrac for comparison of the two types of continued fractions.

To let TeX compute the centered continued fraction of f there is \xintFtoCC:

```
\[\xintFrac {915286/188421}\to\xintFtoCC {915286/188421}\]
```

$$\frac{915286}{188421} \to 5 + -1/7 + 1/39 + -1/53 + -1/13$$

The package macros are expandable and may be nested (naturally \xintCFrac and \xintGCFrac must be at the top level, as they deal with typesetting).

```
\[\xintGCFrac {\xintFtoCC{915286/188421}}\]
```

$$5 - \cfrac{1}{7 + \cfrac{1}{39 - \cfrac{1}{53 - \cfrac{1}{13}}}}$$

The `inline' format expected on input by \xintGCFrac is

$$a_0 + b_0/a_1 + b_1/a_2 + b_2/a_3 + \cdots + b_{n-2}/a_{n-1} + b_{n-1}/a_n$$

Fractions among the coefficients are allowed but they must be enclosed within braces. Signed integers may be left without braces (but the + signs are mandatory). No spaces are allowed around the plus and fraction symbols. The coefficients may themselves be macros, as long as these macros are *f*-expandable.

```
\[ \xintFrac{\xintGCtoF {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}
```

[92] xintcfrac may be used with indeterminates, for basic conversions from one inline format to another, but not for actual computations. See \xintGGCFrac.

```
= \xintGCFrac {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}\]
```

$$\frac{1907}{1902} = 1 - \cfrac{1}{57 - \cfrac{2187}{5}}$$

To compute the actual fraction one has \xintGCtoF:

```
\[\xintFrac{\xintGCtoF {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}\]
```

$$\frac{1907}{1902}$$

For non-numeric input there is \xintGGCFrac.

```
\[\xintGGCFrac {a_0+b_0/a_1+b_1/a_2+b_2/\ddots+\ddots/a_{n-1}+b_{n-1}/a_n}\]
```

$$a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{\ddots + \cfrac{\ddots}{a_{n-1} + \cfrac{b_{n-1}}{a_n}}}}}$$

For regular continued fractions, there is a simpler comma separated format:

```
\[-7,6,19,1,33\to\xintFrac{\xintCstoF{-7,6,19,1,33}}=\xintCFrac{\xintCstoF{-7,6,19,1,33}}\]
```

$$-7, 6, 19, 1, 33 \to \frac{-28077}{4108} = -7 + \cfrac{1}{6 + \cfrac{1}{19 + \cfrac{1}{1 + \cfrac{1}{33}}}}$$

The command \xintFtoCs produces from a fraction f the comma separated list of its coefficients.

```
\[\xintFrac{1084483/398959}=[\xintFtoCs{1084483/398959}]\]
```

$$\frac{1084483}{398959} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 2]$$

If one prefers other separators, one can use the two arguments macros \xintFtoCx whose first argument is the separator (which may consist of more than one token) which is to be used.

```
\[\xintFrac{2721/1001}=\xintFtoCx {+1/(}{2721/1001})\cdots)\]
```

$$\frac{2721}{1001} = 2 + 1/(1 + 1/(2 + 1/(1 + 1/(1 + 1/(4 + 1/(1 + 1/(1 + 1/(6 + 1/(2)\cdots)$$

This allows under Plain TeX with amstex to obtain the same effect as with LaTeX+\amsmath+\xintCFrac:

```
$$\xintFwOver{2721/1001}=\xintFtoCx {+\cfrac1\\ }{2721/1001}\endcfrac$$
```

As a shortcut to \xintFtoCx with separator 1+/, there is \xintFtoGC:

```
2721/1001=\xintFtoGC {2721/1001}
```

2721/1001=2+1/1+1/2+1/1+1/1+1/4+1/1+1/1+1/6+1/2 Let us compare in that case with the output of \xintFtoCC:

```
2721/1001=\xintFtoCC {2721/1001}
```

2721/1001=3+-1/4+-1/2+1/5+-1/2+1/7+-1/2 To obtain the coefficients as a sequence of braced numbers, there is \xintFtoC (this is a shortcut for \xintFtoCx {}). This list (sequence) may then be manipulated using the various macros of xinttools such as the non-expandable macro \xintAssignArray or the expandable \xintApply and \xintListWithSep.

Conversely to go from such a sequence of braced coefficients to the corresponding fraction there is \xintCtoF.

The `\printnumber' (subsection 1.3) macro which we use in this document to print long numbers can also be useful on long continued fractions.

```
\printnumber{\xintFtoCC {35037018906350720204351049/244241737886197404558180}}
```

143+1/2+1/5+-1/4+-1/4+-1/4+-1/3+1/2+1/2+1/6+-1/22+1/2+1/10+-1/5+-1/11+-1/3+1/4+-1/2+1/2+1/4+-1/2+1/23+1/3+1/8+-1/6+-1/9 If we apply \xintGCtoF to this generalized continued fraction, we discover that the original fraction was reducible:

```
\xintGCtoF {143+1/2+...+-1/9}=2897319801297630107/20197107104701740
```

When a generalized continued fraction is built with integers, and numerators are only 1's or -1's, the produced fraction is irreducible. And if we compute it again with the last sub-fraction omitted we get another irreducible fraction related to the bigger one by a Bezout identity. Doing this here we get:

```
\xintGCtoF {143+1/2+...+-1/6}=3281248877106267 29/2287346221788023
```

and indeed:

$$\begin{vmatrix} 2897319801297630107 & 328124887710626729 \\ 20197107104701740 & 2287346221788023 \end{vmatrix} = 1$$

The various fractions obtained from the truncation of a continued fraction to its initial terms are called the convergents. The commands of xintcfrac such as \xintFtoCv, \xintFtoCCv, and others which compute such convergents, return them as a list of braced items, with no separator (as does \xintFtoC for the partial quotients). Here is an example:

```
\[\xintFrac{915286/188421}\to
  \xintListWithSep{,}{\xintApply\xintFrac{\xintFtoCv{915286/188421}}}\]
```

$$\frac{915286}{188421} \to 4, 5, \frac{34}{7}, \frac{1297}{267}, \frac{1331}{274}, \frac{69178}{14241}, \frac{70509}{14515}, \frac{915286}{188421}$$

```
\[\xintFrac{915286/188421}\to
  \xintListWithSep{,}{\xintApply\xintFrac{\xintFtoCCv{915286/188421}}}\]
```

$$\frac{915286}{188421} \to 5, \frac{34}{7}, \frac{1331}{274}, \frac{70509}{14515}, \frac{915286}{188421}$$

We thus see that the `centered convergents' obtained with \xintFtoCCv are among the fuller list of convergents as returned by \xintFtoCv.

Here is a more complicated use of \xintApply and \xintListWithSep. We first define a macro which will be applied to each convergent:

```
\newcommand{\mymacro}[1]{$\xintFrac{#1}=[\xintFtoCs{#1}]$\vtop to 6pt{}}
```

Next, we use the following code:

```
$\xintFrac{49171/18089}\to{}$
\xintListWithSep {, }{\xintApply{\mymacro}{\xintFtoCv{49171/18089}}}
```

It produces:
$\frac{49171}{18089} \to 2 = [2]$, $3 = [3]$, $\frac{8}{3} = [2, 1, 2]$, $\frac{11}{4} = [2, 1, 3]$, $\frac{19}{7} = [2, 1, 2, 2]$, $\frac{87}{32} = [2, 1, 2, 1, 1, 4]$, $\frac{106}{39} = [2, 1, 2, 1, 1, 5]$, $\frac{193}{71} = [2, 1, 2, 1, 1, 4, 2]$, $\frac{1264}{465} = [2, 1, 2, 1, 1, 4, 1, 1, 6]$, $\frac{1457}{536} = [2, 1, 2, 1, 1, 4, 1, 1, 7]$, $\frac{2721}{1001} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 2]$, $\frac{23225}{8544} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8]$, $\frac{49171}{18089} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 2]$

The macro \xintCntoF allows to specify the coefficients as a function given by a one-parameter macro. The produced values do not have to be integers.

```
\def\cn #1{\xintiiPow {2}{#1}}% 2^n
  \[\xintFrac{\xintCntoF {6}{\cn}}=\xintCFrac [l]{\xintCntoF {6}{\cn}}\]
```

$$\frac{3541373}{2449193} = 1 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{8 + \cfrac{1}{16 + \cfrac{1}{32 + \cfrac{1}{64}}}}}}$$

Notice the use of the optional argument [l] to \xintCFrac. Other possibilities are [r] and (default) [c].

```
\def\cn #1{\xintPow {2}{-#1}}%
  \[\xintFrac{\xintCntoF {6}{\cn}}=\xintGCFrac [r]{\xintCntoGC {6}{\cn}}=
      [\xintFtoCs {\xintCntoF {6}{\cn}}]\]
```

$$\frac{3159019}{2465449} = 1 + \cfrac{1}{\frac{1}{2} + \cfrac{1}{\frac{1}{4} + \cfrac{1}{\frac{1}{8} + \cfrac{1}{\frac{1}{16} + \cfrac{1}{\frac{1}{32} + \cfrac{1}{\frac{1}{64}}}}}}} = [1, 3, 1, 1, 4, 14, 1, 1, 1, 1, 79, 2, 1, 1, 2]$$

We used \xintCntoGC as we wanted to display also the continued fraction and not only the fraction returned by \xintCntoF.

There are also \xintGCntoF and \xintGCntoGC which allow the same for generalized fractions. An initial portion of a generalized continued fraction for π is obtained like this

```
\def\an #1{\the\numexpr 2*#1+1\relax }%
\def\bn #1{\the\numexpr (#1+1)*(#1+1)\relax }%
\[\xintFrac{\xintDiv {4}{\xintGCntoF {5}{\an}{\bn}}} =
        \cfrac{4}{\xintGCFrac{\xintGCntoGC {5}{\an}{\bn}}} =
                \xintTrunc {10}{\xintDiv {4}{\xintGCntoF {5}{\an}{\bn}}}\dots\]
```

$$\frac{92736}{29520} = \cfrac{4}{1 + \cfrac{1}{3 + \cfrac{4}{5 + \cfrac{9}{7 + \cfrac{16}{9 + \cfrac{25}{11}}}}}} = 3.1414634146\dots$$

We see that the quality of approximation is not fantastic compared to the simple continued fraction of π with about as many terms:

```
\[\xintFrac{\xintCstoF{3,7,15,1,292,1,1}}=
            \xintGCFrac{3+1/7+1/15+1/1+1/292+1/1+1/1}=
                    \xintTrunc{10}{\xintCstoF{3,7,15,1,292,1,1}}\dots\]
```

$$\frac{208341}{66317} = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1}}}}}} = 3.1415926534\dots$$

When studying the continued fraction of some real number, there is always some doubt about how many terms are valid, when computed starting from some approximation. If $f \leqslant x \leqslant g$ and f, g both have the same first K partial quotients, then x also has the same first K quotients and convergents. The macro \xintFGtoC outputs as a sequence of braced items the common partial quotients of its two arguments. We can thus use it to produce a sure list of valid convergents of π for example, starting from some proven lower and upper bound:

```
$$\pi\to [\xintListWithSep{,}
          {\xintFGtoC {3.14159265358979323}{3.14159265358979324}}, \dots]$$
\noindent$\pi\to\xintListWithSep{,\allowbreak\;}
```

```
        {\xintApply{\xintFrac}
        {\xintCtoCv{\xintFGtoC {3.14159265358979323}{3.14159265358979324}}}}, \dots$
```

$$\pi \to [3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, \dots]$$

$\pi \to 3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}, \frac{4272943}{1360120}, \frac{5419351}{1725033}, \frac{80143857}{25510582}, \frac{165707065}{52746197}, \frac{245850922}{78256779}, \frac{411557987}{131002976}, \dots$

14.2 \xintCFrac

Frac *f* \xintCFrac{f} is a math-mode only, LaTeX with amsmath only, macro which first computes then displays with the help of \cfrac the simple continued fraction corresponding to the given fraction. It admits an optional argument which may be [l], [r] or (the default) [c] to specify the location of the one's in the numerators of the sub-fractions. Each coefficient is typeset using the \xintFrac macro from the xintfrac package. This macro is *f*-expandable in the sense that it prepares expandably the whole expression with the multiple \cfrac's, but it is not completely expandable naturally as \cfrac isn't.

14.3 \xintGCFrac

f \xintGCFrac{a+b/c+d/e+f/g+h/...+x/y} uses similarly \cfrac to prepare the typesetting with the amsmath \cfrac (LaTeX) of a generalized continued fraction given in inline format (or as macro which will *f*-expand to it). It admits the same optional argument as \xintCFrac. Plain TeX with amstex users, see \xintGCtoGCx.

```
    \[\xintGCFrac {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}}\]
```

$$1 + \cfrac{3375 \cdot 10^{-3}}{\frac{1}{7} - \cfrac{\frac{3}{5}}{720}}$$

This is mostly a typesetting macro, although it does provoke the expansion of the coefficients. See \xintGCtoF if you are impatient to see this specific fraction computed.

It admits an optional argument within square brackets which may be either [l], [c] or [r]. Default is [c] (numerators are centered).

Numerators and denominators are made arguments to the \xintFrac macro. This allows them to be themselves fractions or anything *f*-expandable giving numbers or fractions, but also means however that they can not be arbitrary material, they can not contain color changing commands for example. One of the reasons is that \xintGCFrac tries to determine the signs of the numerators and chooses accordingly to use + or -.

14.4 \xintGGCFrac

f \xintGGCFrac{a+b/c+d/e+f/g+h/...+x/y} is a clone of \xintGCFrac, hence again LaTeX specific with package amsmath. It does not assume the coefficients to be numbers as understood by xintfrac. The macro can be used for displaying arbitrary content as a continued fraction with \cfrac, using only plus signs though. Note though that it will first *f*-expand its argument, which may be thus be one of the xintcfrac macros producing a (general) continued fraction in inline format, see \xintFtoCx for an example. If this expansion is not wished, it is enough to start the argument with a space.

```
    \[\xintGGCFrac {1+q/1+q^2/1+q^3/1+q^4/1+q^5/\ddots}\]
```

$$1 + \cfrac{q}{1 + \cfrac{q^2}{1 + \cfrac{q^3}{1 + \cfrac{q^4}{1 + \cfrac{q^5}{\ddots}}}}}$$

14.5 \xintGCtoGCx

nnf ★ \xintGCtoGCx{sepa}{sepb}{a+b/c+d/e+f/...+x/y} returns the list of the coefficients of the generalized continued fraction of f, each one within a pair of braces, and separated with the help of sepa and sepb. Thus

```
\xintGCtoGCx :;{1+2/3+4/5+6/7} gives 1:2;3:4;5:6;7
```

The following can be used byt Plain TeX+amstex users to obtain an output similar as the ones produced by \xintGCFrac and \xintGGCFrac:

```
$$\xintGCtoGCx {+\cfrac}{\\}{a+b/...}\endcfrac$$
$$\xintGCtoGCx {+\cfrac\xintFwOver}{\\\xintFwOver}{a+b/...}\endcfrac$$
```

14.6 \xintFtoC

Frac *f* ★ \xintFtoC{f} computes the coefficients of the simple continued fraction of f and returns them as a list (sequence) of braced items.

```
\fdef\test{\xintFtoC{-5262046/89233}}\texttt{\meaning\test}
```

macro:->{-59}{33}{27}{100}

14.7 \xintFtoCs

Frac *f* ★ \xintFtoCs{f} returns the comma separated list of the coefficients of the simple continued fraction of f. Notice that starting with 1.09m a space follows each comma (mainly for usage in text mode, as in math mode spaces are produced in the typeset output by TeX itself).

```
\[ \xintSignedFrac{-5262046/89233} \to [\xintFtoCs{-5262046/89233}]\]
```

$$-\frac{5262046}{89233} \to [-59, 33, 27, 100]$$

14.8 \xintFtoCx

Frac *n f* ★ \xintFtoCx{sep}{f} returns the list of the coefficients of the simple continued fraction of f separated with the help of sep, which may be anything (and is kept unexpanded). For example, with Plain TeX and amstex,

```
$$\xintFtoCx {+\cfrac1\\ }{-5262046/89233}\endcfrac$$
```

will display the continued fraction using \cfrac. Each coefficient is inside a brace pair { }, allowing a macro to end the separator and fetch it as argument, for example, again with Plain TeX and amstex:

```
\def\highlight #1{\ifnum #1>200 \textcolor{red}{#1}\else #1\fi}
$$\xintFtoCx {+\cfrac1\\ \highlight}{104348/33215}\endcfrac$$
```

Due to the different and extremely cumbersome syntax of \cfrac under LaTeX it proves a bit tortuous to obtain there the same effect. Actually, it is partly for this purpose that 1.09m added \xintGGCFrac. We thus use \xintFtoCx with a suitable separator, and then the whole thing as argument to \xintGGCFrac:

```
\def\highlight #1{\ifnum #1>200 \fcolorbox{blue}{white}{\boldmath\color{red}$#1$}%
    \else #1\fi}
\[\xintGGCFrac {\xintFtoCx {+1/\highlight}{208341/66317}}\]
```

$$3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{\boxed{\mathbf{292}}+\cfrac{1}{2}}}}}$$

14.9 \xintFtoGC

Frac f ★ \xintFtoGC{f} does the same as \xintFtoCx{+1/}{f}. Its output may thus be used in the package macros expecting such an `inline format'.

```
566827/208524=\xintFtoGC {566827/208524}
```

566827/208524=2+1/1+1/2+1/1+1/1+1/4+1/1+1/1+1/6+1/1+1/1+1/8+1/1+1/1+1/11

14.10 \xintFGtoC

Frac Frac f f ★ \xintFGtoC{f}{g} computes the common initial coefficients to two given fractions f and g. Notice that any real number f<x<g or f>x>g will then necessarily share with f and g these common initial coefficients for its regular continued fraction. The coefficients are output as a sequence of braced numbers. This list can then be manipulated via macros from xinttools, or other macros of xintcfrac.

```
\fdef\test{\xintFGtoC{-5262046/89233}{-5314647/90125}}\texttt{\meaning\test}
```

macro:->{-59}{33}{27}

```
\fdef\test{\xintFGtoC{3.141592653}{3.141592654}}\texttt{\meaning\test}
```

macro:->{3}{7}{15}{1}

```
\fdef\test{\xintFGtoC{3.14159265358979323846}{3.14159265358979323847}}\meaning\test
```

macro:->{3}{7}{15}{1}{292}{1}{1}{1}{2}{1}{3}{1}{14}{2}{1}{1}{2}{2}{2}

```
\xintRound {30}{\xintCstoF{\xintListWithSep{,}{\test}}}
```

3.141592653589793238386377506390

```
\xintRound {30}{\xintCtoF{\test}}
```

3.141592653589793238386377506390

```
\fdef\test{\xintFGtoC{1.41421356237309}{1.4142135623731}}\meaning\test
```

macro:->{1}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}{2}

14.11 \xintFtoCC

Frac f ★ \xintFtoCC{f} returns the `centered' continued fraction of f, in `inline format'.

```
566827/208524=\xintFtoCC {566827/208524}
```

566827/208524=3+-1/4+-1/2+1/5+-1/2+1/7+-1/2+1/9+-1/2+1/11

```
\[\xintFrac{566827/208524} = \xintGCFrac{\xintFtoCC{566827/208524}}\]
```

$$\frac{566827}{208524} = 3 - \cfrac{1}{4 - \cfrac{1}{2 + \cfrac{1}{5 - \cfrac{1}{2 + \cfrac{1}{7 - \cfrac{1}{2 + \cfrac{1}{9 - \cfrac{1}{2 + \cfrac{1}{11}}}}}}}}}$$

14.12 \xintCstoF

f ★ \xintCstoF{a,b,c,d,...,z} computes the fraction corresponding to the coefficients, which may be fractions or even macros expanding to such fractions. The final fraction may then be highly reducible. Starting with release 1.09m spaces before commas are allowed and trimmed automatically (spaces after commas were already silently handled in earlier releases).

```
\[\xintGCFrac {-1+1/3+1/-5+1/7+1/-9+1/11+1/-13}=
  \xintSignedFrac{\xintCstoF {-1,3,-5,7,-9,11,-13}}=\xintSignedFrac{\xintGCtoF
```

```
        {-1+1/3+1/-5+1/7+1/-9+1/11+1/-13}}\]
```

$$-1+\cfrac{1}{3+\cfrac{1}{-5+\cfrac{1}{7+\cfrac{1}{-9+\cfrac{1}{11+\cfrac{1}{-13}}}}}}=-\frac{75887}{118187}=-\frac{75887}{118187}$$

```
\[\xintGCFrac{{1/2}+1/{1/3}+1/{1/4}+1/{1/5}}=\xintFrac{\xintCstoF {1/2,1/3,1/4,1/5}}\]
```

$$\frac{1}{2}+\cfrac{1}{\frac{1}{3}+\cfrac{1}{\frac{1}{4}+\cfrac{1}{\frac{1}{5}}}}=\frac{159}{66}$$

A generalized continued fraction may produce a reducible fraction (\xintCstoF tries its best not to accumulate in a silly way superfluous factors but will not do simplifications which would be obvious to a human, like simplification by 3 in the result above).

14.13 \xintCtoF

f ⋆ \xintCtoF{{a}{b}{c}...{z}} computes the fraction corresponding to the coefficients, which may be fractions or even macros.

```
\xintCtoF {\xintApply {\xintiiPow 3}{\xintSeq {1}{5}}}
```

14946960/4805083

```
\[ \xintFrac{14946960/4805083}=\xintCFrac {14946960/4805083}\]
```

$$\frac{14946960}{4805083}=3+\cfrac{1}{9+\cfrac{1}{27+\cfrac{1}{81+\cfrac{1}{243}}}}$$

In the example above the power of 3 was already pre-computed via the expansion done by \xintApply, but if we try with \xintApply { \xintiiPow 3} where the space will stop this expansion, we can check that \xintCtoF will itself provoke the needed coefficient expansion.

14.14 \xintGCtoF

f ⋆ \xintGCtoF{a+b/c+d/e+f/g+......+v/w+x/y} computes the fraction defined by the inline generalized continued fraction. Coefficients may be fractions but must then be put within braces. They can be macros. The plus signs are mandatory.

```
\[\xintGCFrac {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}} =
\xintFrac{\xintGCtoF {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}}} =
\xintFrac{\xintIrr{\xintGCtoF
                  {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}}}}\]
```

$$1+\cfrac{3375\cdot 10^{-3}}{\frac{1}{7}-\cfrac{\frac{3}{5}}{720}}=\frac{88629000}{3579000}=\frac{29543}{1193}$$

```
\[ \xintGCFrac{{1/2}+{2/3}/{4/5}+{1/2}/{1/5}+{3/2}/{5/3}} =
   \xintFrac{\xintGCtoF {{1/2}+{2/3}/{4/5}+{1/2}/{1/5}+{3/2}/{5/3}}} \]
```

$$\frac{1}{2}+\cfrac{\frac{2}{3}}{\frac{4}{5}+\cfrac{\frac{1}{2}}{\frac{1}{5}+\cfrac{\frac{3}{2}}{\frac{5}{3}}}}=\frac{4270}{4140}$$

The macro tries its best not to accumulate superfluous factor in the denominators, but doesn't reduce the fraction to irreducible form before returning it and does not do simplifications which would be obvious to a human.

14.15 \xintCstoCv

f ⋆ \xintCstoCv{a,b,c,d,...,z} returns the sequence of the corresponding convergents, each one within braces.

It is allowed to use fractions as coefficients (the computed convergents have then no reason to be the real convergents of the final fraction). When the coefficients are integers, the convergents are irreducible fractions, but otherwise it is not necessarily the case.

```
\xintListWithSep:{\xintCstoCv{1,2,3,4,5,6}}
```

1/1:3/2:10/7:43/30:225/157:1393/972

```
\xintListWithSep:{\xintCstoCv{1,1/2,1/3,1/4,1/5,1/6}}
```

1/1:3/1:9/7:45/19:225/159:1575/729

```
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintCstoCv {\xintPow
        {-.3}{-5},7.3/4.57,\xintCstoF{3/4,9,-1/3}}}}\]
```

$$\frac{-100000}{243}\to\frac{-72888949}{177390}\to\frac{-2700356878}{6567804}$$

14.16 \xintCtoCv

f ⋆ \xintCtoCv{{a}{b}{c}...{z}} returns the sequence of the corresponding convergents, each one within braces.

```
\fdef\test{\xintCtoCv {11111111111}}\texttt{\meaning\test}
```

macro:->{1/1}{2/1}{3/2}{5/3}{8/5}{13/8}{21/13}{34/21}{55/34}{89/55}{144/89}

14.17 \xintGCtoCv

f ⋆ \xintGCtoCv{a+b/c+d/e+f/g+......+v/w+x/y} returns the list of the corresponding convergents. The coefficients may be fractions, but must then be inside braces. Or they may be macros, too.

The convergents will in the general case be reducible. To put them into irreducible form, one needs one more step, for example it can be done with \xintApply\xintIrr.

```
\[\xintListWithSep{,}{\xintApply\xintFrac
                {\xintGCtoCv{3+{-2}/{7/2}+{3/4}/12+{-56}/3}}}\]
\[\xintListWithSep{,}{\xintApply\xintFrac{\xintApply\xintIrr
                {\xintGCtoCv{3+{-2}/{7/2}+{3/4}/12+{-56}/3}}}}\]
```

$$3,\frac{17}{7},\frac{834}{342},\frac{1306}{542}$$

$$3,\frac{17}{7},\frac{139}{57},\frac{653}{271}$$

14.18 \xintFtoCv

Frac *f* ⋆ \xintFtoCv{f} returns the list of the (braced) convergents of f, with no separator. To be treated with \xintAssignArray or \xintListWithSep.

```
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintFtoCv{5211/3748}}}\]
```

$$1 \to \frac{3}{2} \to \frac{4}{3} \to \frac{7}{5} \to \frac{25}{18} \to \frac{32}{23} \to \frac{57}{41} \to \frac{317}{228} \to \frac{374}{269} \to \frac{691}{497} \to \frac{5211}{3748}$$

14.19 \xintFtoCCv

Frac f ★ \xintFtoCCv{f} returns the list of the (braced) centered convergents of f, with no separator. To be treated with \xintAssignArray or \xintListWithSep.

```
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintFtoCCv{5211/3748}}}\]
```

$$1 \to \frac{4}{3} \to \frac{7}{5} \to \frac{32}{23} \to \frac{57}{41} \to \frac{374}{269} \to \frac{691}{497} \to \frac{5211}{3748}$$

14.20 \xintCntoF

num x f ★ \xintCntoF{N}{\macro} computes the fraction f having coefficients c(j)=\macro{j} for j=0,1,...,N. The N parameter is given to a \numexpr. The values of the coefficients, as returned by \macro do not have to be positive, nor integers, and it is thus not necessarily the case that the original c(j) are the true coefficients of the final f.

```
\def\macro #1{\the\numexpr 1+#1*#1\relax} \xintCntoF {5}{\macro}
```

72625/49902[0]

This example shows that the fraction is output with a trailing number in square brackets (representing a power of ten), this is for consistency with what do most macros of xintfrac, and does not have to be always this annoying [0] as the coefficients may for example be numbers in scientific notation. To avoid these trailing square brackets, for example if the coefficients are known to be integers, there is always the possibility to filter the output via \xintPRaw, or \xintIrr (the latter is overkill in the case of integer coefficients, as the fraction is guaranteed to be irreducible then).

14.21 \xintGCntoF

num x f f ★ \xintGCntoF{N}{\macroA}{\macroB} returns the fraction f corresponding to the inline generalized continued fraction a0+b0/a1+b1/a2+....+b(N-1)/aN, with a(j)=\macroA{j} and b(j)=\macroB{j}. The N parameter is given to a \numexpr.

```
\def\coeffA #1{\the\numexpr #1+4-3*((#1+2)/3)\relax }%
\def\coeffB #1{\the\numexpr \ifodd #1 -\fi 1\relax }% (-1)^n
\[\xintGCFrac{\xintGCntoGC {6}{\coeffA}{\coeffB}} =
                                  \xintFrac{\xintGCntoF {6}{\coeffA}{\coeffB}}\]
```

$$1+\cfrac{1}{2-\cfrac{1}{3+\cfrac{1}{1-\cfrac{1}{2+\cfrac{1}{3-\cfrac{1}{1}}}}}} = \frac{39}{25}$$

There is also \xintGCntoGC to get the `inline format' continued fraction.

14.22 \xintCntoCs

num x f ★ \xintCntoCs{N}{\macro} produces the comma separated list of the corresponding coefficients, from n=0 to n=N. The N is given to a \numexpr.

```
\xintCntoCs {5}{\macro}
```

1, 2, 5, 10, 17, 26

```
\[ \xintFrac{\xintCntoF{5}{\macro}}=\xintCFrac{\xintCntoF {5}{\macro}}\]
```

$$\frac{72625}{49902} = 1+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{10+\cfrac{1}{17+\cfrac{1}{26}}}}}$$

14.23 \xintCntoGC

num x f ★ \xintCntoGC{N}{\macro} evaluates the c(j)=\macro{j} from j=0 to j=N and returns a continued fraction written in inline format: {c(0)}+1/{c(1)}+1/...+1/{c(N)}. The parameter N is given to a \numexpr. The coefficients, after expansion, are, as shown, being enclosed in an added pair of braces, they may thus be fractions.

```
\def\macro #1{\the\numexpr\ifodd#1 -1-#1\else1+#1\fi\relax/\the\numexpr 1+#1*#1\relax}
\fdef\x{\xintCntoGC {5}{\macro}}\meaning\x
\[\xintGCFrac{\xintCntoGC {5}{\macro}}\]
```

macro:->{1/\the \numexpr 1+0*0\relax }+1/{-2/\the \numexpr 1+1*1\relax }+1/{3/\the \numexpr 1+2*2\relax }+1/{-4/\the \numexpr 1+3*3\relax }+1/{5/\the \numexpr 1+4*4\relax }+1/{-6/\the \numexpr 1+5*5\relax }

$$1+\cfrac{1}{\frac{-2}{2}+\cfrac{1}{\frac{3}{5}+\cfrac{1}{\frac{-4}{10}+\cfrac{1}{\frac{5}{17}+\cfrac{1}{\frac{-6}{26}}}}}}$$

14.24 \xintGCntoGC

num x f f ★ \xintGCntoGC{N}{\macroA}{\macroB} evaluates the coefficients and then returns the corresponding {a0}+{b0}/{a1}+{b1}/{a2}+...+{b(N-1)}/{aN} inline generalized fraction. N is givent to a \numexpr. The coefficients are enclosed into pairs of braces, and may thus be fractions, the fraction slash will not be confused in further processing by the continued fraction slashes.

```
\def\an #1{\the\numexpr  #1*#1*#1+1\relax}%
\def\bn #1{\the\numexpr \ifodd#1 -\fi 1*(#1+1)\relax}%
$\xintGCntoGC {5}{\an}{\bn}=\xintGCFrac {\xintGCntoGC {5}{\an}{\bn}} =
\displaystyle\xintFrac {\xintGCntoF {5}{\an}{\bn}}$\par
```

$1+1/2+-2/9+3/28+-4/65+5/126 = 1+\cfrac{1}{2-\cfrac{2}{9+\cfrac{3}{28-\cfrac{4}{65+\cfrac{5}{126}}}}} = \dfrac{5797655}{3712466}$

14.25 \xintCstoGC

f ★ \xintCstoGC{a,b,..,z} transforms a comma separated list (or something expanding to such a list) into an `inline format' continued fraction {a}+1/{b}+1/...+1/{z}. The coefficients are just copied and put within braces, without expansion. The output can then be used in \xintGCFrac for example.

```
\[\xintGCFrac {\xintCstoGC {-1,1/2,-1/3,1/4,-1/5}}=\xintSignedFrac{\xintCstoF {-1,1/2,-1/3,1/4,-1/5}}\]
```

$$-1+\cfrac{1}{\frac{1}{2}+\cfrac{1}{\frac{-1}{3}+\cfrac{1}{\frac{1}{4}+\cfrac{1}{\frac{-1}{5}}}}}=-\frac{145}{83}$$

14.26 \xintiCstoF, \xintiGCtoF, \xintiCstoCv, \xintiGCtoCv

$f\star$ Essentially the same as the corresponding macros without the `i', but for integer-only input. Infinitesimally faster, mainly for internal use by the package.

14.27 \xintGCtoGC

$f\star$ \xintGCtoGC{a+b/c+d/e+f/g+......+v/w+x/y} expands (with the usual meaning) each one of the coefficients and returns an inline continued fraction of the same type, each expanded coefficient being enclosed within braces.

```
\fdef\x {\xintGCtoGC {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/%
                        \xintiiFac {6}+\xintCstoF {2,-7,-5}/16}} \meaning\x
```

macro:->{1}+{3375/1[-3]}/{1/7}+{-3/5}/{720}+{67/36}/{16}

To be honest I have forgotten for which purpose I wrote this macro in the first place.

14.28 Euler's number e

Let us explore the convergents of Euler's number e. The volume of computation is kept minimal by the following steps:

- a comma separated list of the first 36 coefficients is produced by \xintCntoCs,
- this is then given to \xintiCstoCv which produces the list of the convergents (there is also \xintCstoCv, but our coefficients being integers we used the infinitesimally faster \xintiCstoCv),
- then the whole list was converted into a sequence of one-line paragraphs, each convergent becomes the argument to a macro printing it together with its decimal expansion with 30 digits after the decimal point.
- A count register \cnta was used to give a line count serving as a visual aid: we could also have done that in an expandable way, but well, let's relax from time to time...

```
\def\cn #1{\the\numexpr\ifcase \numexpr #1+3-3*((#1+2)/3)\relax
                          1\or1\or2*(#1/3)\fi\relax }
% produces the pattern 1,1,2,1,1,4,1,1,6,1,1,8,... which are the
% coefficients of the simple continued fraction of e-1.
\cnta 0
\def\mymacro #1{\advance\cnta by 1
                \noindent
                \hbox to 3em {\hfil\small\dtt{\the\cnta.} }%
                $\xintTrunc {30}{\xintAdd {1[0]}{#1}}\dots=
                 \xintFrac{\xintAdd {1[0]}{#1}}$}%
\xintListWithSep{\vtop to 6pt{}\vbox to 12pt{}\par}
    {\xintApply\mymacro{\xintiCstoCv{\xintCntoCs {35}{\cn}}}}
```

1. $2.00000000000000000000000000000\cdots = 2$
2. $3.00000000000000000000000000000\cdots = 3$
3. $2.66666666666666666666666666666\cdots = \frac{8}{3}$
4. $2.75000000000000000000000000000\cdots = \frac{11}{4}$
5. $2.71428571428571428571428571428 5\cdots = \frac{19}{7}$
6. $2.71875000000000000000000000000\cdots = \frac{87}{32}$
7. $2.71794871794871794871794871794 8\cdots = \frac{106}{39}$
8. $2.71830985915492957746478873239 4\cdots = \frac{193}{71}$
9. $2.71827956989247311827956989247 3\cdots = \frac{1264}{465}$
10. $2.71828358208955223880597014925 3\cdots = \frac{1457}{536}$
11. $2.71828171828171828171828171828 1\cdots = \frac{2721}{1001}$
12. $2.71828183520599250936329588014 9\cdots = \frac{23225}{8544}$
13. $2.71828182294394971189104243059 1\cdots = \frac{25946}{9545}$
14. $2.71828182873569572668472552379 8\cdots = \frac{49171}{18089}$
15. $2.71828182844540131803502507417 2\cdots = \frac{517656}{190435}$
16. $2.71828182847058372177782893096 2\cdots = \frac{566827}{208524}$
17. $2.71828182845856341127785060620 2\cdots = \frac{1084483}{398959}$
18. $2.71828182845906511407452954664 8\cdots = \frac{13580623}{4996032}$
19. $2.71828182845902801320706559102 6\cdots = \frac{14665106}{5394991}$
20. $2.71828182845904585140462108494 9\cdots = \frac{28245729}{10391023}$
21. $2.71828182845904521352198375822 1\cdots = \frac{410105312}{150869313}$
22. $2.71828182845904525462479502709 2\cdots = \frac{438351041}{161260336}$
23. $2.71828182845904523475756063147 9\cdots = \frac{848456353}{312129649}$
24. $2.71828182845904523537901337277 2\cdots = \frac{14013652689}{5155334720}$
25. $2.71828182845904523534353553278 7\cdots = \frac{14862109042}{5467464369}$
26. $2.71828182845904523536075323018 8\cdots = \frac{28875761731}{10622799089}$
27. $2.71828182845904523536027459394 1\cdots = \frac{534625820200}{196677847971}$
28. $2.71828182845904523536029912091 1\cdots = \frac{563501581931}{207300647060}$
29. $2.71828182845904523536028717990 0\cdots = \frac{1098127402131}{403978495031}$
30. $2.71828182845904523536028747861 1\cdots = \frac{22526049624551}{8286870547680}$
31. $2.71828182845904523536028746472 6\cdots = \frac{23624177026682}{8690849042711}$
32. $2.71828182845904523536028747150 3\cdots = \frac{46150226651233}{16977719590391}$
33. $2.71828182845904523536028747134 9\cdots = \frac{1038929163353808}{382200680031313}$

34. $2.718281828459045235360287471355\cdots = \frac{1085079390005041}{399178399621704}$

35. $2.718281828459045235360287471352\cdots = \frac{2124008553358849}{781379079653017}$

36. $2.718281828459045235360287471352\cdots = \frac{5206128467061 7417}{1915227631129 4112}$

One can with no problem compute much bigger convergents. Let's get the 200th convergent. It turns out to have the same first 268 digits after the decimal point as e - 1. Higher convergents get more and more digits in proportion to their index: the 500th convergent already gets 799 digits correct! To allow speedy compilation of the source of this document when the need arises, I limit here to the 200th convergent.

```
\fdef\z {\xintCntoF {199}{\cn}}%
\begingroup\parindent 0pt \leftskip 2.5cm
\indent\llap {Numerator = }\printnumber{\xintNumerator\z}\par
\indent\llap {Denominator = }\printnumber{\xintDenominator\z}\par
\indent\llap {Expansion = }\printnumber{\xintTrunc{268}\z}\dots\par\endgroup
```

Numerator = 568964038871896267597523892315807875293889017667917446057232024547192296961118
2301752438601749953108177313670124170860974963432938 2906

Denominator = 331123817669737619306256360816356753365468823729314438156205615463246659728581
8654613376920631489160195506145705925533766114264521 7223

Expansion = 1.718281828459045235360287471352662497757247093699959574966967627724076630353 5
475945713821785251664274274663919320030599218174135966290435729003342952605956
307381323286279434907632338298807531952510190115738341879307021540891499348841
675092447614606680822648001684774118...

One can also use a centered continued fraction: we get more digits but there are also more computations as the numerators may be either 1 or -1.

This documentation has been compiled without the source code, which is available in the separate file:
sourcexint.pdf,
which should be among the candidates proposed by texdoc --list xint. To produce a single file including both the user documentation and the source code, run tex xint.dtx to generate xint.tex (if not already available), then edit xint.tex to set the \NoSourceCode toggle to 0, then run thrice latex on xint.tex and finally dvipdfmx on xint.dvi. Alternatively, run pdflatex either directly on xint.dtx, or on xint.tex with \NoSourceCode set to 0.